Problem Set 5

CS373 - Spring 2011

Due: Tuesday May 03 at 2:00 PM in class (151 Everitt Lab)

Please follow the homework format guidelines posted on the class web page:

`http://www.cs.uiuc.edu/class/sp11/cs373/`

1. Decidable, Recognizable, or Neither?
   [**Category**: Proof, **Points**: 28]

   Prove whether the following languages are Turing decidable, recognizable (but not decidable), or unrecognizable.

   (a) $\{\langle M, w\rangle \mid M$ retuns to the same configuation at least twice on input $w\}$ (7 Points)

   (b) The set of pairs $\langle M, w\rangle$ such that Turing Machine M (with one tape, which moves left or right on every step, and the read head starts at the left end of the input string) makes three consecutive right moves at some time when given input w. (7 Points)

   (c) $\{\langle M\rangle \mid M$ accepts at least 5 strings$\}$ (7 Points)

   (d) $\{\langle M\rangle \mid |L(M)|$ is prime$\}$ (7 Points)

2. A and B
   [**Category**: Proof, **Points**: 12]

   Prove that Turing recognizable languages A and B are decidable given the following assumptions:

   (a) $A \cap B = \emptyset$, and $\overline{A \cup B}$ is recognizable. (6 Points)

   (b) $A \cup B = \Sigma^*$, and $(A \cap \overline{B}) \cup (\overline{A} \cap B)$ is recognizable. (6 Points)

3. Recognizers as Deciders
   [**Category**: Proof, **Points**: 12]

   Assume that the following statement is true: A is Turing recognizable iff there is a decidable language B such that $A = \{w \in \Sigma^* \mid \exists x \in \Sigma^*, \langle w, x\rangle \in B\}$.

   (a) Let A be Post's Correspondence Problem. Describe a language B that fulfills the above statement. (6 Points)

   (b) Let there be a decidable B such that $C = \{w \in \Sigma^* \mid \forall x \in \Sigma^*, \langle w, x\rangle \in B\}$. What can be said about C in terms of recognizability? Explain. (6 Points)

4. Set of Deciders
   [**Category**: Proof, **Points**: 15]

   Let A be a Turing recognizable set of deciders. That is, every $\langle M\rangle \in A$ decides the language $L(M)$. Prove that there exists some decider that A does not contain.

5. Regular Reverse
[**Category**: Proof, **Points**: 15]

Consider the set REV_{DFA} of DFAs with languages that contain the reverse of their strings. That is, let $REV_{DFA} = \{\langle M \rangle \mid M \text{ is a DFA and } x \in L(M) \Rightarrow x^R \in L(M)\}$.
Prove whether or not REV_{DFA} is decidable. (*Hint*: Is $\{\langle M, M' \rangle \mid M \text{ and } M' \text{ are DFAs and } L(M) = L(M')\}$ decidable?)

6. Computational Party Analysis
[**Category**: Construction, **Points**: 18]

Imagine that we build a "social network" graph $G = (V, E)$ of interactions at a party. Each node $p \in V$ represents a unique person, and an undirected edge $\{p, p'\} \in E$, $p \neq p'$ represents whether p and p' talked to each other at the party.

(a) Consider the following problem: "Given the graph G with $|V| = n$, did everyone talk to everyone else during the party?" Describe an algorithm that computes the answer given G. (4 Points)

(b) Argue that the previous problem is in P. (4 Points)

(c) Consider the following problem: "Given the graph G with $|V| = n$ and some $k \in \mathbb{N}, k \leq n$, is there a group of exactly k people that all talked to each other during the party?" Describe an algorithm that computes the answer given G and k. (5 Points)

(d) Argue that the previous problem is in NP. (5 Points)