Lecture 5:

Closure under complement

Nondeterministic finite automata

Are regular languages over Σ closed under complement?

$RegLang_\Sigma = \{ L_1, L_2, L_3, \ldots \}$

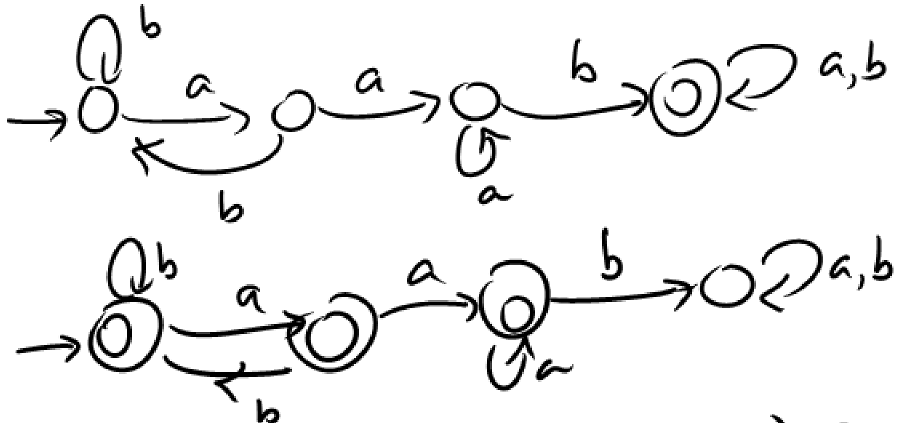
If L_1 and L_2 are in $RegLang_\Sigma$,

then $L_1 \cup L_2 \in RegLang_\Sigma$

$L_1 \cap L_2 \in RegLang_\Sigma$.

If $L \in RegLang_\Sigma$ then is $\bar{L} \in RegLang_\Sigma$.

$\bar{L} = \Sigma^* \setminus L$



Formally $A = (Q, \Sigma, \delta, q_0, F)$ DFA

$A' = (Q, \Sigma, \delta, q_0, F')$

$F' = Q \setminus F$

Lemma. $\forall w \in \Sigma^*.\ w \in L(A)$ iff $w \notin L(A')$

i.e. $L(A') = \Sigma^* \setminus L(A)$

Proof. δ is the same in A & A'

So δ^* is also the same.

$w \in L(A)$ iff $\delta^*(q_0, w) \in F$

iff $\neg(\delta^*(q_0, w) \notin F)$

iff $\neg(\delta^*(q_0, w) \in Q \setminus F)$

iff $\neg(\delta^*(q_0, w) \in F')$

iff $w \notin L(A')$

Concatenation

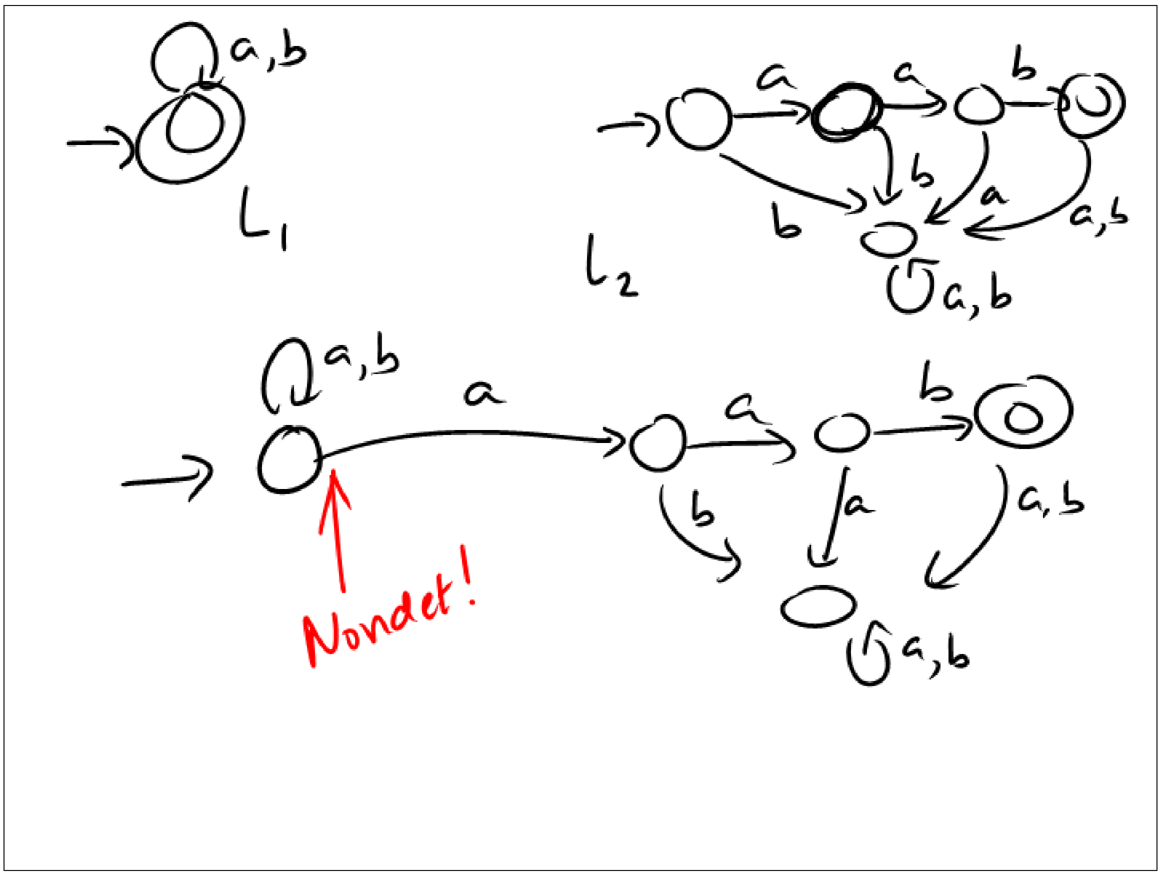
$L_1 \, , \, L_2 \subseteq \Sigma^*$

$$L_1 . L_2 = \left\{ w \in \Sigma^* \;\middle|\; \begin{array}{l} \exists w_1, w_2 \in \Sigma^* \\ w = w_1 w_2 , \\ w_1 \in L_1 , w_2 \in L_2 \end{array} \right\}$$

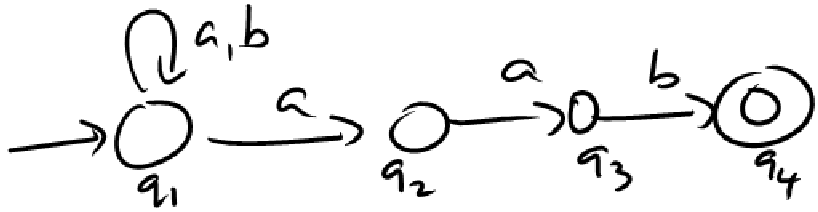
$L_1 = \{ a, bb, ab \}$

$L_2 = \{ aba , \epsilon \}$

$L_1 \circ L_2 = \{ aaba, a, bbaba, bb, ababa, ab \}$


a,b
L_1
a
a
b
b
b
a
a,b
a,b
L_2
a,b
a
a
b
b
a
a,b
a,b
Nondet!

Nondeterministic finite automata



ab a ab

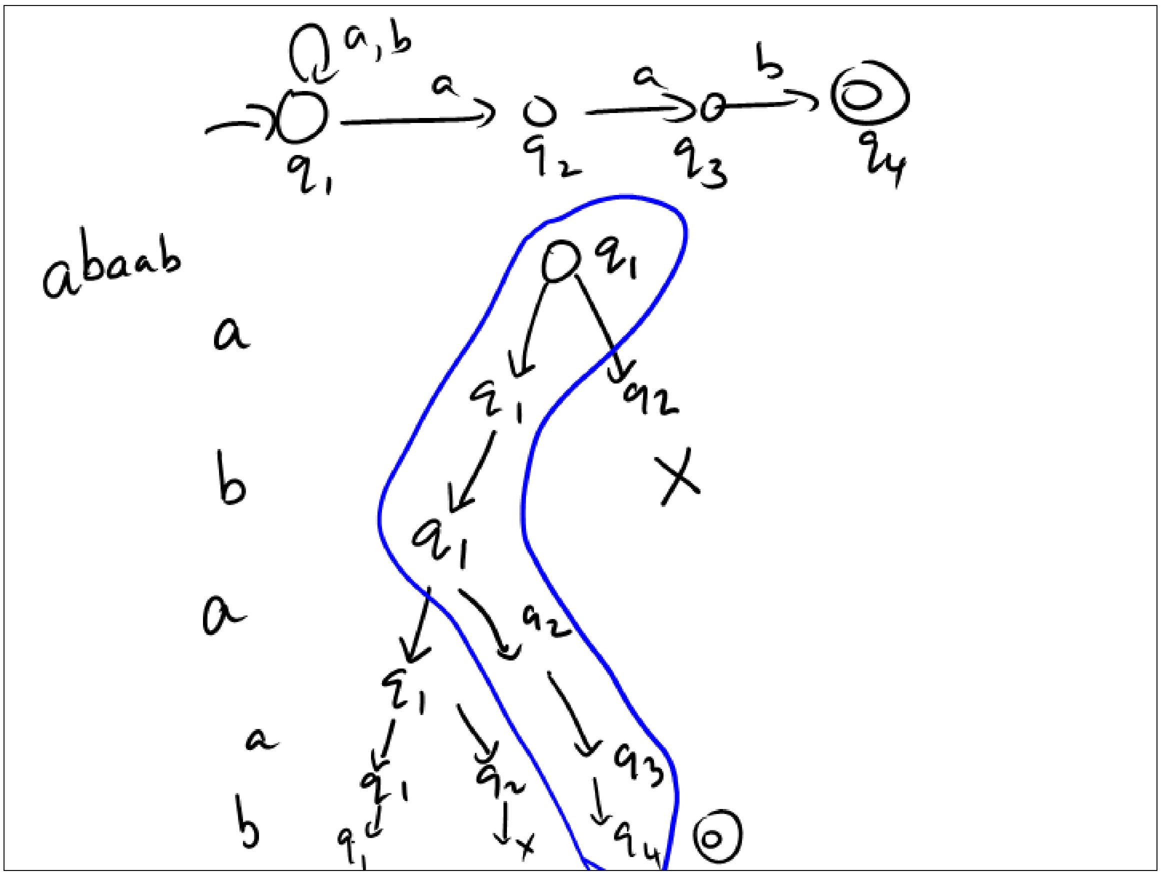
$q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \xrightarrow{b} q_4$

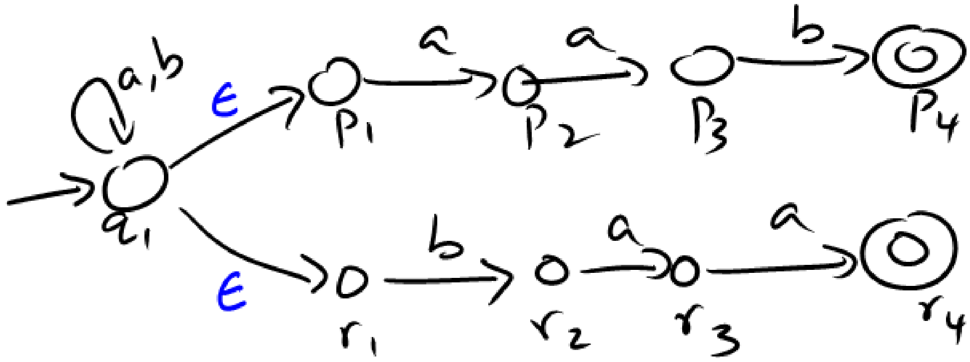
Accept.

$q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_1 \xrightarrow{a} q_1 \xrightarrow{a} q_3 \xrightarrow{a} q_4$

Reject.

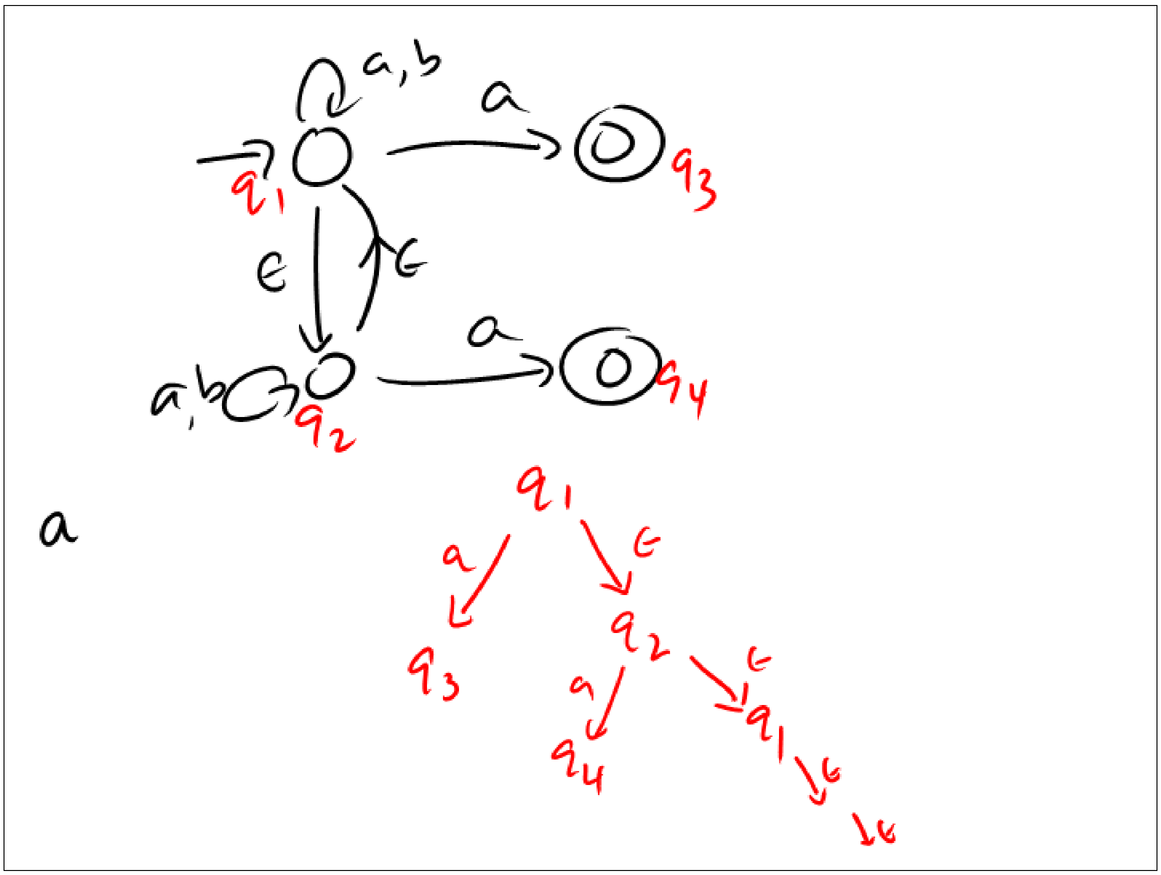
$q_1 \xrightarrow{a} q_2 \not\xrightarrow{b}$


a,b
a
a
b
q1
q2
q3
q4
abaab
a
b
a
a
b
q1
q1
q2
q1
q1
q2
q1
q2
q3
q1
q2
q1
q4



abaa

$$q_1 \xrightarrow{a} q_1 \xrightarrow{\epsilon} r_1 \xrightarrow{b} r_1 \xrightarrow{a} r_3 \xrightarrow{a} r_4$$


a,b
a
q_1
q_3
ε
ε
a
a,b
q_2
q_4
a
q_1
a
ε
q_3
q_2
a
ε
q_4
q_1

Defining NFAs

$$\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$$

NFA is a tuple $(Q, \Sigma, \delta, q_0, F)$

- Q is a finite set (of states)
- Σ is a finite set (alphabet)
- $q_0 \in Q$ (initial state)
- $F \subseteq Q$ (set of accepting/final states)
- $\delta : Q \times \Sigma_\epsilon \rightarrow 2^Q$

$\boxed{2^Q \equiv \mathcal{P}(Q)}$

For DFAs, $\delta : Q \times \Sigma \rightarrow Q$

$A = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$F = \{q_3\}$

δ: $\delta(q_0, a) = \{q_0, q_1\}$ $\delta(q_0, b) = \{q_0\}$ $\delta(q_0, \epsilon) = \emptyset$	. .
$\delta(q_1, a) = \emptyset$ $\delta(q_1, b) = \{q_2\}$ $\delta(q_1, \epsilon) = \{q_2\}$	. .

$N = (Q, \Sigma, \delta, q_0, F)$

$w \in \Sigma^*$

N accepts w if

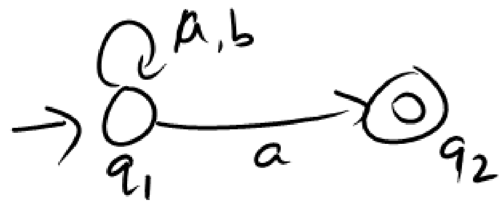
$w = y_1 y_2 \ldots y_m$ where each $y_i \in \Sigma_\epsilon$

and there is a sequence of states

$r_0, r_1, r_2, \ldots r_m$

where
- $r_0 = q_0$
- $r_m \in F$
- $r_{i+1} \in \delta(r_i, y_{i+1})$

DFA
$r_{i+1} = \delta(r_i, y_{i+1})$



aba is accepted:

aba = a b a

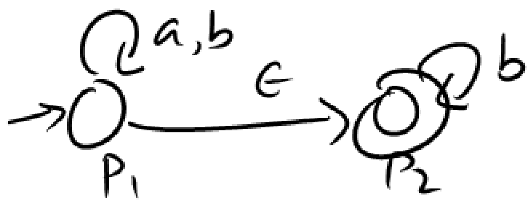
q_1, q_1, q_1, q_2

initial state

$q_1 \in \delta(q_p, a)$

$q_1 \in \delta(q_1, b)$

$q_2 \in \delta(q_1, a)$



abb

$abb = ab\epsilon b$

P_1, P_1, P_2, P_2