CS 373: Theory of Computation (Summer 2011)

Extra Practice Proving Not Recognizable

1. *Consider three languages L, L_1, L_2 such that $L = L1 \setminus L2$. Show that if L is not Turing-recognizable and L_2 is Turing-recognizable then L_1 is not Turing-recognizable.*

   This problem is incorrect!

   For a counter example, take $L_1 = \Sigma^*$ (decidable) and $L_2 = A_{TM}$ (recognizable). Then $L = L_1 \setminus L_2 = \overline{A_{TM}}$ (unrecognizable). We have met the constraints, but L_1 is recognizable.

   Corrected version: Consider three languages L, L_1, L_2 such that $L = L_1 \setminus L_2$. Show that if L is not Turing-recognizable and L_2 is Turing-decidable then L_1 is not Turing-recognizable.

2. Prove the following are not Turing-recognizable:

   (a) $L_1 = \{\langle M \rangle \mid \text{M is a TM and on input MIDTERM2, M never changes the contents of the odd-numbered positions on its tape}\}$. (That is, it can read the odd-numbered positions, but not write a different symbol onto them.)

   (b) $\overline{L_{Pal}} = \{\langle M \rangle \mid M \text{ is a TM that does not accept any palindrome}\}$.

   (c) $L_{ALL} = \{\langle M \rangle \mid M \text{ is a TM with input alphabet } \Sigma \text{ and } L(M) = \Sigma^*\}$. (Informally, L_{ALL} is the set of TMs that accept every input string.)

   (d) $\overline{L_{ALL}} = \{\langle M \rangle \mid M \text{ is a TM with input alphabet } \Sigma \text{ and } L(M) \neq \Sigma^*\}$.