Lecture #7

NFAs $\equiv$ DFAs

NFAs without ϵ-transitions

$N = (Q, \Sigma, \delta, q_0, F)$ $\delta(q, \epsilon) = \phi$ $\forall q \in Q$.

$\delta^* : Q \times \Sigma^* \to 2^Q$

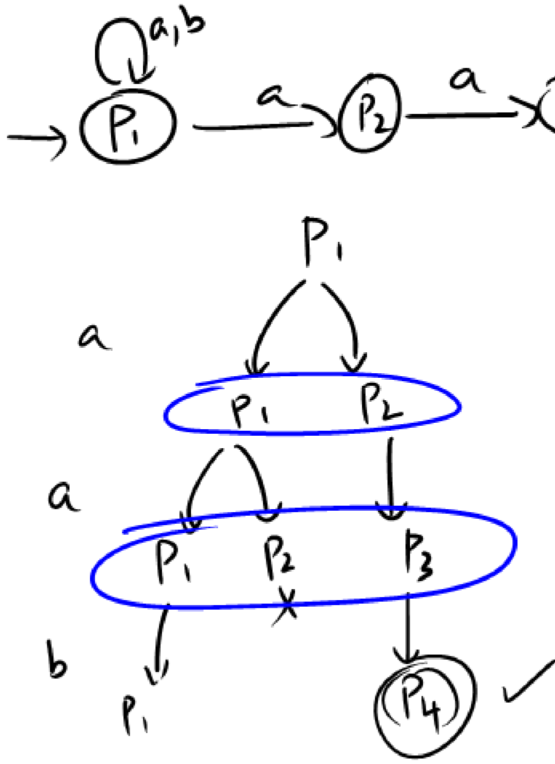
$\delta^*(q, \epsilon) = \{q\}$

$\delta^*(q, wa) = \delta\cancel{(\delta^*(q, w), a)}$

$\delta^*(q, a) = \delta(q, a)$

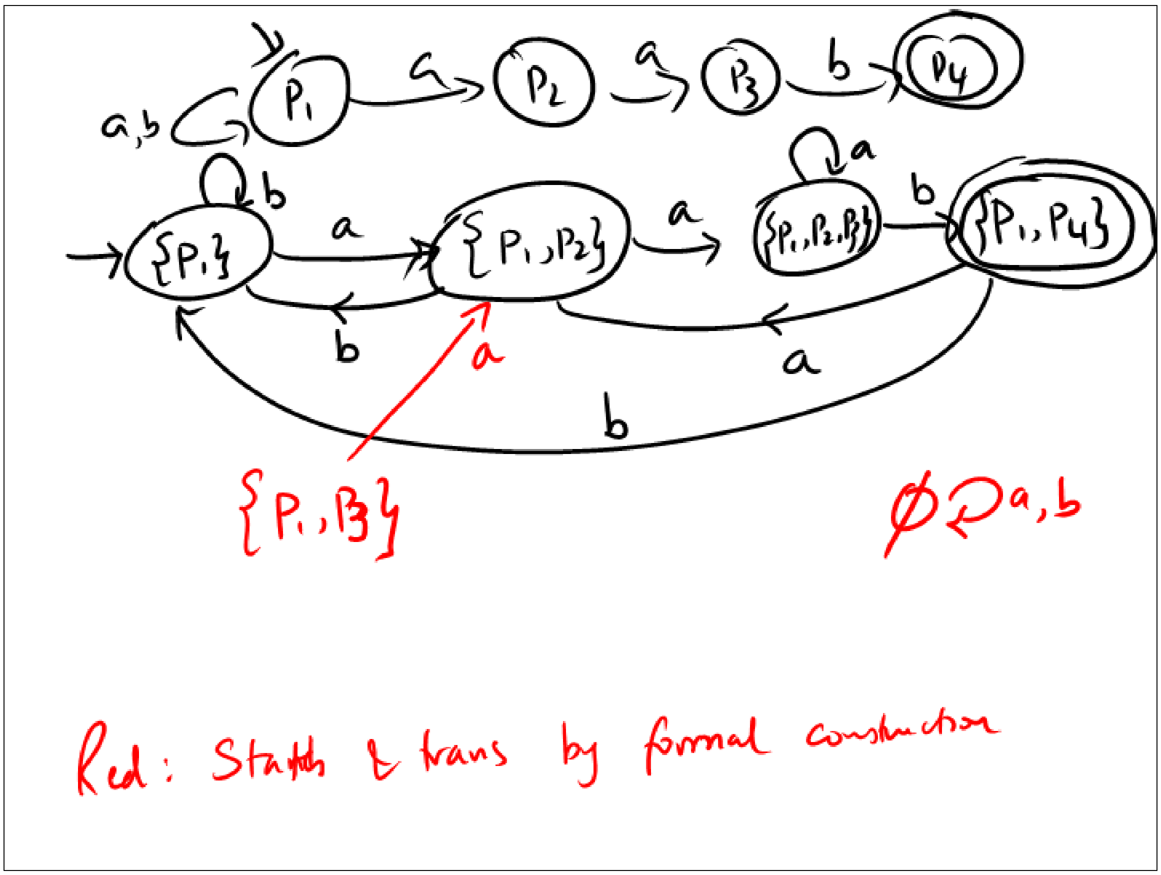
$= \bigcup_{q' \in \delta^*(q, w)} \delta(q', a)$

$w \in L(N)$ iff $\delta^*(q_0, w) \cap F \neq \phi$



Idea

DFA will keep track, after reading w, the precise set of states the NFA could be in after reading w.


a,b
P1
a
P2
a
P3
b
P4
b
{P1}
a
{P1,P2}
a
{P1,P2,P3}
a
b
{P1,P4}
b
a
b
a
{P1,P3}
∅ a,b
Red: States & trans by formal construction

Formal construction $\text{NFA}_{\not\epsilon} \rightarrow \text{DFA}$

$$N = (Q, \Sigma, \delta, q_0, F)$$

$$D = (2^Q, \Sigma, \delta', \{q_0\}, F')$$

$$F' = \{ X \subseteq Q \mid X \cap F \neq \emptyset \}$$

$$\delta'(X, a) = \bigcup_{q \in X} \delta(q, a) \qquad \forall X \subseteq Q. \quad \forall a \in \Sigma.$$

$$N = (Q, \Sigma, \delta, q_0, F)$$

$$D = (2^Q, \Sigma, \mu, \{q_0\}, F')$$

$$F' = \{X \mid X \cap F \neq \emptyset\} \; ; \; \mu(X, a) = \bigcup_{q \in X} \delta(q, a)$$

Then $L(N) = L(D)$.

<u>Claim</u> $\mu^*(\{q_0\}, w) = \delta^*(q_0, w) \quad \forall w \in \Sigma^*$

Proof by induction on $|w|$.

$|w| = 0$; $w = \epsilon$, $\mu^*(\{q_0\}, \epsilon) = \{q_0\} = \delta^*(q_0, \epsilon)$

$|w| \geq 1$; $w = xa$.

$$\mu^*(\{q_0\}, xa) = \mu(\mu^*(\{q_0\}, x), a)$$

$$= \mu(\delta^*(q_0, x), a) = \bigcup_{q \in \delta^*(q_0, x)} \delta(q, a) = \delta^*(q_0, xa)$$

We have proved $\mu^*(\{q_0\}, w) = \delta^*(q_0, w)\ \forall w.$

$L(N) = L(D)$

$w \in L(N)$ iff $\underline{\delta^*(q_0, w)} \cap F \neq \emptyset$

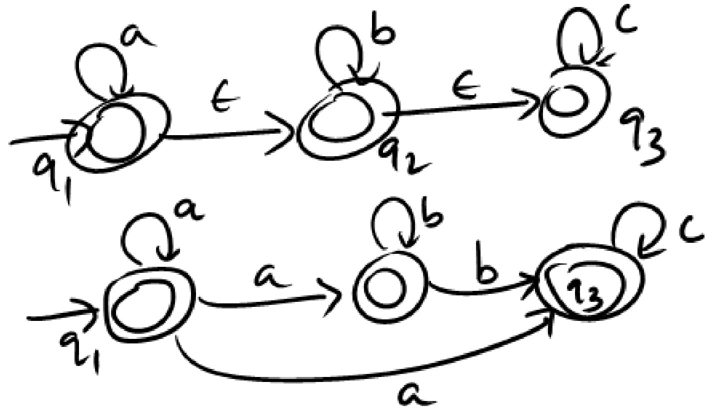
iff $\mu^*(\{q_0\}, w) \cap F \neq \emptyset$

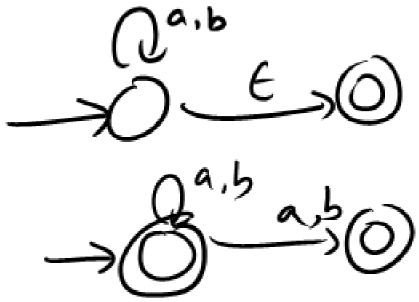
iff $\mu^*(\{q_0\}, w) \in F'$ (by def F')

iff $w \in L(D)$.

	∪	∩	·	*	rev	complem
2^n DFA	✓	✓	✓	✓	✓	✓
NFA	✓	✓	✓	✓	✓	✓
Regexp	✓	?	✓	✓	?	?

NFA with ε-transitions




a,b
ε
a,b
a,b

$N = (Q, \Sigma, \delta, q_0, F)$ — NFA with ε-tr

EpsCl(q) = set of all states that can be reached from q following ε-transitions only, including q.

$= \{ p \mid \exists r_1 \ldots r_n \quad n \geq 1$

$q = r_1 \; ; \; p = r_n \; ,$

$r_i \xrightarrow{\epsilon} r_{i+1} \quad \forall i \leq n-1 \}$.

$M' = (Q, \Sigma, \delta', q_0, F')$

$F' = F \cup \{q_0\}$ if $EpsCl(q_0) \cap F \neq \emptyset$

$= F$ otherwise

$$\delta'(q, a) = \bigcup_{p \in \delta(q,a)} \mathrm{Eps\,cl}(p)$$

$L(M) = L(N)$

Claim $\delta'^{*}(q_0, x) = \delta^{*}(q_0, x)$ ~~unless~~ if $x \neq \epsilon$

Induction on x.

Base case $|x| = 1$