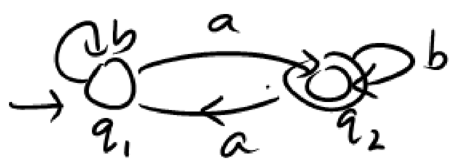
CS 373 : Lecture #4

Closure properties,
product construction



$\Sigma = \{a, b\}$

$A = (Q, \Sigma, \delta, q_1, F)$

$Q = \{q_1, q_2\}$

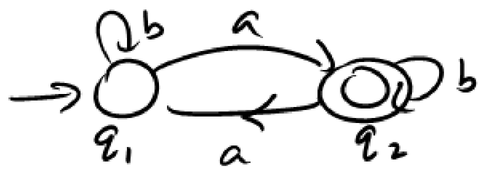
$\Sigma = \{a, b\}$

$F = \{q_2\}$

δ :

	a	b
q_1	q_2	q_1
q_2	q_1	q_2

A



Why is abaa accepted by A.

abaa is accepted because there is

this seq. $\cancel{p_0, p_1, p_2, p_3, p_4}$

i.e. $q_\phi, q_2, \underline{\underline{q_2}}, \underline{q_1}, q_2$

q_1 - initial state

$q_2 = \delta(q_1, a)$; $q_2 = \delta(q_2, b)$

$q_1 = \delta(q_2, a)$; $q_2 = \delta(q_1, a)$

$q_2 \in F$.

Regular languages over Σ

$\equiv$ the class of languages accepted by DFA over Σ.

$L_1, L_2 \in \text{Reg-lang}(\Sigma)$

$\Rightarrow L_1 \cup L_2 \in \text{Reg-lang}(\Sigma)$

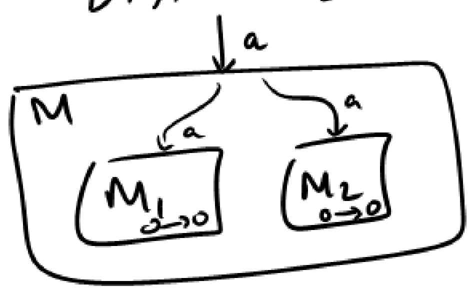
$\Rightarrow L_1 \cap L_2 \in \text{Reg-lang}(\Sigma)$

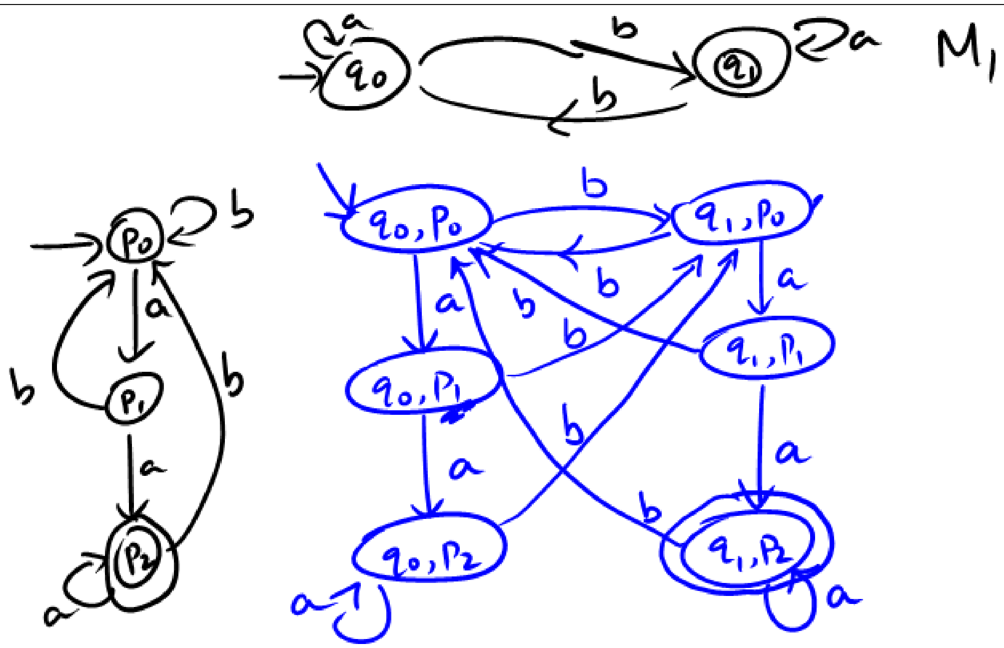
$\Rightarrow \overline{L_1} \in \text{Reg}(\Sigma)$.

Question. If L_1 and L_2 are regular languages over Σ, is $L_1 \cap L_2$ a regular language over Σ?

L_1 - DFA $M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$

L_2 - DFA $M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$





If $F = \{(q_0, p_2), (q_1, p_2), (q_1, p_0), (q_1, p_1)\}$
then automaton accepts the union.

Product construction

Let $M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$

$M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$

Then $N = (Q_1 \times Q_2, \Sigma, \delta, (q_0^1, q_0^2), F)$

is a product automaton of M_1 and M_2 if

$$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$$

and $F \subseteq Q_1 \times Q_2$

For intersection : $F = F_1 \times F_2$

For union : $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$

$$A = (Q, \Sigma, \delta, q_0, F)$$

$\delta^*(q, w)$: the state you land up in when you start from q and read w.

<u>Def</u>

$$\delta^*(q, \epsilon) = q$$

$$\delta^*(q, wa) = \delta(\delta^*(q, w), a)$$

$$q \xrightarrow{w} q_1 \xrightarrow{a} q_2$$

When is w accepted by A?

$$\delta^*(q_0, w) \in F.$$

w is rejected by A if

$$\delta^*(q_0, w) \notin F.$$

$M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$

$M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$

$N = (Q_1 \times Q_2, \Sigma, \delta, (q_0^1, q_0^2), F)$ PA

Lemma. $\forall w \in \Sigma^*.$

$$\delta^*((q_0^1, q_0^2), w) = (\delta_1^*(q_0^1, w), \delta_2^*(q_0^2, w))$$

Induction on length of w

$|w| = 0$ Base-case : i.e. $w = \epsilon$

$$\delta^*((q_0^1, q_0^2), \epsilon) = (q_0^1, q_0^2) \quad \text{(by def of } \delta^*)$$

$$= (\delta_1^*(q_0^1, \epsilon), \delta_2^*(q_0^2, \epsilon)) \quad \text{(by def of } \delta_1^* \text{ and } \delta_2^*)$$

<u>Inductive step</u>. $|w| > 0$ So $w = w_1 a$

$\delta^*((q_0^1, q_0^2), w_1 a) = \delta(\delta^*((q_0^1, q_0^2), w_1), a)$

by def by δ^*

$= \delta((\delta_1^*(q_0^1, w_1), \delta_2^*(q_0^2, w_1)), a)$

by inductive hypothesis, since $|w_1| < |w|$.

$= (\delta_1(\delta_1^*(q_0^1, w_1), a), \delta_2(\delta_2^*(q_0^2, w_1), a))$

by def. of δ in prod. const.

$= (\delta_1^*(q_0^1, w_1 a), \delta_2^*(q_0^2, w_1 a))$ by def. of δ_1^*, δ_2^*

$= (\delta_1^*(q_0^1, w), \delta_2^*(q_0^2, w))$

<u>Lemma</u>. If $M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$

and $M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$

and $N_1 = (Q_1 \times Q_2, \Sigma, \delta, (q_0^1, q_0^2), F_1 \times F_2)$

and $N_2 = (Q_1 \times Q_2, \Sigma, \delta, (q_0^1, q_0^2), (Q_1 \times F_2) \cup (F_1 \times Q_2))$

then $L(N_1) = L(M_1) \cap L(M_2)$

and $L(N_2) = L(M_1) \cup L(M_2)$.

$\cap$ $w \in L(N_1)$ iff $\delta^*((q_0^1, q_0^2), w) \in F_1 \times F_2$

iff $(\delta_1^*(q_0^1, w), \delta_2^*(q_0^2, w)) \in F_1 \times F_2$

iff $\delta_1^*(q_0^1, w) \in F_1$ and $\delta_2^*(q_0^2, w) \in F_2$

iff $w \in L(M_1)$ and $w \in L(M_2)$ $\square$