1. Design a one-tape TM that computes the function $f(n) = 2n$ when n is given in unary notation. More specifically, design a TM with input alphabet $\{0, 1, \$\}$, that on input $\$0^n$ changes the tape contents to $\$0^{2n}$ and halts. On halting, the "Instantaneous Description" (or "ID") of your TM should be $q_{\text{halt}}\$0^{2n}$. (The initial ID is $q_{\text{start}}\$0^n$.)

   Could this be done more easily with a two-tape TM?

   Can you modify your solution to obtain a TM such that $q_{\text{start}}0^n \Rightarrow^* q_{\text{halt}}0^{2n}$ (i.e., without the \$ symbol in the input or output).

2. Give a reasonably detailed description of a TM that computes the exponential function to the base 2 for unary input: i.e., it should convert an input string of the form $\$0^n$ to the string $\$0^{2^n}$. That is, you should ensure that $q_{\text{start}}\$0^n \Rightarrow^* q_{\text{halt}}\$0^{2^n}$. You don't have to completely design the TM; just provide enough detail that a TM programmer would know what states and transitions to use. Multiple tapes will be convenient.

3. Think about at home... How would a (multi-tape) TM compute $\lceil \log n \rceil$? That is, if the initial ID is $q_0\$0^n$, then the final ID should be $q_{\text{halt}}\$0^{\lceil \log n \rceil}$