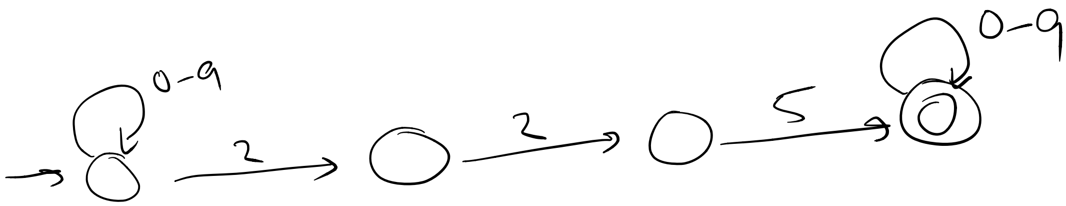
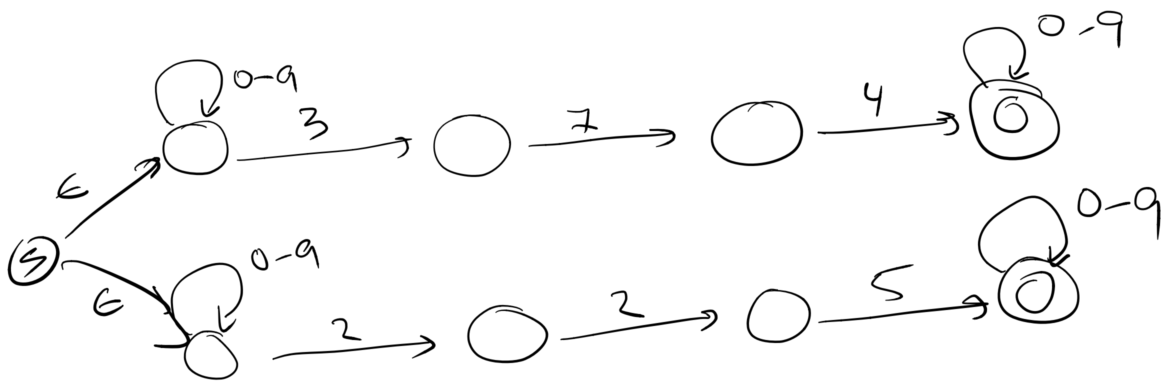
Regular Languages, DFAs and (ϵ-) NFAs

Midterm 2: 7–9 p.m. on Nov 5 (Mid 1 is Sep 30)

L_{374}

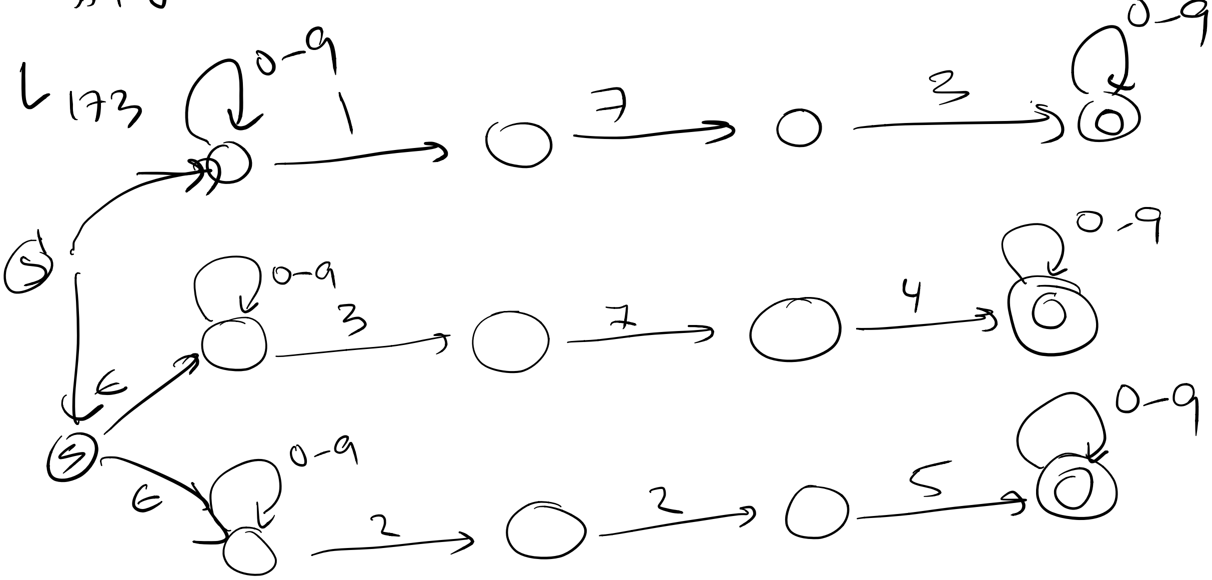
L_{225}





$L_{374} \cup L_{225}$

L_{173}



$L_{173 \cup 225 \cup 374}$

$\mathcal{L}_{NFA}$ = all languages accepted by <u>some</u> NFA

$\mathcal{L}_{NFA}$ is closed under union

For any NFAs N_1, N_2, exists NFA N_3 st $L(N_3) = L(N_1) \cup L(N_2)$

$\mathcal{L}_{DFA}$ - lang's accepted by DFAs

DFAs M_1, M_2

$M_3 =$, $L(M_3) = L(M_1) \cap L(M_2)$

$$M_3 = (Q_1 \times Q_2, \Sigma, \delta_3, (s_1, s_2), A_1 \times A_2)$$

M_4 $L(M_4) = L(M_1) \cup L(M_2)$

$$M_4 = (Q_1 \times Q_2, \Sigma, \delta_3, (s_1, s_2), A_1 \times Q_2 \cup Q_1 \times A_2)$$

$\mathcal{L}_{DFA}$ is closed under union, intersection, complement.

$\mathcal{L}_{NFA}$ is closed under union.

$$N_1 = (Q_1, \Sigma, \delta_1, s_1, A_1)$$
$$N_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$$

$$N_3 = (Q_1 \cup Q_2 \cup \{s^*\}, \Sigma, \delta_3,$$

$s^*, A_1 \cup A_2)$

$\delta_3(s^*, \epsilon) = \{s_1, s_2\}$

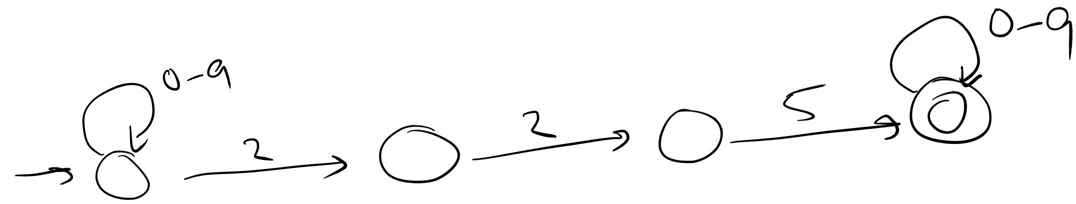
$\delta_3(s^*, c) = \phi$ for $c \in \Sigma$

$\delta_3(s, c) = \delta_1(s, c)$ for $c \in \Sigma + \{\epsilon\}$, $s \in Q_1$

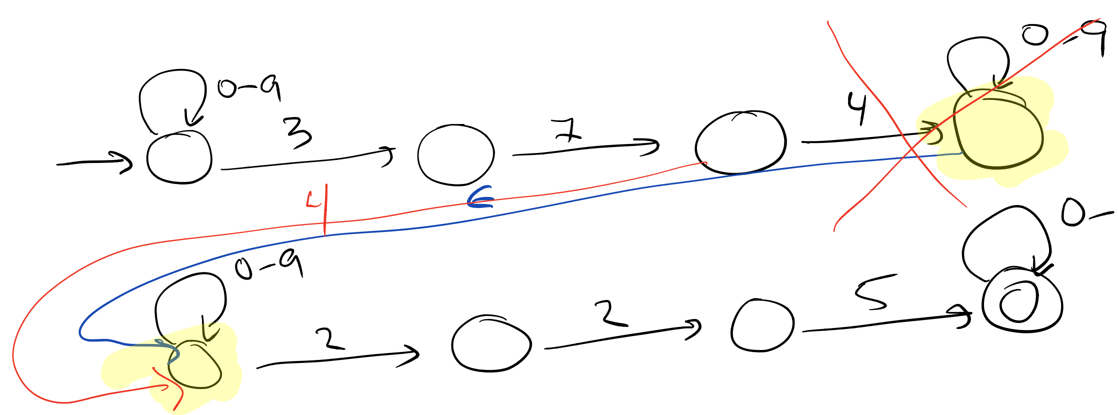
$\delta_3(s, c) = \delta_2(s, c)$ for $c \in \Sigma + \{\epsilon\}$, $s \in Q_2$

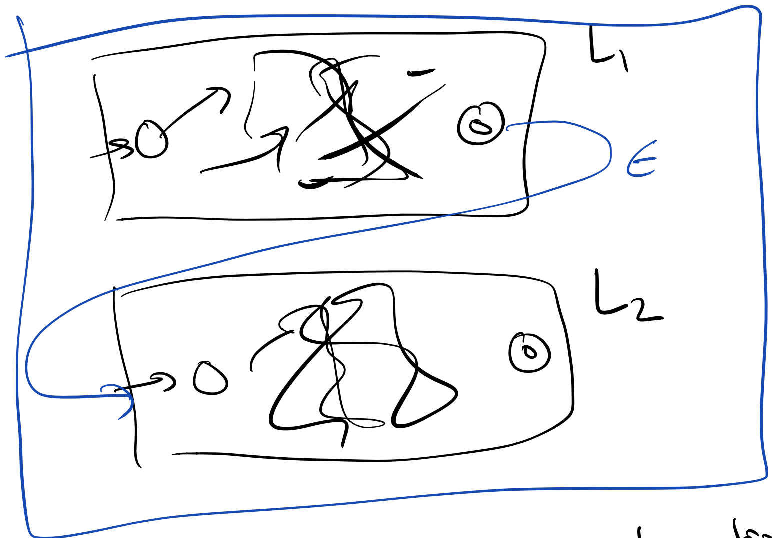
L_{374}

L_{225}



$L_{374} \cdot L_{225}$



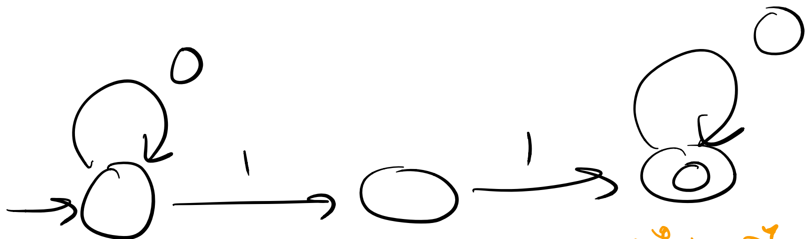


$L_1 \circ L_2$

closed under concatenation

$\mathcal{L}_{NFA}$

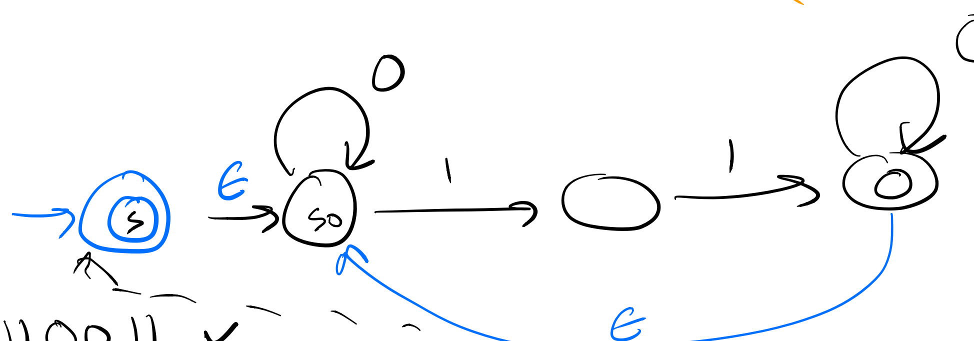
L_1



0^*110^*

$(0^*110^*)^*$

L_1^*



0110011 ✓

0111111 ✓

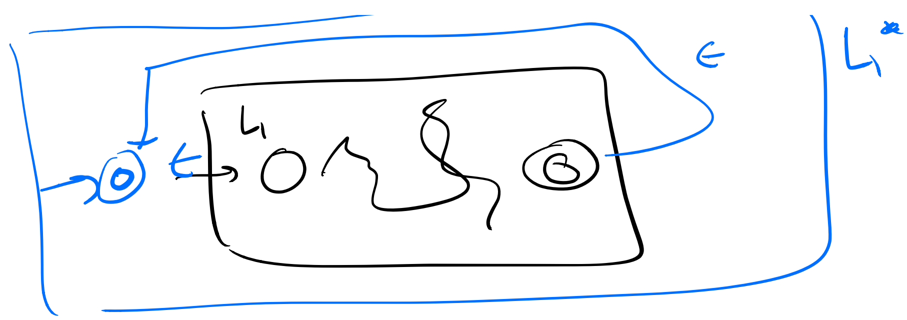
ϵ ✓

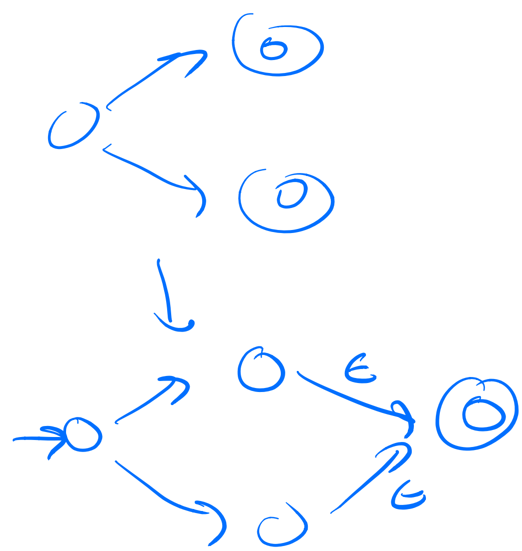
0 ✗

$\epsilon \in L_1^*$

$0 \notin L_1^*$

$\delta^*(s, \epsilon) = \epsilon\text{-reach}(s) = \{s, s_0\}$





$\therefore$ NFA closed under Kleene-$*$

Regex

$\emptyset$

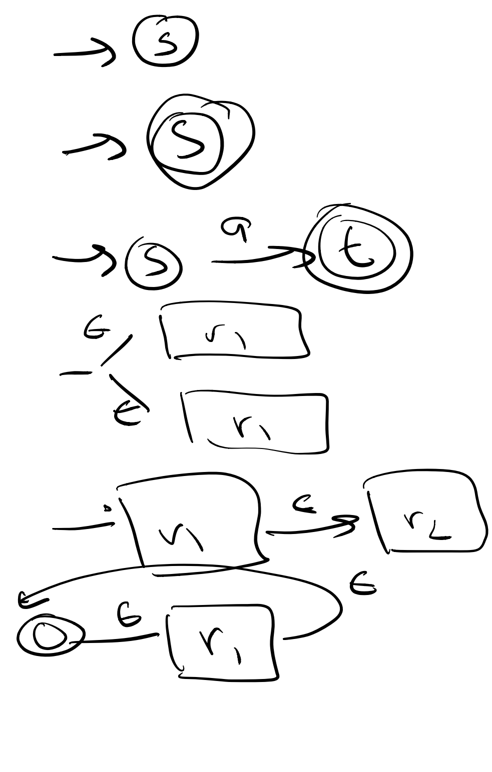
ϵ

a

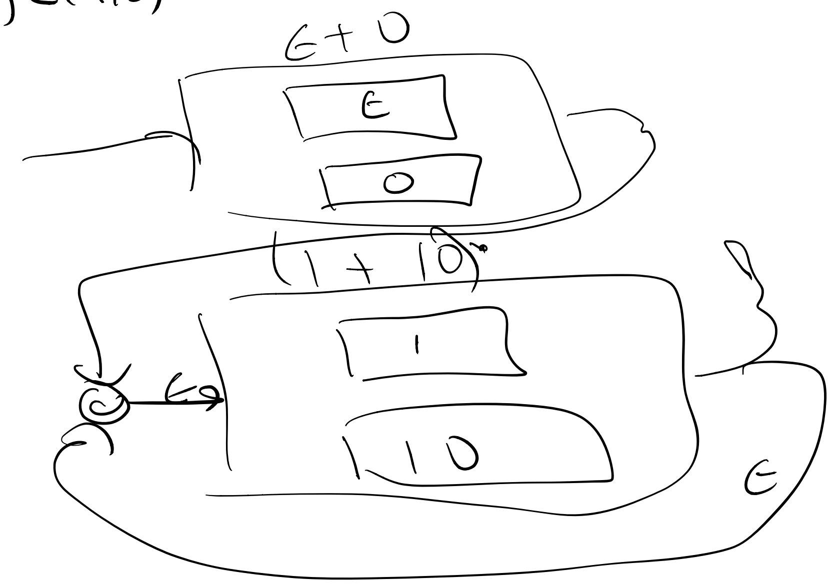
$r_1 + r_2$

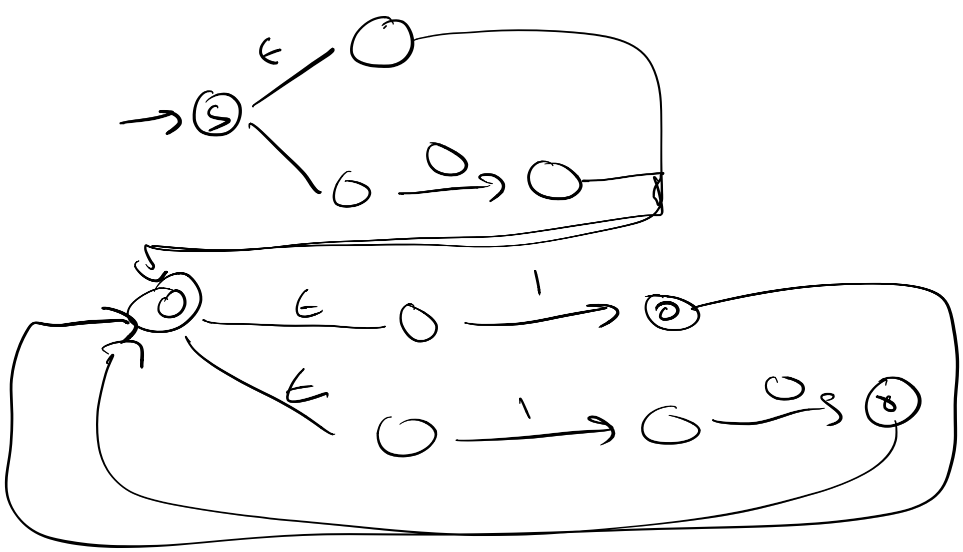
$r_1 r_2$

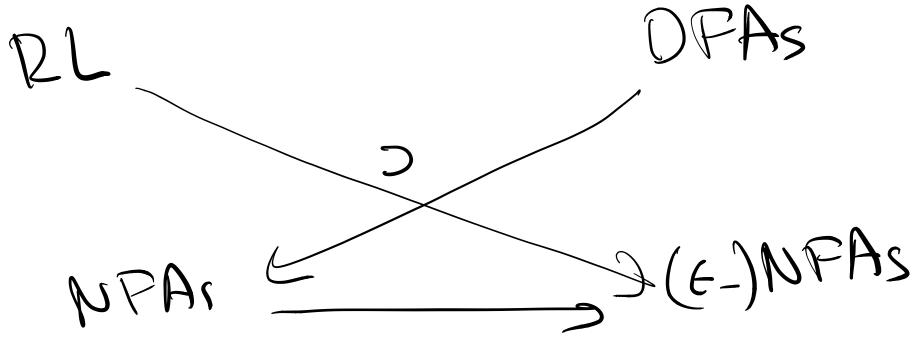
r_1^*

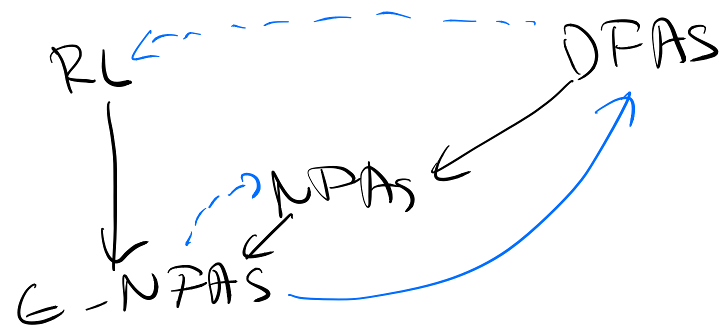


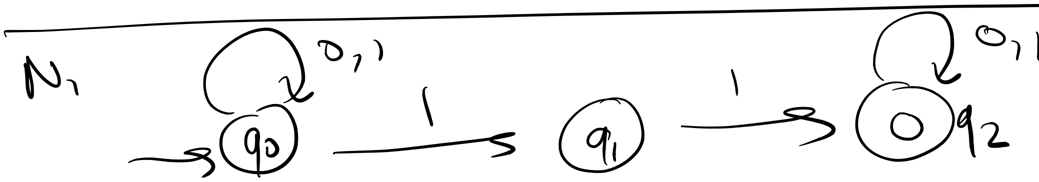
$(\epsilon+0)(1+10)^*$

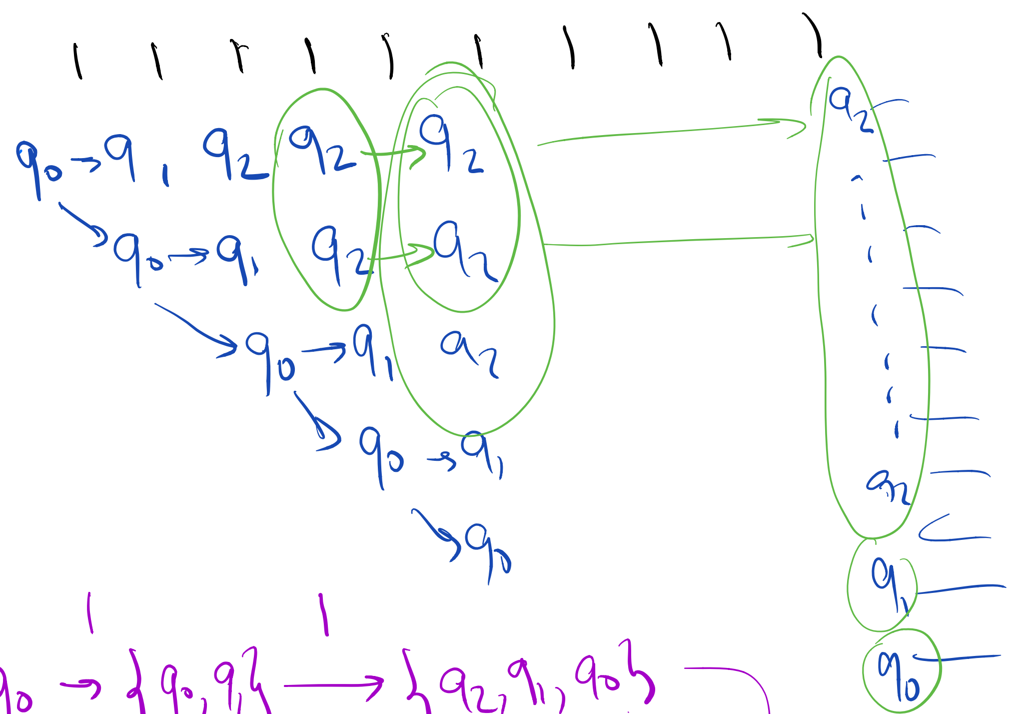












$$q_0 \xrightarrow{1} \{q_0, q_1\} \xrightarrow{1} \{q_2, q_1, q_0\}$$

$\delta(q_0, 1) = \{q_0, q_1\}$

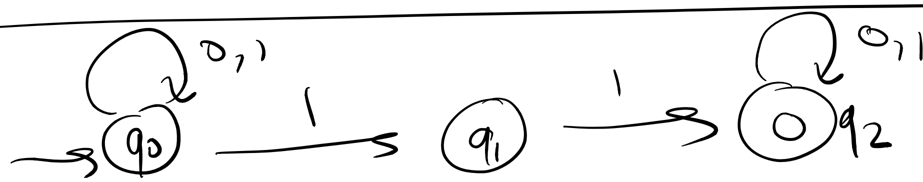
$\delta(q_1, 1) = \{q_2\}$

$$\xrightarrow{1} \{q_2, q_1, q_0\} \xrightarrow{1}$$

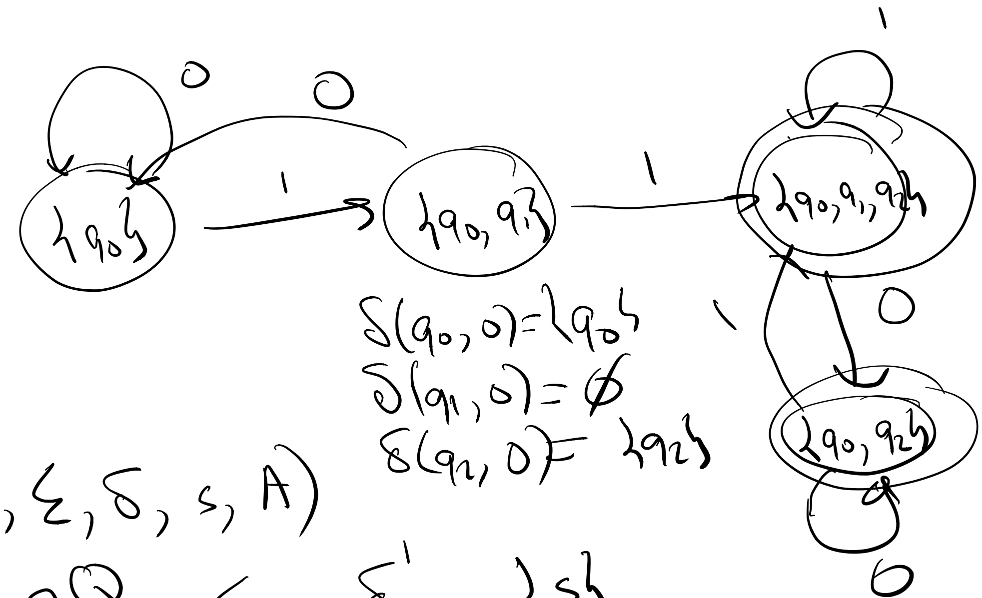
$\delta(q_2, 1) = \{q_2\}$

$\delta(q_1, 1) = \{q_2\}$
$\delta(q_0, 1) = \{q_0\}$

N_1



M_1



$\delta(q_0, 0) = \{q_0\}$
$\delta(q_1, 0) = \emptyset$
$\delta(q_2, 0) = \{q_2\}$

$N_1 = (Q, \Sigma, \delta, s, A)$

$M_1 = \left(2^Q, \Sigma, \delta', \{s\}, \{S \in 2^Q \mid S \cap A \neq \emptyset\}\right)$

$$\delta'(\underset{2^Q}{S}, a) = \bigcup_{p \in S} \delta(p, a)$$

$$\bigcup_{p \in \epsilon\text{-reach}(S)} \epsilon\text{-reach}(\delta(p, a))$$

$$\epsilon\text{-reach}(S) = \bigcup_{p \in S} \epsilon\text{-reach}(p)$$