You saw the following context-free grammars in class on Thursday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

  $S \to \varepsilon \mid S(S)$ — properly nested parentheses

  Here is a different grammar for the same language:

  $S \to \varepsilon \mid (S) \mid SS$ — properly nested parentheses

- $\{0^m1^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of 0s followed by a *different* number of 1s.

$S \to A \mid B$	$\{0^m1^n \mid m \neq n\}$
$A \to 0A \mid 0C$	$\{0^m1^n \mid m > n\}$
$B \to B1 \mid C1$	$\{0^m1^n \mid m < n\}$
$C \to \varepsilon \mid 0C1$	$\{0^m1^n \mid m = n\}$

Give context-free grammars for each of the following languages. For each grammar, describe the language for each non-terminal, either in English or using mathematical notation, as in the examples above. We probably won't finish all of these during the lab session.

1. $\{0^{2n}1^n \mid n \geq 0\}$

2. $\{0^m1^n \mid m \neq 2n\}$

   [Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string 01.]

3. $\{0,1\}^* \setminus \{0^{2n}1^n \mid n \geq 0\}$

   [Hint: Extend the previous grammar. What's missing?]

Work on these later:

4. $\{w \in \{0,1\}^* \mid \#(0,w) = 2 \cdot \#(1,w)\}$ — Binary strings where the number of 0s is exactly twice the number of 1s.

*5. $\{0,1\}^* \setminus \{ww \mid w \in \{0,1\}^*\}$.

   *[Anti-hint: The language $\{ww \mid w \in 0,1^*\}$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]*