CS373 Lec#3

Languages, machines, finite-state automata

Σ - finite alphabet

$L \subseteq \Sigma^*$ $\qquad$ Σ^* - set of all strings over Σ.

$L(M) = \{ x \in \Sigma^* \mid M \text{ says "YES" on input } x \}$

What kinds of languages are computable by machines?

$\Sigma = \{a, b\}$

$L = \{ x \mid x \text{ ends in aab} \}$

$= \{ aab, aaab, baab, \ldots \}$

$= \{ x \mid \exists z \in \Sigma^* . \; x = zaab \}$

$L = \{ 0^n \mid n \text{ is prime} \}$

$\underbrace{0 \ldots 0}_{n}$

Lin prgm.

$$\left[\begin{array}{l} \text{Max } \underline{3x+4y} \\ x \le 4y+3z \quad , v_x \quad , v_y \\ y > 0 \\ \vdots \end{array}\right]$$

$\langle v_x, v_y \rangle$ maximizes the obj. fn under the constraints.

string.

$\langle \text{Max}[3x+4y] ; x \le 4y+3z ; y>0, v_x, v_y \rangle$

$\longrightarrow \boxed{M} \rightarrow$ Yes / $\rightarrow$ No.

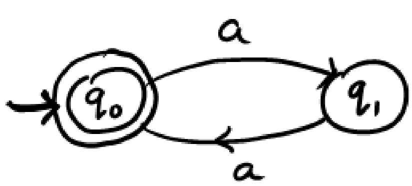
$\Sigma = \{a\}$

$L = \{w \in \Sigma^* \mid |w| \text{ is even}\}$

```
main () {
    n := 0;
    while (there are letters to read) {
        read a letter
        n := n+1;   (n := (n+1) % 2
                     or  n := 1 - n)
    }
    if (n % 2 == 0)
        say Yes
    else
        say No.
```

$$L = \{a^n \mid n \text{ is even}\}$$
$$= \{w \mid |w| \text{ is even}\}$$

$a^i = aaa\ldots a$

A: 

$\Sigma = \{a\}$

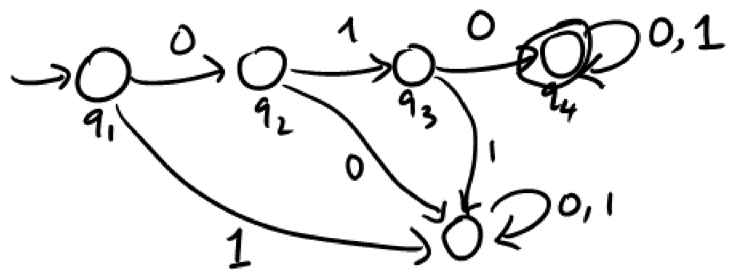
aaa $\quad q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_0 \xrightarrow{a} q_1$ reject

aa $\quad q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_0$ accept.

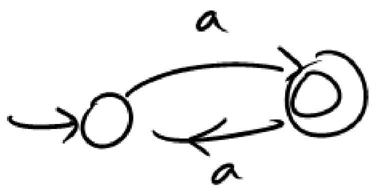
$$L(A) = \{\cancel{w \mid |w| \text{ is even}}\}$$
$$= \{a^n \mid n \text{ is even}\}$$

$$\Sigma = \{0,1\}$$

$$L = \{\, 010w \mid w \in \Sigma^* \,\}$$



Deterministic finite automata.

A 

$L_1 = \{ a^i \mid i$ is odd $\}$.

$L_1 = \{ a^{4i+1} \mid i \in \mathbb{N} \}$

$= \{ \underbrace{aaaaa}_{5}, \underbrace{aaaaaaaaa}_{9}, \dots \}$

$L(A) = L_1$

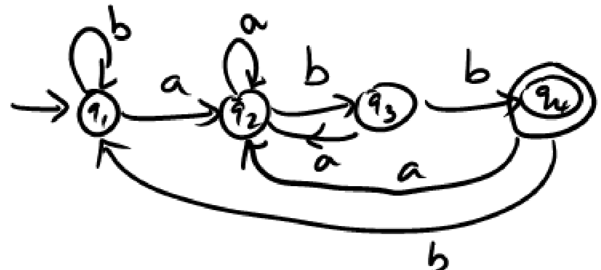
$L(A) \neq L_2$

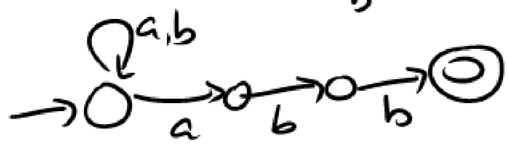
Is $L = \{ a^p \mid p \text{ is prime} \}$.

$\Sigma = \{a, b\}$

Eg. $L = \{ w \mid w \text{ ends in } abb \}$

$= \{ w \mid \exists z \in \Sigma^* .\ w = z\,abb \}$

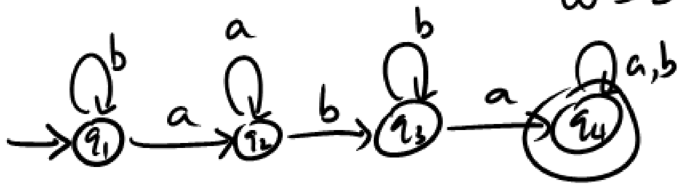


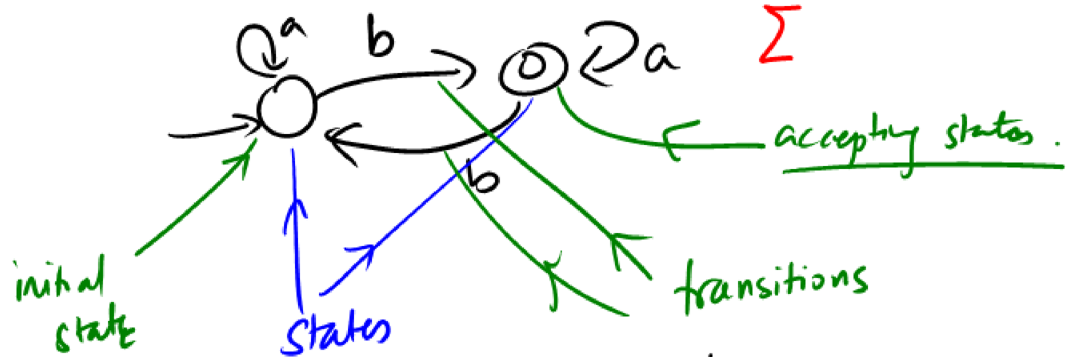


X Non-deterministic

$$L = \{ w \mid w \text{ has a } \underline{\text{subseq}} \text{ aba} \}$$

$$= \{ w \mid \exists z_1, z_2, z_3, z_4 \in \Sigma^* \quad w = z_1 a z_2 b z_3 a z_4 \}$$





A finite automaton is a tuple

$(Q, \Sigma, \delta, q_0, F)$

where Q is a finite set (of states)

Σ is a finite set (alphabet)

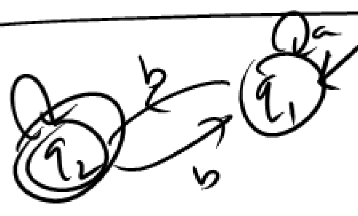
$q_0 \in Q$ - (initial state)

$F \subseteq Q$ - (set of final/accept states)

$\delta : Q \times \Sigma \to Q$ (transition function)

$$A = (\{q_1, q_2\}, \{a, b\}, \delta, q_1, \{q_2\})$$

$$\delta(q_1, a) = q_1$$
$$\delta(q_1, b) = q_2$$
$$\delta(q_2, a) = q_2$$
$$\delta(q_2, b) = q_1$$



$$A = (Q, \Sigma, \delta, q_0, F) \qquad \delta : Q \times \Sigma \to Q$$

A word $w \in \Sigma^*$ ($w = a_1 \dots a_n$) is <u>accepted</u> by A

if there is a sequence of states

$$p_0, p_1, p_2, \dots p_n$$

such that $p_0 = q_0$

$$p_{i+1} = \delta(p_i, a_{i+1}) \qquad \forall i \in \{0, \dots n-1\}$$

and $p_n \in F$

The language accepted by A is

$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid w \text{ is accepted by } A \}$$

Machine
Class
DFA

- What languages are accepted by these machines?
- What languages <u>cannot</u> be accepted by these machines?
- Alternate characterizations.