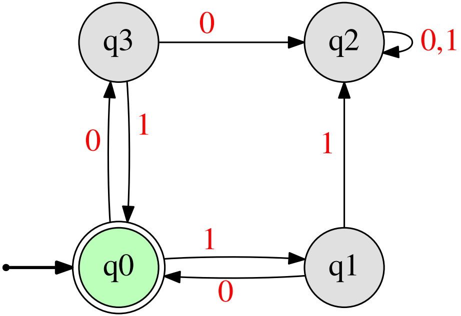
3.3
Complement language

Complement

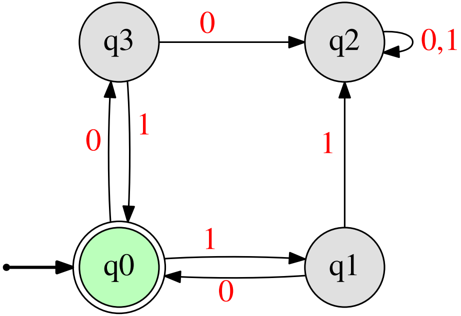
Question: If M is a DFA, is there a DFA M' such that $L(M') = \Sigma^* \setminus L(M)$? That is, are languages recognized by DFAs closed under complement?

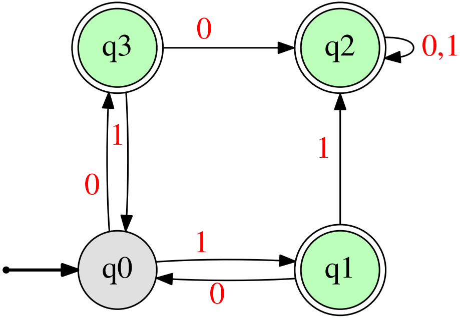


Complement

Example...

Just flip the state of the states!





Complement

Theorem

Languages accepted by DFAs are closed under complement.

Proof.

Let $M = (Q, \Sigma, \delta, s, A)$ such that $L = L(M)$.
Let $M' = (Q, \Sigma, \delta, s, Q \setminus A)$. Claim: $L(M') = \bar{L}$. Why?
$\delta^*_M = \delta^*_{M'}$. Thus, for every string w, $\delta^*_M(s, w) = \delta^*_{M'}(s, w)$.
$\delta^*_M(s, w) \in A \Rightarrow \delta^*_{M'}(s, w) \notin Q \setminus A$. $\delta^*_M(s, w) \notin A \Rightarrow \delta^*_{M'}(s, w) \in Q \setminus A$. $\square$

Complement

Theorem

Languages accepted by DFA*s are closed under complement.*

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THE END

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(for now)