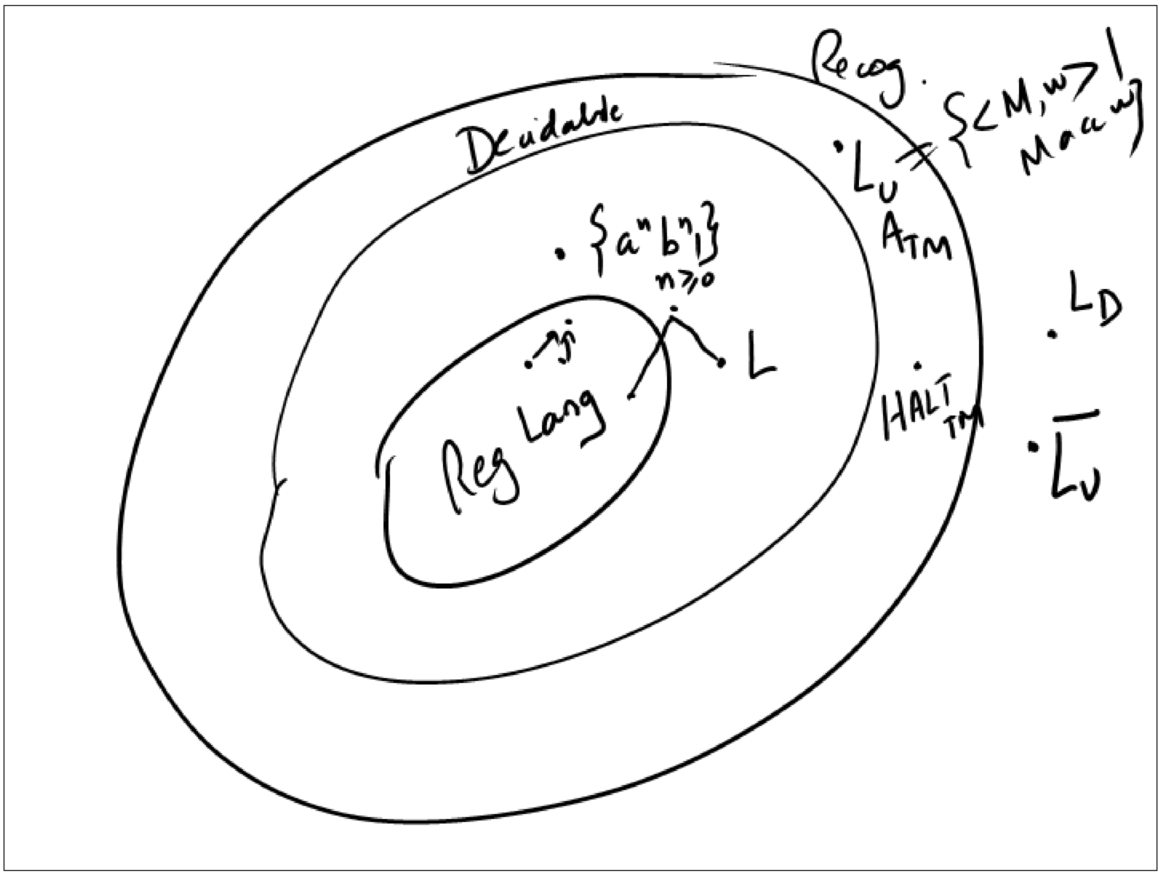
Lecture # 17

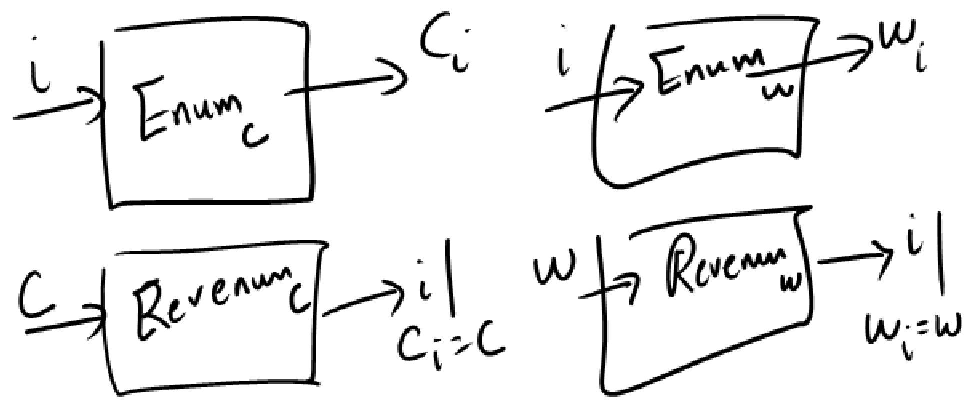
Reductions


Recog.
Decidable
Reg Lang
$\{a^n b^n \mid n \geq 0\}$
L
$L_U = \{\langle M, w\rangle \mid M \text{ acc } w\}$
A_{TM}
$HALT_{TM}$
L_D
$\overline{L_U}$

Let $L_U^C = \{ \langle C, w \rangle \mid C$ is a C-program that accepts $w \}$

$C_1, C_2, C_3, \ldots\ldots$

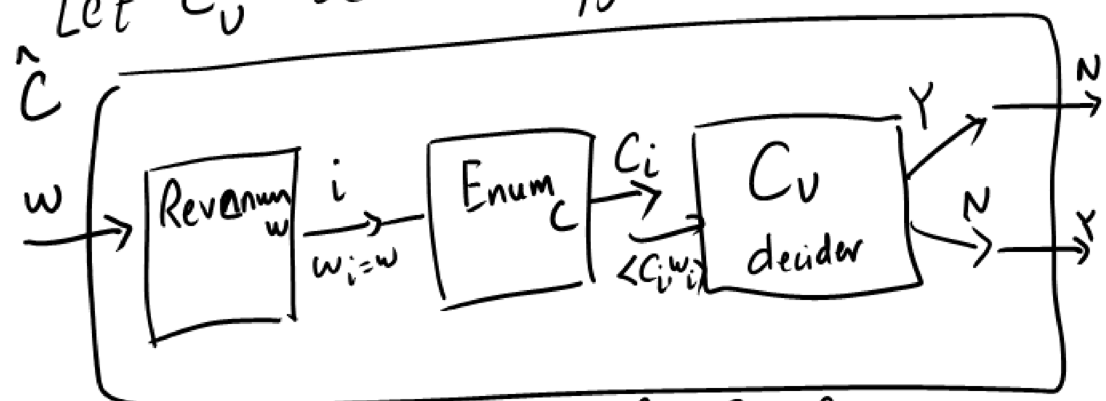
$w_1, w_2, \ldots\ldots$



$L_U = \{\langle C, w\rangle \mid C$ is a C-pgm that acc $w\}$

cannot be decided by any C-pgm.

Let C_U be a C-pgm deciding L_U.



Since $\hat{C}$ is a C-pgm, $\hat{C} = C_j$ for some j.

w_j cannot be accepted or rejected by $\hat{C}$.

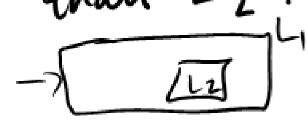
$$HALT_{TM} = \{\langle M, w\rangle \mid M \text{ halt on } w\}$$

L_1 reduces to L_2

if given a decider for L_2,
there is a decider for L_1.

L_1 is solvable if L_2 is solvable.

L_1 is not harder than L_2.



P1 : Find the minimum number in a list L of numbers.

P2 : Sort a list of numbers.

L_1 reduces to L_2

If L_2 is decidable, then L_1 is decidable.

If L_1 is undecidable, then L_2 is undecidable.

L_U is undecidable — I know this

L is undecidable?

L reduces to L_U

L coulde be decidable

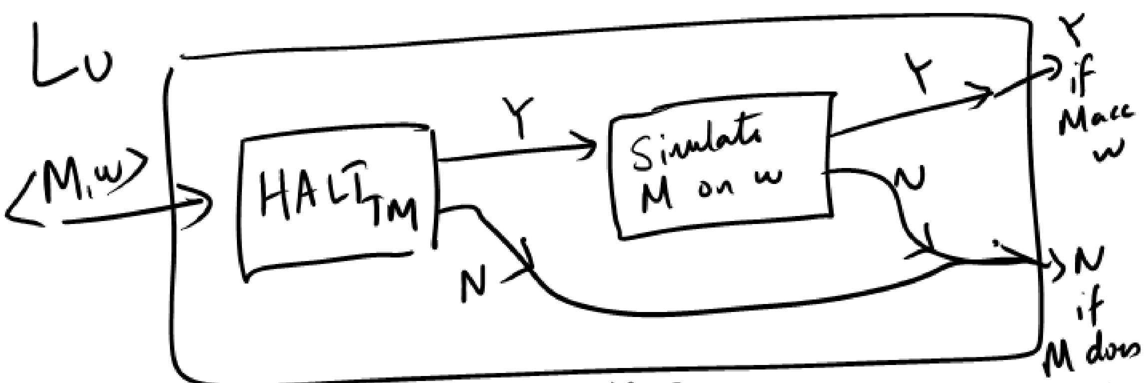
L_U reduces to L

Shows L is undecidable.

$HALT_{TM} = \{\langle M,w\rangle \mid M$ ~~acc~~ halts on $w\}$ is undecidable.

L_U reduces to $HALT_{TM}$.

$L_U = \{\langle M,w\rangle \mid M \text{ acc } w\}$



Hence L_U reduces to $HALT_{TM}$ & hence $HALT_{TM}$ is undecidable

So $\{a\}$ reduces to L_u

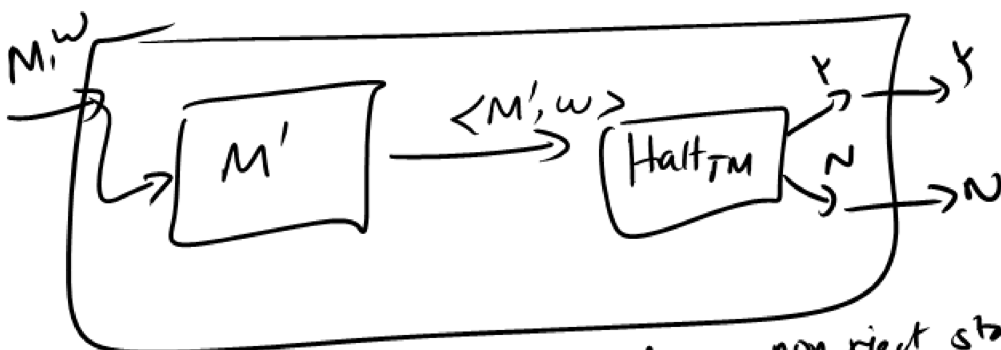
TM deciding $\{a\}$

A reduces to B trivially

if A is known to be decidable
or B is known to be undecidable.

$$HALT_{TM} = \{ \langle M, w \rangle \mid M \ \underset{\text{halts on}}{\underline{\text{acc}}} \ w \}$$

L_U



M': M with q_{rej} made a non-reject state

q_{rej} self-loops on q_{rej},

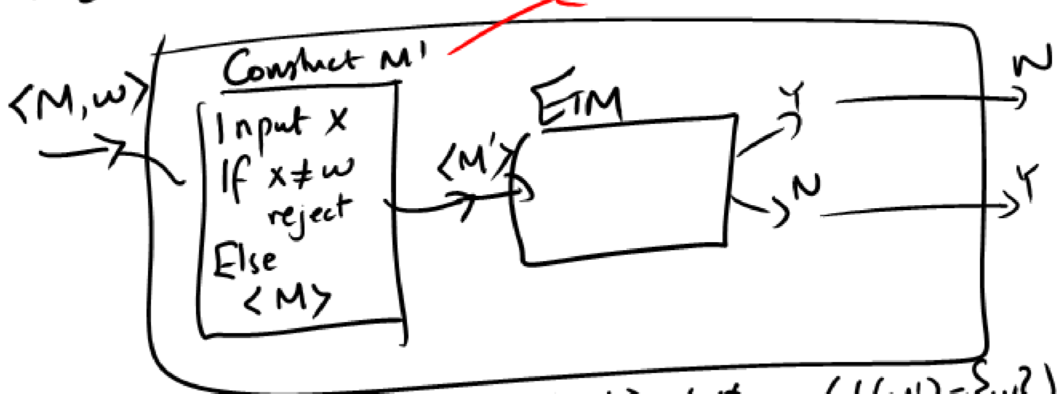
Add a new q_{rej}' state that is not reachable

$$E_{TM} = \{ \langle M \rangle \mid L(M) = \phi \}$$

is undecidable

L_U reduces E_{TM}

M' depends on w



M acc w $\Rightarrow \cancel{\text{if}}$ $L(M') \neq \phi$ $\quad (L(M') = \{w\})$

M does not acc w $\Rightarrow L(M') = \phi$