Lecture #24

Closure properties of CFLs

Recursive automata

Pushdown automata.

$G_1 = (V_1, \Sigma, P_1, S_1)$ $\quad V_1 \cap V_2 = \emptyset$

$G_2 = (V_2, \Sigma, P_2, S_2)$

- $L(G_1) \cdot L(G_2)$ is a CFL? Yes.

$$G = (V_1 \cup V_2 \cup \{S\}, \Sigma, P_1 \cup P_2 \cup \{S \to S_1 S_2\}, S)$$

- $(L(G_1))^*$ is a CFL?

$$G = \left(V_1 \cup \{S\}, \Sigma, P_1 \cup \left\{\begin{matrix} S \to \epsilon \\ S \to SS_1 \end{matrix}\right\}, S\right)$$

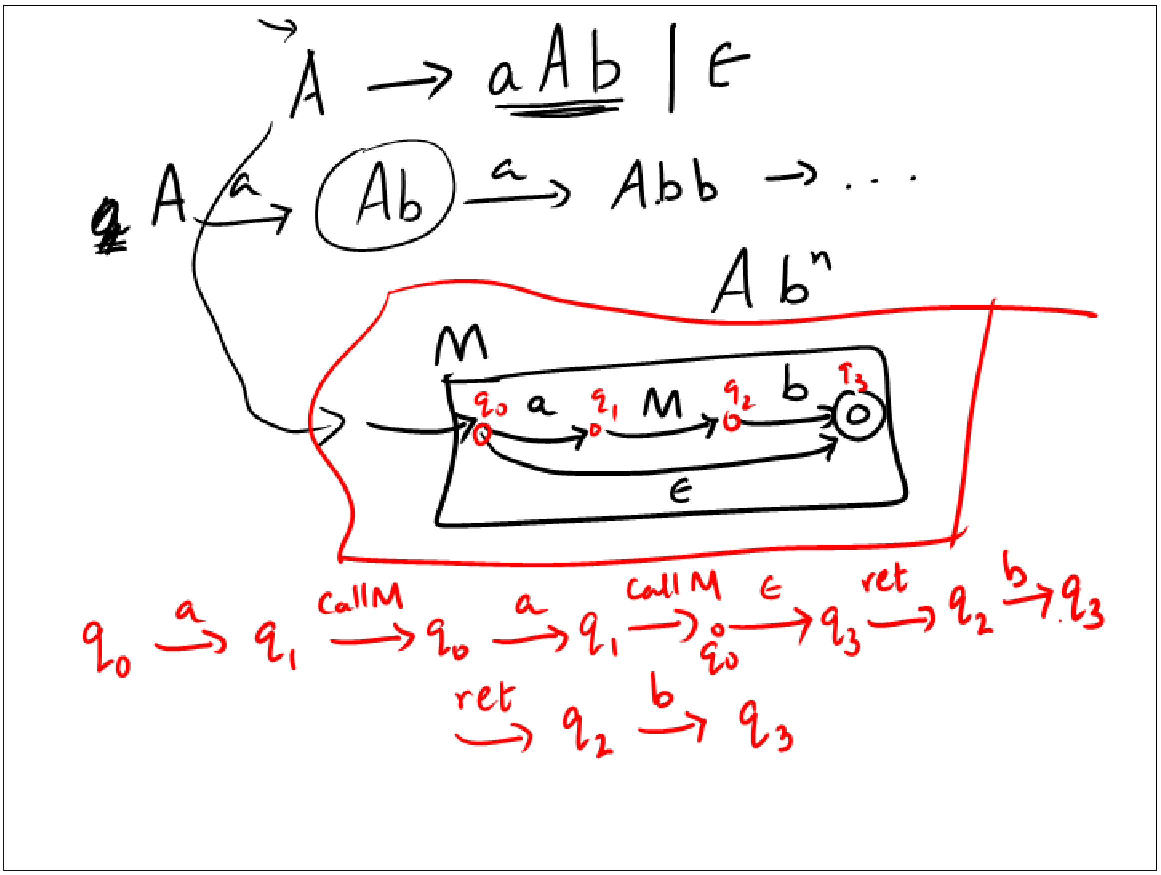
$S \to SS \mid S_1 \mid \epsilon$

- $h : \Sigma \to \Pi^*$

$h(L(G_1))$ a CFL? Replace every "a" on right-hand side of rules by $h(a)$.

- $(L(G_1))^R$ a CFL? Yes

  Reverse right-hand side of every rule.

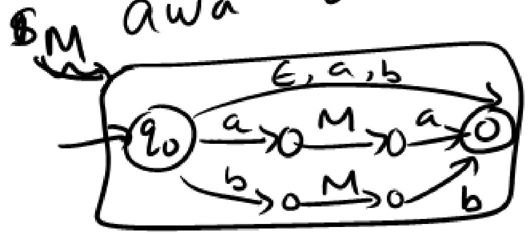


A → aAb | ϵ
A → Ab → Abb → . . .
Abⁿ
M
q0 a q1 M q2 b q3
ϵ
q0 → q1 → q0 → q1 → q0 → q3 → q2 → q3
a callM a callM ϵ ret b
ret → q2 → q3
b

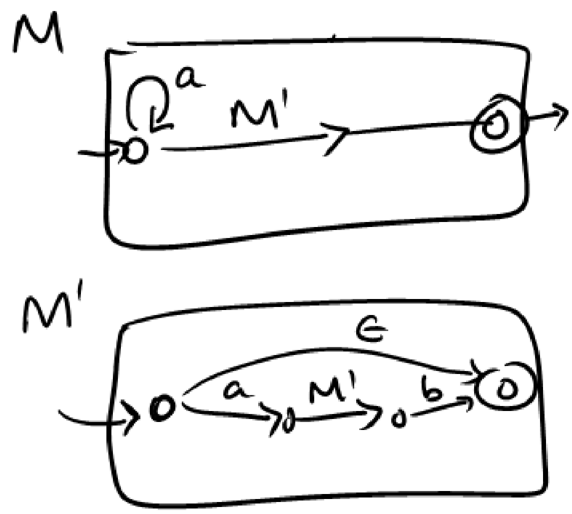
Palindromes: $\Sigma = \{a, b\}$

ϵ, a, b

If w is a palindrome then awa and bwb are palindromes



$$L = \{ a^i b^j \mid \cancel{a}\, i \geq j \}$$



A recursive automaton over Σ is a tuple

$$(M, \text{main}, \{A_m\}_{m \in M})$$

M - finite set of module names

$\text{main} \in M$ - initial module

A_m is a NFA over $\Sigma \cup M$.

$$A_m = (Q_m, \Sigma \cup M, \delta_m, q_0^m, F_m)$$

$$\delta_m : Q_m \times (\Sigma \cup M \cup \{\epsilon\}) \to 2^{Q_m}.$$

$$Q_m \cap Q_{m'} = \phi \quad \forall m, m' \in M, m \neq m'.$$

Configuration of the RA

is a pair (q, s)

where $q \in \bigcup_{m \in M} Q_m$

and $s \in \left(\bigcup_{m \in M} Q_m \right)^*$

A word w is accepted by RA

if $w = y_1 \ldots y_k$ $\quad (y_i \in \Sigma \cup \{\epsilon\})$

and there is a sequence of configurations

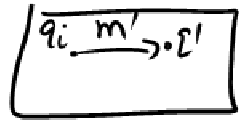
$(q_0, s_0), (q_1, s_1), \ldots (q_k, s_k)$

where

- (q_0, s_0) : $s_0 = \epsilon$ $\quad q_0$: initial state of main
- (q_k, s_k) : $s_k = \epsilon$ $\quad q_k \in$ Final state of main
- $\forall i < k$:
  - Internal: $\delta_m(q_i, y_{i+1}) = q_{i+1}$ $\quad q_i \in Q_m$

    $s_{i+1} = s_i$

- **Call** $q_i \in Q_m$, $y_{i+1} = \epsilon$



$$\delta_m(q_i, m') = q'$$

$$q_{i+1} = q_0^{m'}$$

$$S_{i+1} = q' S_i$$

- **Return** $q_i \in Q_{m'}$ and $q_i \in F_{m'}$

$$S_i = q' S$$

$$q_{i+1} = q' \quad S_{i+1} = S$$

$$y_{i+1} = \epsilon$$

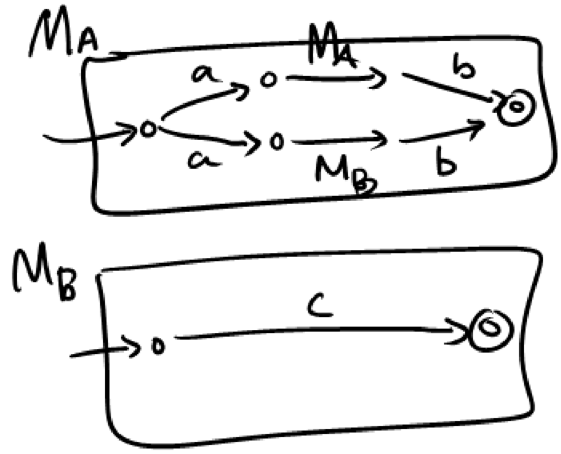
$L(A) = \{ \underset{\in \Sigma^*}{w} \mid w \text{ is accepted by } A \}$

CFLs $\equiv$ Lang. acc by RAs

CFG $\hookrightarrow$ RAs

$$A \to a\underline{Ab} \mid a\underline{Bb}$$

$$B \to c$$



Recursive automata to CFGs.

$q \xrightarrow{a} q'$ ✓

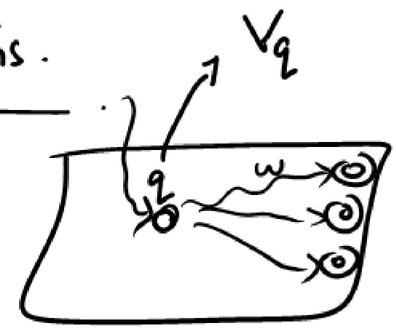
$V_q \longrightarrow a V_{q'}$

$q \xrightarrow{\epsilon} q'$ ✓

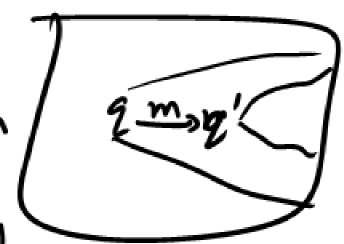
$V_q \longrightarrow V_{q'}$

$q \xrightarrow{m} q'$ ✓

$V_q \longrightarrow V_{q_0^m} V_{q'}$

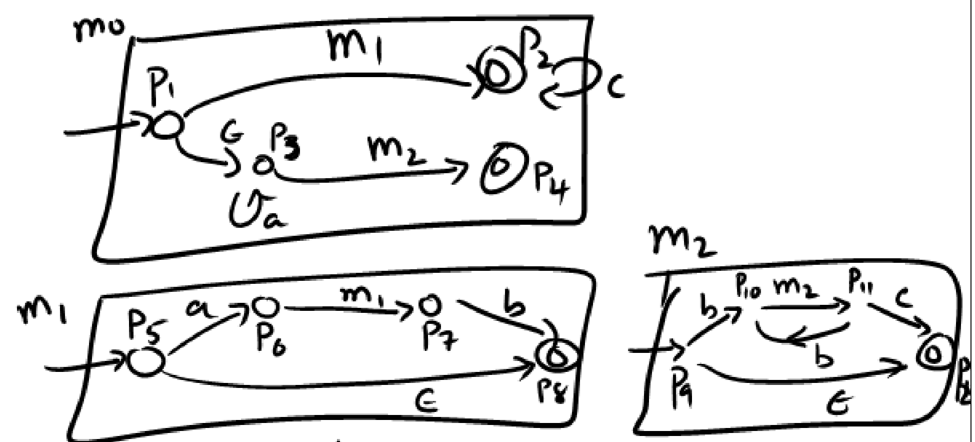


Start var: $V_{q_0^{main}}$



For every final state p

$V_p \to \epsilon$



$$V_{p_1} \rightarrow V_{p_3} \mid V_{p_5} V_{p_2}$$

$$V_{p_2} \rightarrow c\, V_{p_2} \mid \epsilon$$

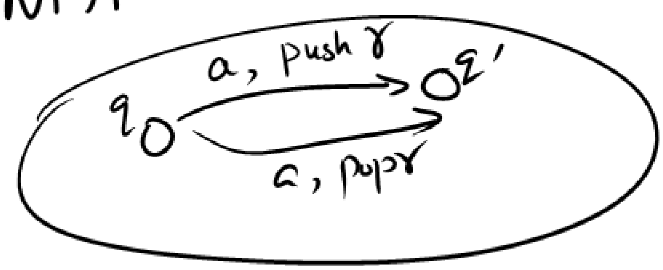
$$V_{p_3} \rightarrow a\, V_{p_3} \mid V_{p_9} V_{p_4}$$

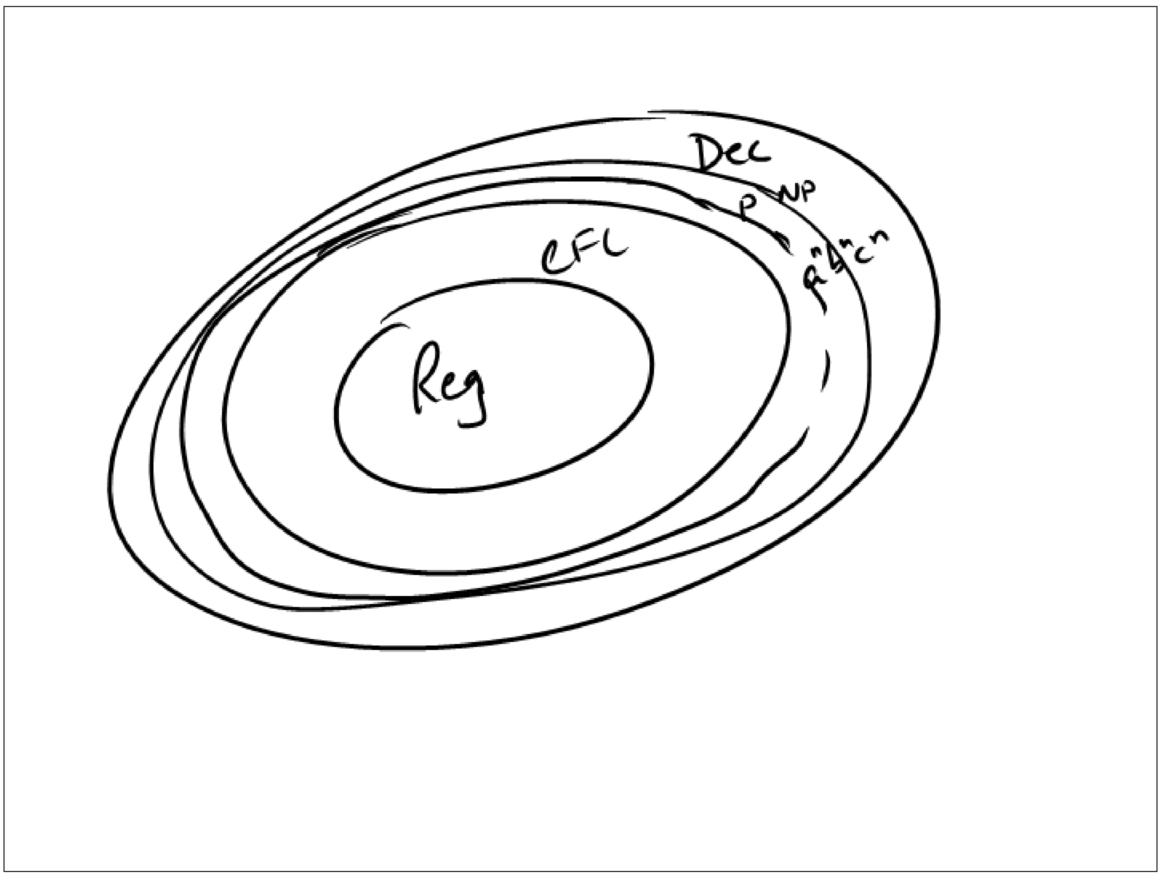
$$V_{p_4} \rightarrow \epsilon$$

. . .

Pushdown automata.

NFA + stack




Dec
P
NP
CFL
$a^n b^n c^n$
Reg