

1. For each of the languages below indicate whether it is (a) decidable (a.k.a. *recursive*), (b) recognizable (a.k.a. *recursively enumerable*) but not decidable, or (c) not recognizable.

You may use the following facts:

- the language  $\overline{D} = \{w \mid M_w \text{ does not accept } w\}$  is not recognizable;
- the language  $ACCEPT = \{(z, w) \mid z, w \in \{0, 1\}^* \text{ and } M_z \text{ accepts } w\}$  is recognizable but not decidable.
- $L_1 = \{(z, w) \mid M_z \text{ does not halt on input } w\}$ .
- $L_2 = \{(z, w, n) \mid M_z \text{ accepts } w \text{ within } n \text{ steps}\}$ .
- $L_3 = \{w \mid M_w \text{ accepts } w\}$ .

2. We saw in class that the language  $ACCEPT = \{(z, w) \mid z, w \in \{0, 1\}^* \text{ and } M_z \text{ accepts } w\}$  is undecidable. Use a reduction to argue that the language  $L = \{(z_1, z_2, w) \mid \text{exactly one of } M_{z_1} \text{ and } M_{z_2} \text{ accepts } w\}$  is undecidable.

3. Can you show that  $L$  from the previous problem is not recognizable?

Hint: The previous problem shows  $ACCEPT$  reduces to  $L$ . Show that  $\overline{ACCEPT}$  reduces to  $L$  using a similar technique. Then prove and use the fact that  $\overline{ACCEPT}$  is not recognizable to complete the proof.