20.4.2
The safe edges form the MST

Safe Edges form a connected graph

Lemma 20.3.

Let G be a connected graph with distinct edge costs, then the set of safe edges form a connected graph.

Proof.

1. Suppose not. Let S be a connected component in the graph induced by the safe edges.
2. Consider the edges crossing S, there must be a safe edge among them since edge costs are distinct and so we must have picked it.

□

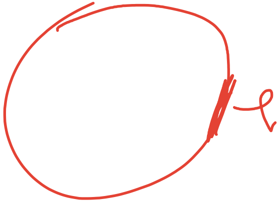
Safe Edges do not contain a cycle

Lemma 20.4.

Let G be a connected graph with distinct edge costs, then the set of safe edges does not contain a cycle.

Proof.

Proposition 20.5 : proved every edge in graph is either safe or unsafe. If $\exists$ cycle, then by definition the most expensive edge in the cycle is unsafe. Contradiction. $\square$



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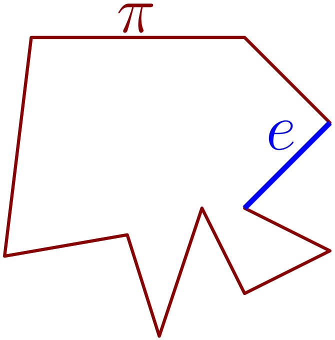
Lemma 20.4.

Let G be a connected graph with distinct edge costs, then the set of safe edges does not contain a cycle.

Proof.

Assume false, and let π a cycle made of safe edges.
e: Most expensive edge in the cycle π.
$\mathcal{C} = (S, V \setminus S)$: Cut that e is safe for.
π must have at least two edges in $\mathcal{C}$.
f: cheapest edge in $\pi \cap \mathcal{C}$.
e is not cheapest edge in $\mathcal{C}$.
A contradiction. $\square$



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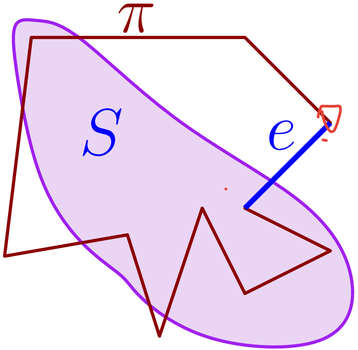

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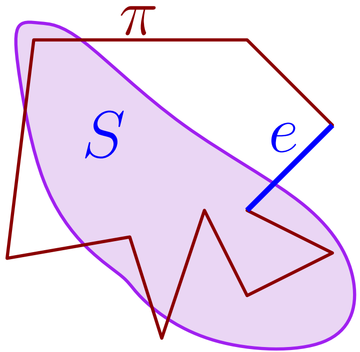

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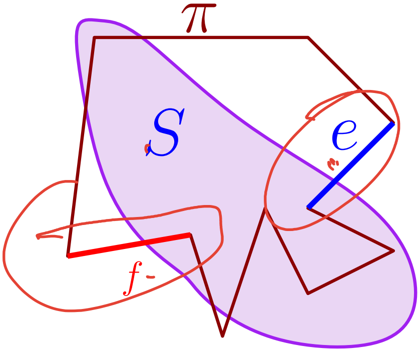

Safe Edges form an MST

Corollary 20.5.

Let G be a connected graph with distinct edge costs, then set of safe edges form the unique MST *of G.*

Consequence: Every correct MST algorithm when G has unique edge costs includes exactly the safe edges.

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THE END

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(for now)