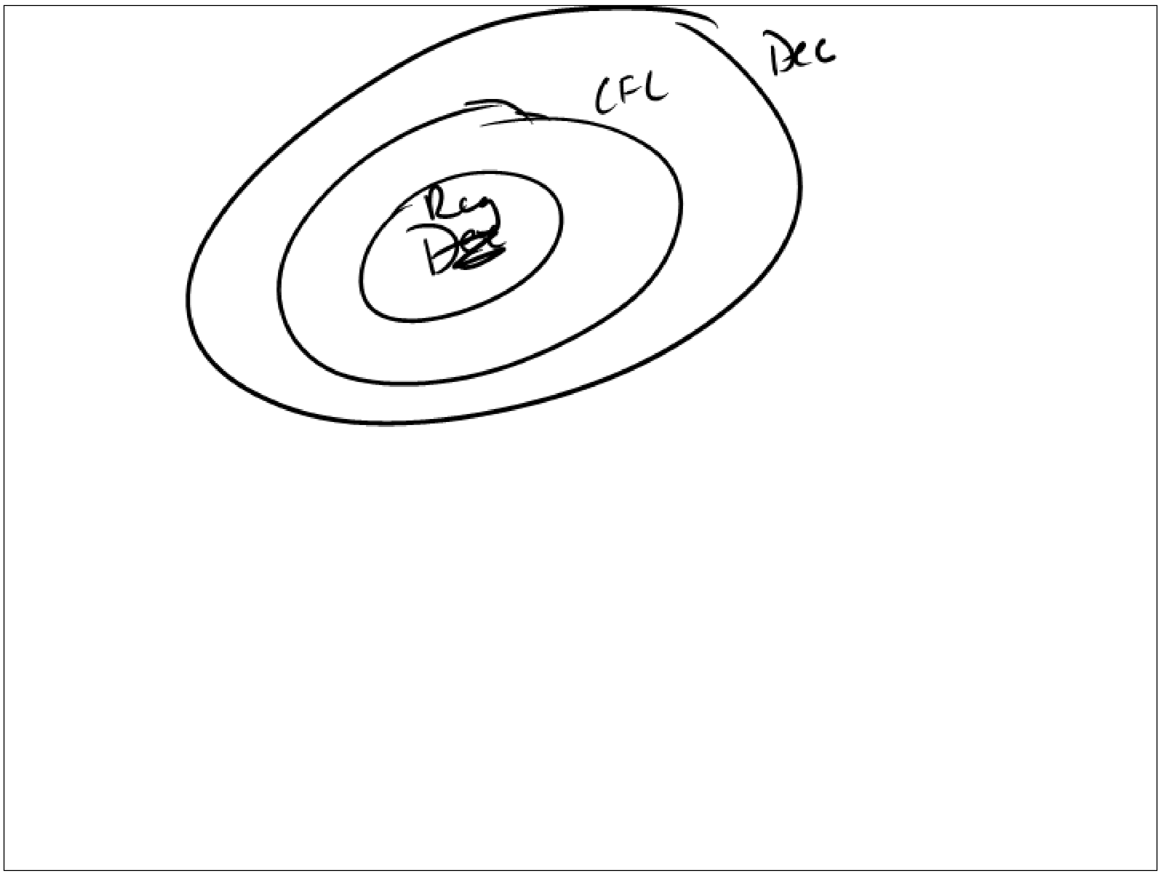
Context-free languages



Dec
CFL
Reg
Dec

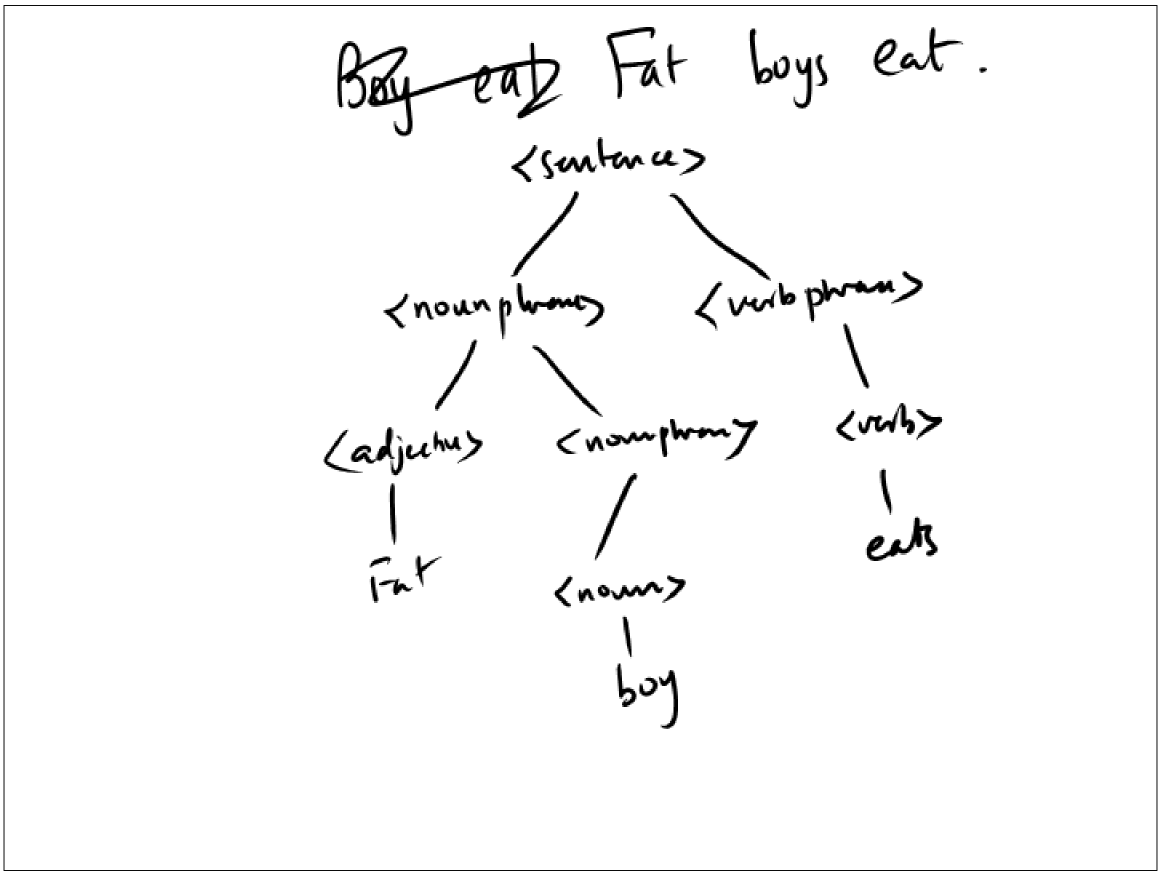
<Sentence> :-> <noun phrase> <verb phrase>

<nounphrase> :- <adjective> <nounphrase>

<nounphrase> :- <noun>

<noun> :- boy | girl | cat | dragon | ...

<adjective> :- big | little | ugly | ...


~~Boy eat~~ Fat boys eat.
<sentence>
<nounphrase>
<verbphrase>
<adjective>
<nounphrase>
<verb>
Fat
<noun>
eats
boy

<u>Well-bracked words</u>

$$\Sigma = \{ (\, ,\,) \}$$

$L = \{$ well-bracketed expressions: $()$, $()()$, $(())$, $(()())$, $\ldots \}$

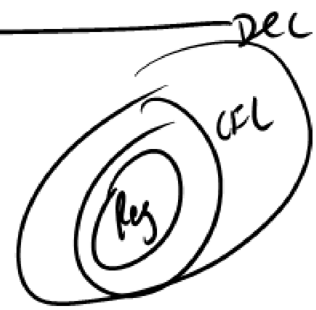
$((\notin L$

L is <u>not</u> regular.

L is the smallest set of words s.t.

- $\varepsilon \in L$
- If $x \in L$, $(x) \in L$
- If $x, y \in L$, $xy \in L$

$$S \to \varepsilon$$
$$S \to (S)$$
$$S \to SS$$



$$L = \{ w \mid w^R = w \}$$

$$S \rightarrow \epsilon \mid aSa \mid bSb \mid a \mid b$$

Context-free grammar.

$$G = (V, \Sigma, P, S)$$

$\left.\begin{array}{l} V \text{ - nonterminals} \\ \Sigma \text{ - finite alphabet (terminals)} \end{array}\right\} \begin{array}{c} V \cap \Sigma \\ = \emptyset \end{array}$

P - set of productions/rules

$$P \subseteq_{fin} V \times (\Sigma \cup V)^*$$

$S \in V$ - start non-terminal.

$$S \to 0S0$$
$$S \to T$$
$$T \to 1T1$$
$$T \to a$$

$$S \to 0S0 \mid T$$
$$T \to 1T1 \mid a$$
$$L = \{0^i 1^j a 1^j 0^i \mid i,j \geq 0\}$$

$$G = (V, \Sigma, P, S)$$
$$V = \{S, T\}$$
$$\Sigma = \{0, 1, a\}$$
$$P = \{(S, 0S0), (S, T), (T, 1T1), (T, a)\}$$

$G = (V, \Sigma, P, S)$

Yields-relation.

$$\alpha X \beta \Rightarrow_G \alpha \gamma \beta \quad \text{if} \quad X \rightarrow \gamma \in P$$

(i.e. $(X, \gamma) \in P$)

$\Rightarrow_G^*$: reflexive transitive closure of $\Rightarrow$

$\alpha \Rightarrow^* \beta$ if $\exists n \geq 0$

$$\alpha = \alpha_0 \Rightarrow \alpha_1 \Rightarrow \alpha_2 \dots \Rightarrow \alpha_n = \beta$$

$$L(G) = \{ w \in \Sigma^* \mid S \Rightarrow_G^* w \}$$

$$S \to \underline{0S0} \mid 1$$

$$L = \{ 0^n 1 0^n \mid n \geq 0 \}$$

00100

$$S \Rightarrow 0\underline{S}0 \Rightarrow 0\,\underline{0S0}\,0 \Rightarrow 00100$$

So $00100 \in L$.

$$S \Rightarrow^* 00100$$

$0010 \in L$? $\quad 0010 \notin L$.

L = well-formed Boolean expressions over T, F.

$\Sigma = \{ (,), T, F, \vee, \wedge, \neg \}$

$(T \vee F)$ $\quad (T \vee (F \wedge F))$ ....

$\neg(F)$ $\quad \neg(F) \vee T$

$$S \to T \mid F \mid (S \vee S) \mid \neg(S) \mid (S \wedge S) \mid (S)$$

$f : \mathbb{N} \to \{T, F\}$

Boolean expressions over arithmetic terms

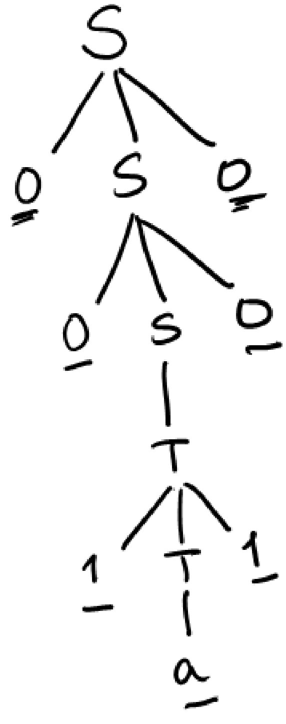
$$\Sigma = \{f, 0, 1, \ldots 9, +, *, \wedge, \vee, \neg, (,)\}$$

$$S \to f(A) \mid T \mid \perp \mid (S \vee S) \mid (S \wedge S) \mid \neg(S) \mid (S)$$

$$A \to 0 \mid 1 \mid \ldots 9 \mid (A + A) \mid (A * A) \mid (A)$$

$$S \Rightarrow^* \text{"} f(((1+2)*3)) \vee f(5) \text{"}$$

Parse trees



$S \to 0S0 \mid T$

$T \to 1T1 \mid a$

$L = \{ 0^i 1^j a 1^j 0^i \mid i,j \geq 0 \}$

001a100

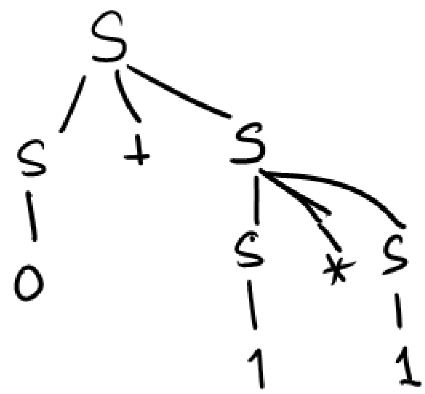
$$S \rightarrow 0 \mid 1 \mid S+S \mid S*S$$

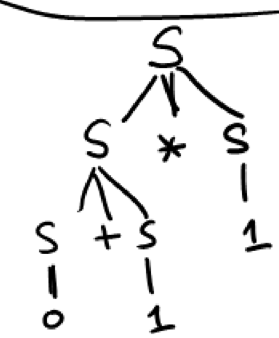
$$0+1*1$$

$$S \Rightarrow S+S \Rightarrow 0+S \Rightarrow 0+S*S$$

$$\Rightarrow 0+1*S$$

$$\Rightarrow 0+1*1$$





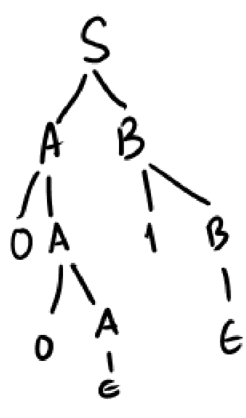
$$S \to AB$$
$$A \to 0A \mid \epsilon$$
$$B \to 1B \mid \epsilon$$

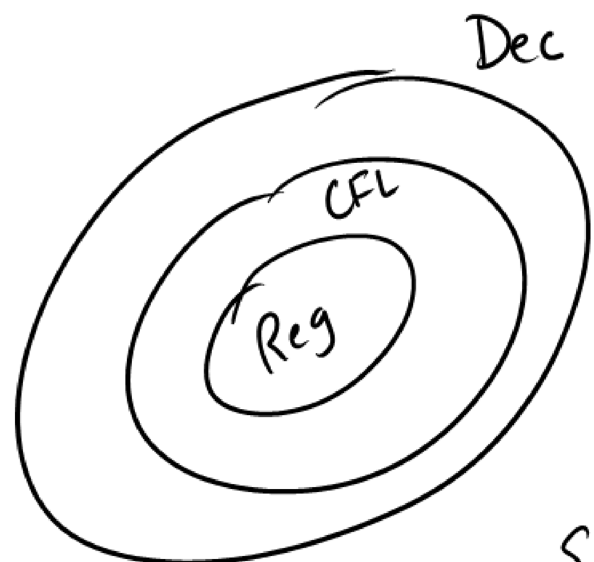
$L = \{0^* 1^*$

001

$$S \Rightarrow AB \Rightarrow 0AB \Rightarrow 00AB$$
$$\Rightarrow 00B \Rightarrow 001B \Rightarrow 001$$

$$S \Rightarrow AB \Rightarrow A1B \Rightarrow A1$$
$$\Rightarrow 0A1 \Rightarrow 00A1 \Rightarrow 001$$





$\underset{\sim}{\text{Reg}} \subsetneq \underline{\text{CFL}} \subsetneq \text{Dec}$

$\qquad\quad\;\; \|$

$\quad\;\;$ PA/RA

$\{a^n b^n c^n \mid n \geq 0\}$

↳ Dec but <u>not</u> CF.