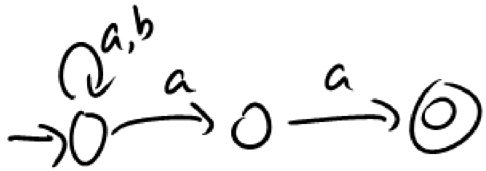
Lecture #6:

- Closure of languages accepted by NFAs under $\cup$, $\cdot$, $*$, reversal
- Regular Expressions

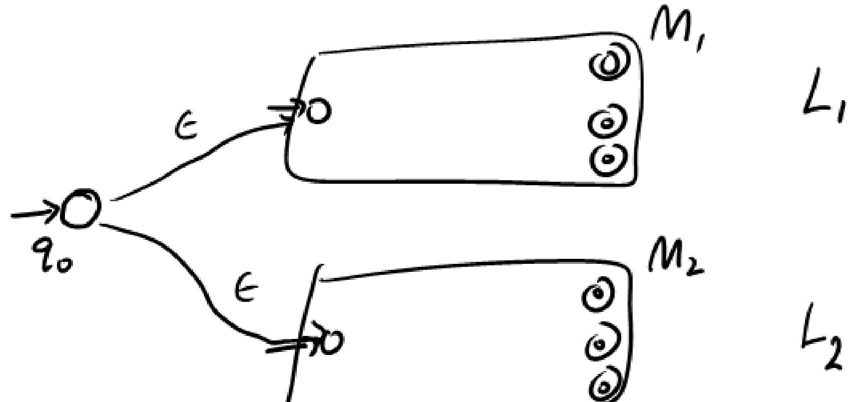


$$L = \{ waa \mid w \in \Sigma^* \}.$$

Are languages accepted by NFAs closed under $\cup$.

i.e. if L_1 & L_2 are languages over Σ, accepted by NFAs

is $L_1 \cup L_2$ also accepted by some NFA?



$w \in L_1 \cup L_2$, then $w \in L_i$, so $q_0 \rightsquigarrow \odot$ M_i

Union.

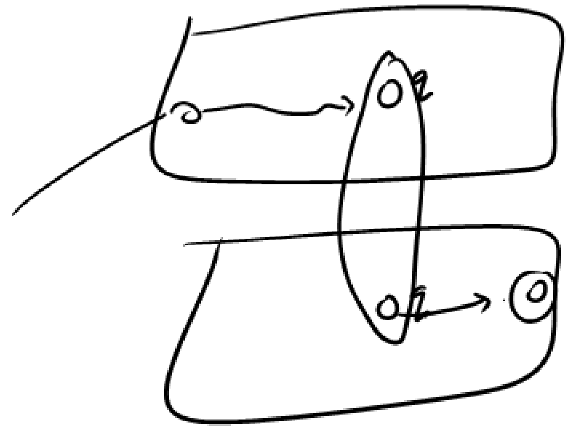
$$M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$$
$$M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$$

Let $q_0 \notin Q_1 \cup Q_2$; Assume wlog $Q_1 \cap Q_2 = \emptyset$

$$M = (Q_1 \cup Q_2 \cup \{q_0\}, \Sigma, \delta, q_0, F_1 \cup F_2)$$

$$\delta(q, a) = \begin{cases} \delta_1(q,a) & \text{if } q \in Q_1, a \in \Sigma_\epsilon \\ \delta_2(q,a) & \text{if } q \in Q_2, a \in \Sigma_\epsilon \\ \{q_0^1, q_0^2\} & \text{if } q = q_0, a = \epsilon \\ \emptyset & \text{if } q = 0, a \neq \epsilon \end{cases}$$

Thm. $L(M) = L(M_1) \cup L(M_2)$.



Concatenation

$L_1, L_2 \subseteq \Sigma^*$

$$L_1 \circ L_2 = \{ w \mid \exists w_1, w_2 \in \Sigma^* \; w_1 \in L_1, w_2 \in L_2, w = w_1 w_2 \}$$

$$\{a, aa, aaa, \ldots a^i, \ldots\} \circ \{b, bb, bbb, \ldots b^i \ldots\} = \{a^i b^j \mid i \in \mathbb{N}, j \in \mathbb{N}\}.$$

$$\{a^i \mid i \in \mathbb{N}\} \circ \phi = \phi$$

$$\{a^i \mid i \in \mathbb{N}\} \circ \{\epsilon\} = \{a^i \mid i \in \mathbb{N}\}$$

$$L \circ \emptyset = \emptyset$$

$$L \circ \{\epsilon\} = L$$

$$\emptyset \circ L = \emptyset$$

$$\{\epsilon\} \circ L = L$$

$$L \circ L = \{ w_1 w_2 \mid w_1, w_2 \in L \}$$

$$L \circ L = \{ ww \mid w \in L \} \quad X$$

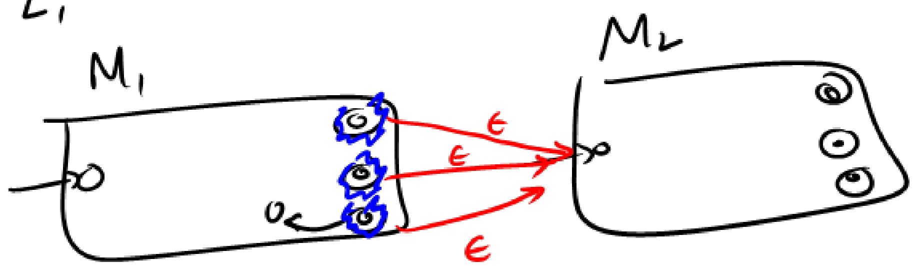
$$\{ab^i \mid i \in \mathbb{N}\} \circ \{ab^i \mid i \in \mathbb{N}\}$$

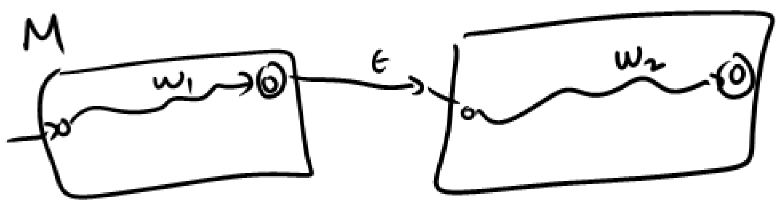
$$= \{ab^i ab^j \mid i, j \in \mathbb{N}\}$$

$$\neq \{ab^i ab^i \mid i \in \mathbb{N}\}$$

Closure under concatenation

$L_1 - M_1 \qquad L_2 - M_2$

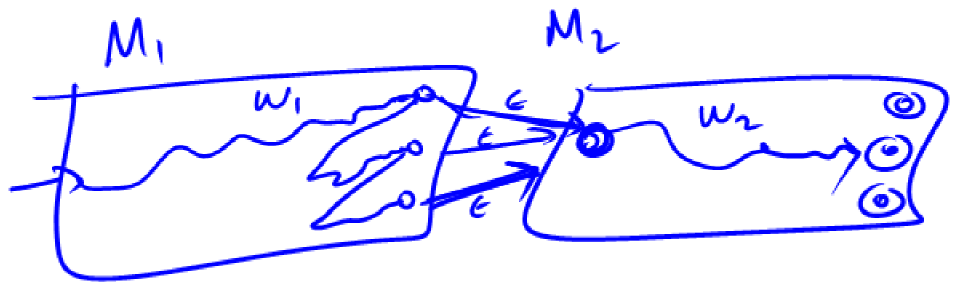




$w \in L_1 \circ L_2$

$w_1 \in L_1 \quad w_2 \in L_2$

$w = w_1 w_2$



M_1
M_2
w_1
w_2
ϵ
ϵ
ϵ

Kleene-*

$*$ is a unary operator

$$L^* = \{ w \mid w = w_1 \ldots\ldots w_n \ , n \in \mathbb{N}_0 , \ w_i \in L \ \ \forall 1 \leq i \leq n \}$$

$$L^i = \underbrace{L \circ L \circ L \circ \ldots \circ L}_{i\text{-times}} .$$

$$L^* = \bigcup_{i \geq 0} L^i .$$

L^* contains ϵ always.

$\{a,b\}^* =$ all words over $\{a,b\}$

$$\{aa\}^* = \{\epsilon, aa, aaaa, aaaaaa, \ldots\}$$
$$= \{a^{2i} \mid i \in \mathbb{N}_0\}.$$

$$\emptyset^* = \{\epsilon\}$$

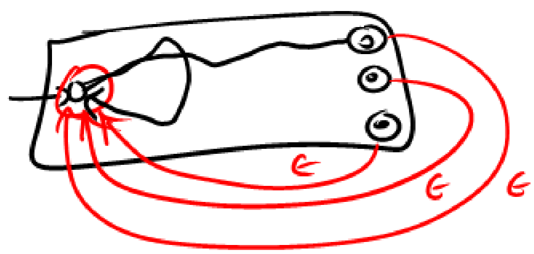
$$\{\epsilon\}^* = \{\epsilon\}$$

$$\{a\}^* = \{\epsilon, a, aa, \ldots\}$$

Closure under *

If L is acc by an NFA, is L^* acc by an NFA?

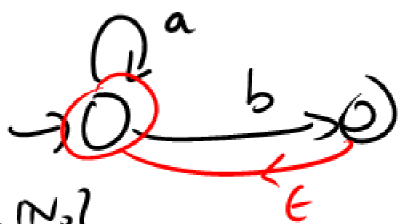
$\hat{M}$



Argue why $L(\hat{M}) = (L(M))^*$.

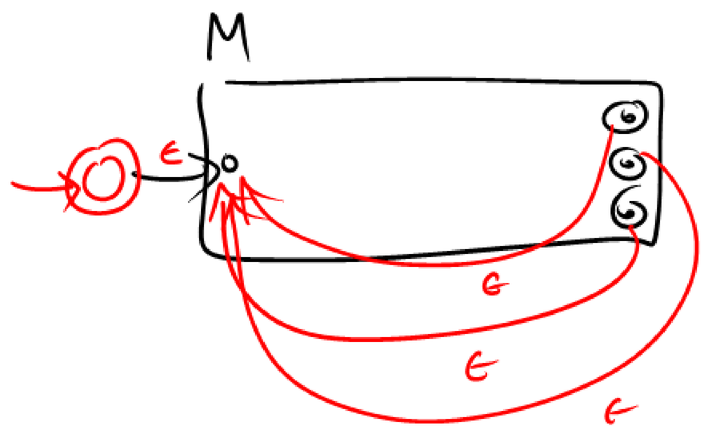
$w \in (L(M))^*$ $\quad w = w_1 w_2 \ldots w_k \Rightarrow w \in L(\hat{M})$

$(L(M))^* \subseteq L(\hat{M})$



$L = \{ a^i b \mid i \in \mathbb{N}_0 \}$

$a \in L(\hat{M})$


M
ε
ε
ε
ε

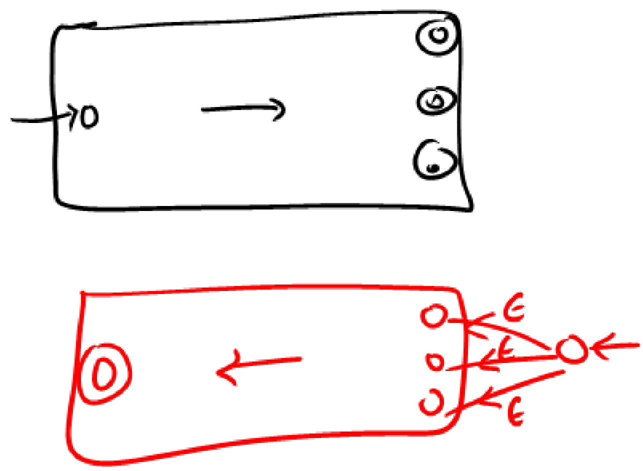
Closure under reversal

$w \in \Sigma^*$, $w = a_1 \dots a_n$

$w^R = a_n \dots a_1$

$L^R = \{ w^R \mid w \in L \}$

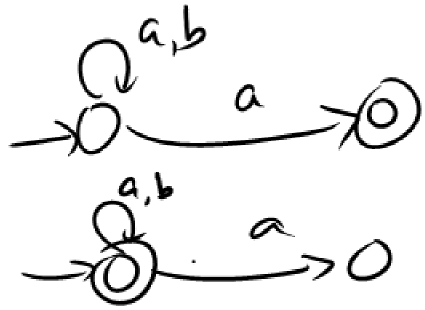
If L is acc by an NFA,
is L^R also acc by some NFA?



	$\cup$	$\cap$	concat	$*$	Complement
DFA	Doable product	Doable product	Hard	Hard	Easy
NFA	Easy	Doable product	Easy	Easy	Hard

new way

$\cup$, concat, $*$ - simple for NFAs.


a,b
a
a,b
a

$$R ::= \underset{a \in \Sigma}{a} \mid R_1 + R_2 \mid \epsilon \mid \phi \mid R_1 \circ R_2 \mid R^*$$

$\Sigma = \{a, b\}$

$$(a+b)^* aab = \{ waab \mid w \in \Sigma^* \}$$