For each of the following languages over the alphabet  $\Sigma = \{0, 1\}$ , either prove the language is regular (by giving an equivalent regular expression, DFA, or NFA) or prove that the language is not regular (using a fooling set argument). Exactly half of these languages are regular.

- 1.  $\{\mathbf{0}^n \mathbf{10}^n \mid n \ge 0\}$
- 2.  $\{\mathbf{0}^{n}\mathbf{10}^{n}w \mid n \geq 0 \text{ and } w \in \Sigma^{*}\}$
- 3.  $\{w \mathbf{0}^n \mathbf{1} \mathbf{0}^n x \mid w \in \Sigma^* \text{ and } n \ge 0 \text{ and } x \in \Sigma^*\}$
- 4. Strings in which the number of 0s and the number of 1s differ by at most 2.
- 5. Strings such that in every prefix, the number of 0s and the number of 1s differ by at most 2.
- 6. Strings such that in every substring, the number of 0s and the number of 1s differ by at most 2.