Problem Set 1

Spring 10

Due: Tuesday Feb 9 in class before the lecture.

Please follow the homework format guidelines posted on the class web page:

`http://www.cs.uiuc.edu/class/sp10/cs373/`

1. DFA building [**Category**: Construction, **Points**: 10]

   Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Let L be the set of strings s in Σ^* such that s has an even number of $\mathtt{a}$'s and exactly two $\mathtt{b}$'s. Construct a deterministic finite automaton for L that has at most 7 states. Make sure that your DFA is complete.

   If you find this hard, you can also give a DFA with more states that accepts L for partial credit.

2. [**Category**: Construction, **Points**: 20]

   Your goal is to design an automatic lights control system. The system has a sensor that detects motion in the room and sends the data to controller every 2 seconds; controller turns the lights on when somebody enters the room, and turns off when nobody is in the room. However, due to some uncertainty in the sensor, it may not detect the motion sometimes. That is why we do not want the lights to turn off immediatly after the sensor tells that there's nobody in the room. We want our controller to wait for additional 2 seconds (till the next signal from the sensor), and only then turn the lights off, if the sensor still thinks that the room is empty. We want to ensure that the system works as desired. Your task is to build an automaton to verify the behaviour of the system. Assume that the system starts with the light beign off. Your automaton is given a sequence of events and actions in the alphabet $\Sigma = \{on, off, yes, no\}$, where every odd event is either *no* when the sensor thinks that the room is empty, or *yes* otherwise. Every even event is the response from controller: *on*, if it decides to turn the lights on, and *off* otherwise. The given sequence is of even length. Your automaton should decide whether or not the behaviour is legitimate.

   Examples of good sequences:

   (a) no, off, yes, on

   (b) $yes, on, yes, on, no, on, yes, on$

   (c) $no, off, yes, on, yes, on, no, on, no, off, yes, on$

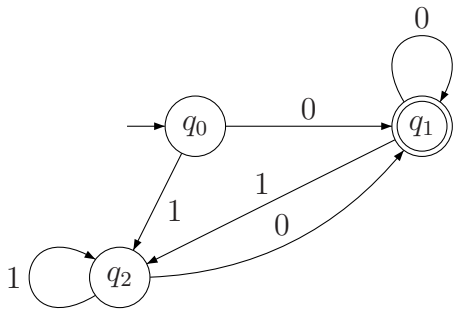
   Examples of bad sequences:

   (a) yes, on, no, off

   (b) $no, off, yes, on, yes, off$

3. [**Category**: Analysis, **Points**: 20]

   What is the language of the following DFA. Try to describe its language clearly and as simple as you can (for example in one short sentence).



4. [**Category**: Notation, **Points**: 20]

   Write down the DFA in the previous problem using formal notation (make sure to clearly describe all five important pieces of a DFA).

5. [**Category**: Notation, **Points**: 10]

   Draw the state-diagram of the DFA described below.

   DFA $A = (Q, \Sigma, \delta, q_0, F)$ where

   - $\Sigma = \{a, b, c\}$
   - $Q = \{q_0, (a, p_0, q_0), (b, p_0, q_0), (c, p_0, q_0), somnambulist, \{a, p_0, q_0\}, \emptyset\}$
   - $F = \{\emptyset\}$
   - δ is defined as follows:
     - For every $d \in \Sigma$, $\delta(\{a, p_0, q_0\}, d) = (d, p_0, q_0)$.
     - For every $e, d \in \Sigma$, $\delta((e, p_0, q_0), d) = (d, p_0, q_0)$.
     - For every $q \in \{q_0, somnambulist, \emptyset\}$, $\delta(q, c) = \{\}$.
     - For every $q \in \{q_0, somnambulist, \emptyset\}$, $\delta(q, a) = \{a, p_0, q_0\}$.
     - For every $q \in \{q_0, somnambulist, \emptyset\}$, $\delta(q, b) = q$.

6. Extra Credit/Honors [**Category**: Construction, **Points**: 20]

   Present a DFA over $\Sigma = \{0, 1\}$ that accepts the set of all strings that, when interpreted in reverse as a binary number, is divisible by 3. Examples: 011 (110 = 6), 0, 0011 (1100 = 12), 01001 (10010 = 18), etc. Prove, that your DFA accepts exactly this language.