Final: Monday, December 18, 8-11am, 2017

A	B	C	D	E	F	G	H	J	K
9am	10am	11am	noon	1pm	1pm	2pm	2pm	3pm	3pm
Rucha	Rucha	Srihita	Shant	Abhishek	Xilin	Shalan	Phillip	Vishal	Phillip
101 Armory	101 Armory	101 Armory	151 Loomis	151 Loomis	151 Loomis	ECE 1002	ECE 1002	ECE 1002	ECE 1002

Name:	
NetID:	
Name on Gradescope:	

- ***Don't panic!***
- If you brought anything except your writing implements, your double-sided **handwritten** (in the original) 8½" × 11" cheat sheet, and your university ID, please put it away for the duration of the exam. In particular, please turn off and put away *all* medically unnecessary electronic devices.
  - Submit your cheat sheet together with your exam. An exam without your cheat sheet attached to it will not be graded.
  - If you are NOT using a cheat sheet, please indicate so in large friendly letters on this page.
- **Best answer.** Choose best possible choice if multiple options seems correct to you – for algorithms, faster is always better.
- Please ask for clarification if any question is unclear.
- **This exam lasts 170 minutes.**
- Fill your answers in the Scantron form using a pencil. We also recommend you circle/mark your answer in the exam booklet.
- Please return ***all*** paper with your answer booklet: your cheat sheet, and all scratch paper. We will **not** return the cheat sheet.
- Do not fill more than one answer on the Scantron form - such answers would not be graded. Also, fill your answer once you are sure of your answer – erasing an answer might make the form unscannable.
- ***Good luck!***

Before doing the exam...

- Fill your name and netid in the back of the Scantron form, and also on the top of this page.
- **Fill in the pattern shown on the right in the Scantron form.**

  This encodes which version of the exam you are taking, so that we can grade it.

91	A	B	C	D	●
92	A	B	C	●	E
93	A	B	●	D	E
94	A	B	●	D	E
95	A	B	C	D	●
96	A	●	C	D	E

1. (10 points) (This question is *way harder* than the questions on the exam, but it is a good practice problem.) Given a directed graph G with n vertices and m edges, a vertex v in G is a ***leader***, if for every vertex x in G, either:

(I) There is a path from x to v in G.

(II) Or, there is a path from v to x in G.

Deciding if there is a leader in G can be done by an algorithm in time (faster is way better):

(A) $O(1)$.

(B) $O(n)$.

(C) $O(n + m)$.

(D) $O(n^2(n + m))$.

(E) $O(n(n + m))$.

---

2. (5 points) You are given a set $\mathcal{I} = \{I_1, I_2, \ldots, I_n\}$ of n weighted intervals on the real line. Consider the problem of computing the maximum weight set $X \subseteq \mathcal{I}$, such that every point on the line is covered by at most two intervals. This problem:

(A) Can be solved in linear time by a greedy algorithm.

(B) NP-Complete.

(C) Can be solved in polynomial time.

(D) Undecidable.

(E) NP-Hard.

---

3. (3 points) You are given an unsorted sets X, Y, Z of numbers. Each set has exactly n numbers in it. Deciding if there is number $x \in X$, $y \in Y$ in $z \in Z$, such that $x + y = z$ can be solved in (faster is better):

(A) $O(n \log n)$ time.

(B) $O(n^2 \log n)$ time.

(C) $O(n^2)$ time.

(D) $O(n^3 \log n)$ time.

(E) $O(n^3 \log^2 n)$ time.

---

4. (3 points) Given a DAG G, and two vertices u, v in G, and a parameter t. Consider the problem of deciding if there is a path from u to v in G of length exactly t. This problem can be solved in (faster is better):

(A) $O(n \log n + m)$ time.

(B) only exponential time since this problem is NP-Complete.

(C) $O(n^2 m)$ time.

(D) $O(t(n+m))$ time.

(E) None of the other answers is correct.

---

5. (5 points) There are n people living along Purple street in Shampoo-Banananana. The n people live in locations $1, \ldots, n$ along Purple street (which is as straight as a ruler),
It is time for redistricting Purple street. A district can have between Δ and 2Δ people living in it, for some prespecified parameter Δ (where $n/3 > \Delta > 0$). A district is a consecutive interval along Purple street. Every person is assigned to a district containing it. The districts are disjoint.
For every person in Purple street, you know their vote in the last election. Specifically, you are given an array $v[1 \,..\, n]$, where the vote of the ith person is $v[i]$, which is either equal to 0 or 1. A set $S \subseteq \{1, \ldots, n\}$ of people is t-good, if $|\#_0(S) - \#_1(S)| \leq t$, where $\#_0(S)$ (resp. $\#_1(S)$) is the number of people in S that voted for 0 (resp 1) in S.
Given $\Delta, v[1 \,..\, n]$ and t as input, consider the fastest possible algorithm that outputs TRUE if there is a way to redistrict Purple street so that every district is t-good (the algorithm outputs FALSE otherwise). The running time of the algorithm is:

(A) $O(n)$.

(B) $O(n^2\Delta^2)$.

(C) $O(n^2\Delta)$.

(D) $O(n\Delta^2)$.

(E) $O(n\Delta)$.

---

6. (5 points) You are given a string s with n characters over a finite alphabet Σ. A k *coloring* assigns every character in s a number between 1 and k. Given such a coloring of s, let $f(s, i)$, be the substring resulting for deleting all the characters in s that a color different than i. Given strings $s_1, \ldots, s_k$ (in addition to s), consider the problem of deciding if there is a k coloring of s such that $f(s, i) = s_i$, for all i, and computing the coloring in such a case.
This problem can be solved in (faster is better):

(A) $O(k^{O(n)})$.

(B) The problem is undecidable.

(C) $O(kn^k)$.

(D) $O(kn^{k+1})$.

(E) $O(kn \log n)$.

---

7. (2 points) Consider the language $L = \left\{ 1^i \;\middle|\; i^2 \text{ is even} \right\}$. This language is

(A) Decidable.
(B) Context-free.
(C) Regular.
(D) Finite.
(E) Undecidable.

---

8. (3 points) Consider the recurrence $f(n) = f\Big(\lfloor (3/4)n \rfloor\Big) + f\Big(\lfloor (1/4)n \rfloor\Big) + O(n^2)$, where $f(n) = O(1)$ if $n < 10$. The solution to this recurrence is

(A) $O(n \log n)$.
(B) $O(n)$.
(C) $O(1)$.
(D) $O(n^2)$.
(E) None of the above.

---

9. (3 points) Let L_1 be a context-free language, and let L_2 be a regular language. Then the language $L_1 \cap L_2$ is

(A) undecidable.
(B) context-free.
(C) None of the other answers is correct.
(D) regular.

---

10. (3 points) You are given a set S of n numbers $S = \{x_1, \ldots, x_n\} \subseteq [\![m]\!] = \{1, 2, 3. \ldots, m\}$, where m is a parameter, and you are given a target number $t \leq m$. Consider the problem of deciding if there is a subset $X \subseteq S$, such that $\sum_{x \in X} x = t$. This problem is (faster is better if applicable):

(A) Solvable in $O(nt)$ time.
(B) NP-Complete, even if $t = n^{O(1)}$.
(C) Solvable in $O(n^2 t)$ time.
(D) Solvable in $O(n^2)$ time.
(E) Solvable in $O(2^n n)$ time.

11. (3 points) Consider the problem of checking if there is a Hamiltonian path in a graph with n vertices and m edges. This problem can be solved in (faster is better if applicable)

(A) This problem is NP-Complete, so it can not be solved.

(B) $O(n \cdot n!)$ time.

(C) $O(n \cdot 2^{n \log^2 n})$ time.

(D) $O(2^{n \log^2 n})$ time.

(E) None of the other answers are correct.

12. (3 points) You are given k DFAs $D_1, \ldots, D_k$ over the alphabet $\{0, 1\}$, where each DFA has at most n states. Consider the problem of deciding if there is a word $w \in \{0, 1\}^*$ that is accepted by all these k DFAs. This problem can be solved in (faster is better):

(A) None of the other answers is correct.

(B) $O(k^{n+1})$ time.

(C) $O(kn^k)$ time.

(D) $O(nk)$ time.

(E) $O(k^n)$ time.

13. (5 points) You are given k arrays $A_1, \ldots, A_k$ of sorted numbers (the total number of elements in these arrays is n (you can assume they are all distinct). Given a parameters t, one can compute the number of rank t in $A_1 \cup A_2 \cup \cdots \cup A_k$ in (faster is better):

(A) $O(k)$.

(B) $O(k \log^2 n)$.

(C) $O(n)$.

(D) $O(k \log k)$.

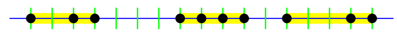
(E) $O(n \log k)$.

14. (5 points) There are n people living along Purple street in Shampoo-Banananana. The n people live in locations specified by a given array $t[1 \, .. \, n]$, where $0 < t[1] < \cdots < t[n]$. Here $t[i]$ is the location of the ith person, which is the distance in meters from the start of Purple street,

The district needs to break the street into blocks. A block can have between Δ and 2Δ people living in it, for some prespecified parameter Δ (where $n/3 > \Delta > 1$). A block is a consecutive interval along Purple street. Every person is assigned to a block containing it. A portion of Purple street that has no people living on it, does not necessarily has to be in a block.

The price of a block, covering the ith to jth person, is the total length of the block (in meters), which is $t[j] - t[i]$ (the next block would start at $t[j+1]$). The *total price* of a solution is the total length of the blocks in the solution.

Here is an instance and a suggested solution (of total price 10), with $\Delta = 2$:

$t = [\underline{1, 3, 4}, \underline{8, 9, 10, 11}, \underline{13, 16, 17}]$ 

One can compute the minimum price of breaking the street into blocks (faster is better):

(A) $O(n^2\Delta^2)$.
(B) $O(n\Delta^2)$.
(C) $O(n\Delta)$.
(D) $O(n^2\Delta)$.
(E) $O(n)$.

---

15. (1 point) All problems in P are decidable.

(A) True.
(B) False.

---

16. (3 points) The set of all finite languages over $\{0, 1\}$ is

(A) undecidable.
(B) uncountable.
(C) countable.
(D) None of the other answers is correct.
(E) $2^{\mathbb{R}}$.