Problem Set 3

CS373 - Spring 2011

Due: Thursday Mar 17 at 2:00 PM in class (151 Everitt Lab)

Please follow the homework format guidelines posted on the class web page:

`http://www.cs.uiuc.edu/class/sp11/cs373/`

1. Mirrors

   [**Category**: Proof, **Points**: 18]

   Let $A = \{w \in \{a,b\}^* \mid w = w^R\}$.

   (a) Determine the set of equivalence classes of $\sim_A$ on A. Using this, conclude that A is not regular. (4 Points)

   (b) Prove that $\overline{A}$ cannot be generated by any CFG in which $S \to aSa|bSb$ are the only production rules that have terminals on the right side. (6 Points)

   (c) A *mirror* grammar is a CFG (V, Σ, R, S) where the only production rules are of the form

$$X \to aYb$$

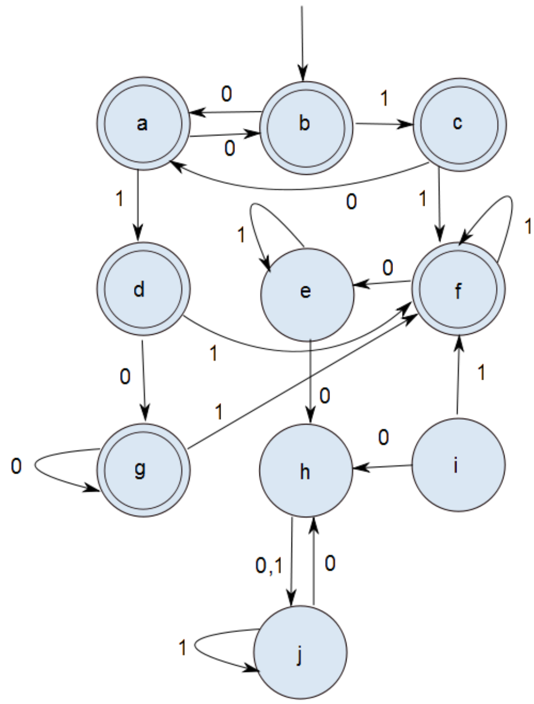
$$X \to a$$

$$X \to \varepsilon$$

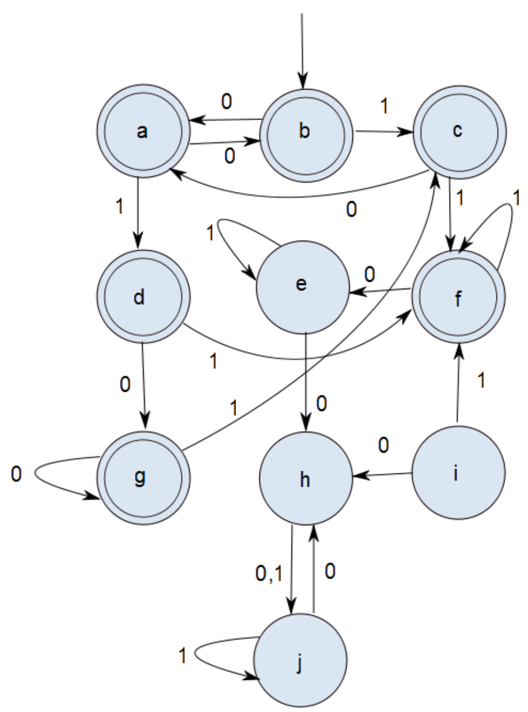
   for $X, Y \in V$, $a, b \in \Sigma$. Prove that every regular language has a mirror grammar, but not all CFLs with mirror grammars are regular. *Hint*: Recall that a rule of the form $X \to aY$ can be seen as a transition from X to Y in a finite automaton. For mirror grammars, think of Y as being more than just one state. (8 Points)

2. Minimize
[**Category**: Construction, **Points**: 10]

Minimize the following DFA. For each state in the minimal DFA, provide a regular expression for the equivalence class of $\sim_A$ that corresponds to that state.



[CORRECTION] There was a mistake in the original DFA that made it more complicated than we expected. Here is what we intended. You can solve either for full credit:



3. Perfect NFAs
[**Category**: Proof, **Points**: 10]

Given a regular language A and an NFA $N = (Q_N, \Sigma, \delta_N, q_{N0}, F_N)$ for A, we can convert N into a DFA with $2^{|Q_N|}$ states. Let $M = (Q_M, \Sigma, \delta_M, q_{M0}, F_M)$ be the minimal DFA for A.

(a) Given minimal M with $L(M) = A$, give a lower bound on the number of states in any NFA N with $L(N) = A$. (5 Points)

(b) Find A, N, and M with $|Q_M| \geq 4$ such that the subset construction using N yields M, the minimal DFA. Prove that $L(M) = L(N) = A$ and that M is minimal. (5 Points)

4. Construct CFGs
[**Category**: Construction, **Points**: 18]

Construct CFGs for the following languages. Be sure to explain your grammar and the purpose of each nonterminal.

(a) $B_1 = \{a^m b^n \in \{a,b\}^* \mid m, n \geq 0, m \neq n\}$ (6 Points)

(b) $B_2 = \{a^n b^{2n} \in \{a,b\}^* \mid n \geq 0\}$ (6 Points)

(c) $B_3 = \{a^m b^n \in \{a,b\}^* \mid m < n < 2m\}$ (6 Points)

5. Construct PDAs
[**Category**: Construction, **Points**: 12]

Construct PDAs for the following languages and explain their correctness.

(a) $B_1 = \{w \in \{a,b\}^* \mid w$ has twice as many a's as b's$\}$ (6 Points)

(b) $B_2 = \{w_1\#w_2\#w_3\#...\#w_k\#\#w_j^R \in \{a,b,\#\}^* \mid j,k \in \mathbb{N}, 1 \leq j \leq k,$ and $w_1, ..., w_k \in \{a,b\}^* \backslash \{\varepsilon\}\}$ (6 Points)

6. Binary sequence
[**Category**: Construction, **Points**: 10]

Let $\Sigma = \{0, 1, \#\}$, and let $A = \{b_1\#b_2\#b_3 \ldots \#b_n \mid n \geq 1\}$, where b_i is the representation of i in binary with no leading zeros. As examples, the four shortest members of A are 1, 1#10, 1#10#11, and 1#10#11#100. Construct a CFG or PDA for the language $\overline{A}$.

7. Chomsky Normal Form
[**Category**: Construction, **Points**: 7]

Give a grammar in Chomsky normal form for $\{0^n 1^n 2^k \in 0^* 1^* 2^* \mid n, k \geq 0\}$.

8. Unary
[**Category**: Proof, **Points**: 15]

A language $A \subseteq \Sigma^*$ is *unary* if $|\Sigma| = 1$.

(a) Prove that every unary CFL is regular. *Hint*: Find a way to describe the CFL as a finite union of regular languages. The lengths of the strings have a particular pattern that might help. You might want to solve the later parts of this question first. (8 Points)

(b) Give an example of a nonregular unary language. Prove that it is nonregular and whether or not it is context-free. (3 Points)

(c) Consider bijections that convert from binary langauges to unary languages. One example would be $f : Pow(\{0,1\}^*) \rightarrow Pow(\{0\}^*)$ such that $f(A) = \{0^k \in \{0\}^* \mid$ the kth binary string in the lexicographic order is in $A\}$ for $A \subseteq \{0,1\}^*$. (By lexciographic order, we mean the sequence ε, 0, 1, 00, 01, 10, 11, 000, ... This order exists because $\{0,1\}^*$ is countable.) Prove that there is no way to convert binary languages to unary languages that preserves the classes of regular and context-free languages. That is, there is no bijection f such that both 1. A is regular iff $f(A)$ is regular, and 2. A is context free iff $f(A)$ is context free. (4 Points)