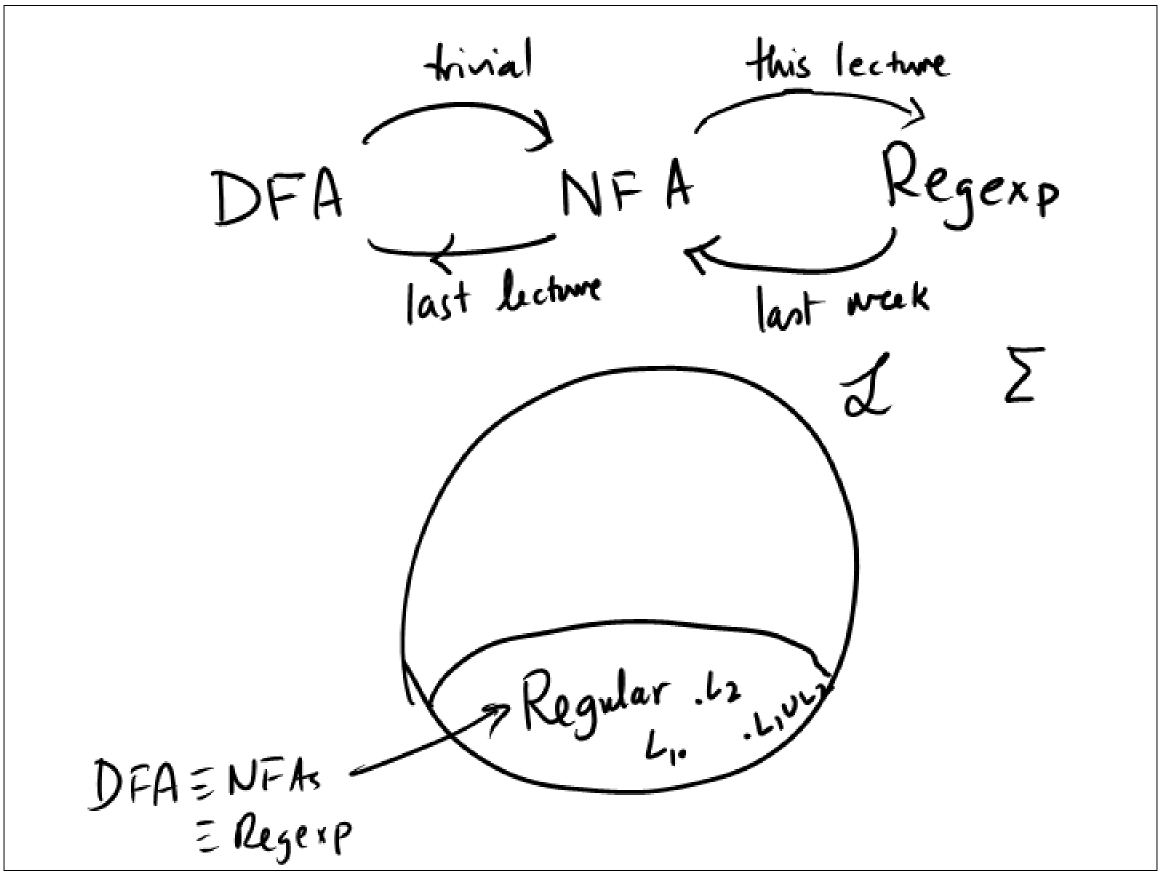
Lecture #8

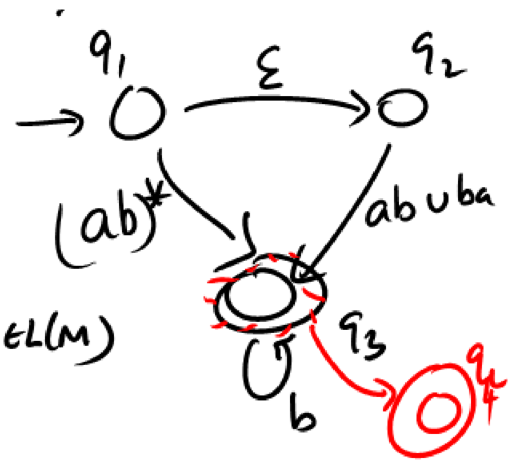
NFA/DFA $\hookrightarrow$ regexp


trivial
this lecture
DFA
NFA
Regexp
last lecture
last week
$\mathcal{L}$
Σ
Regular
L_2
L_1
$L_1 \cup L_2$
DFA ≡ NFAs
≡ Regexp

Generalized NFA

$$q_1 \xrightarrow{ab} q_3 \xrightarrow{b} q_3$$

$abb \in L(M)$



$$q_1 \xrightarrow{abababab} q_3$$

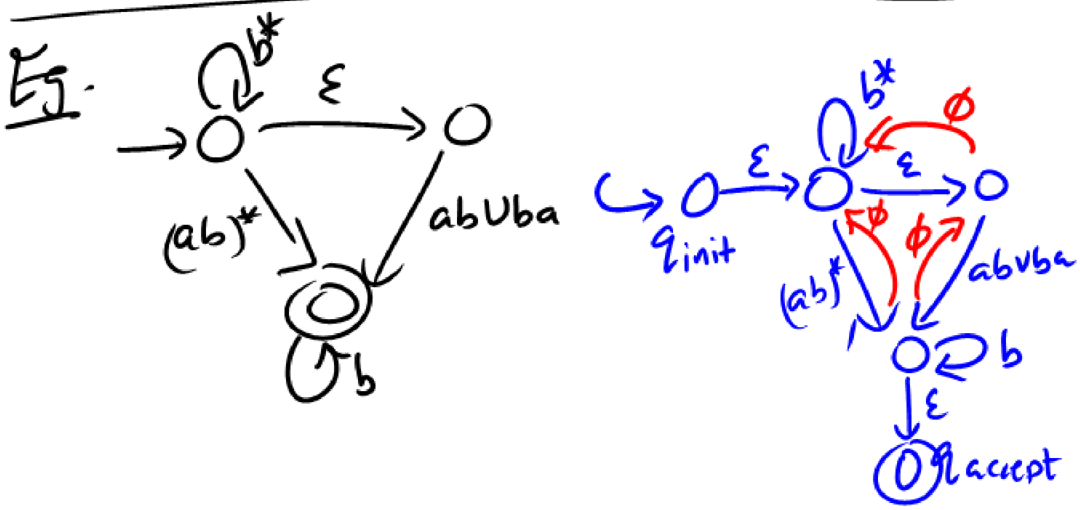
(C1) There are no transitions going into the initial state

(C2) There are no transitions from the accepting states.

(C3) There is only one accept state.

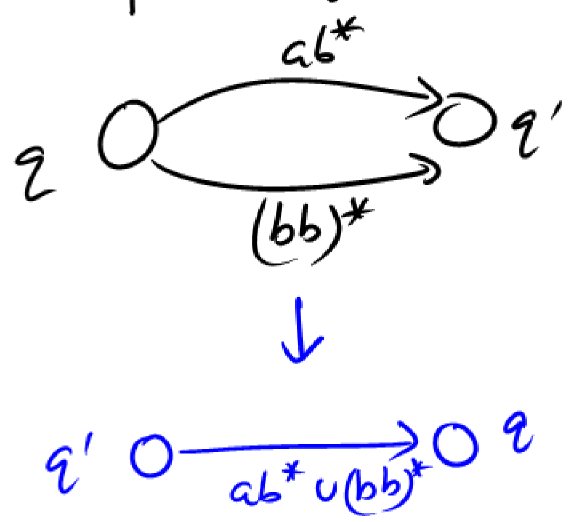
Also, for every q, q',
there is a transition from q to q'.
(unless q is accept-state or q' is initial state)

Eg.



One more condition.

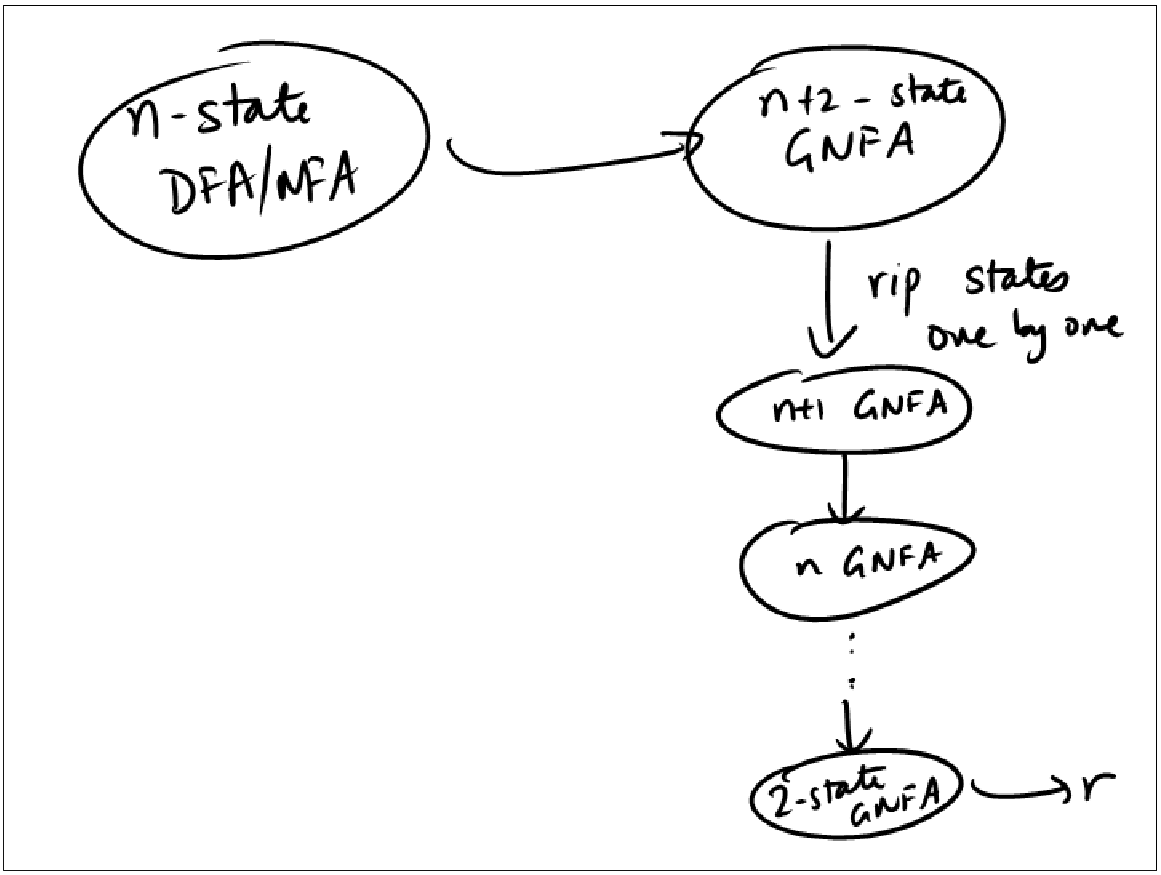
There is at most one transition from q to q' $(\forall q, q' \in Q)$



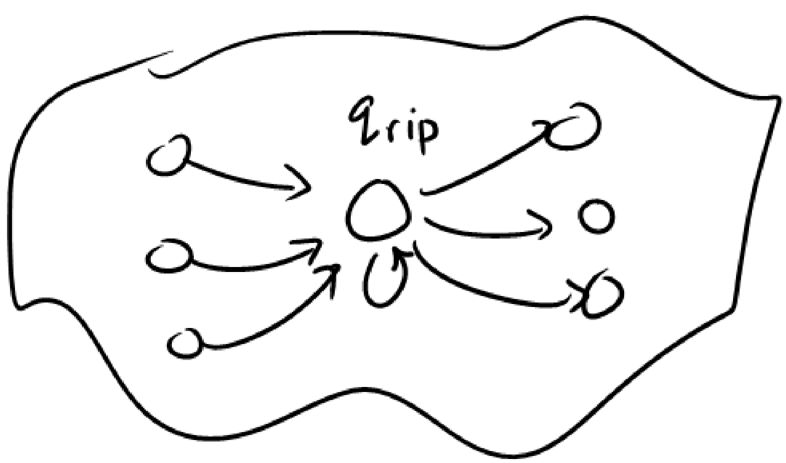
How does a 2-state GNFA look like?

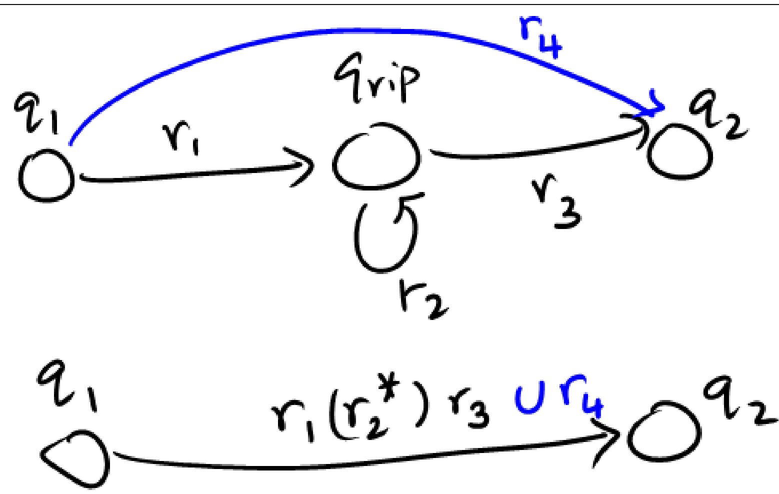
$\mathcal{A}$ 

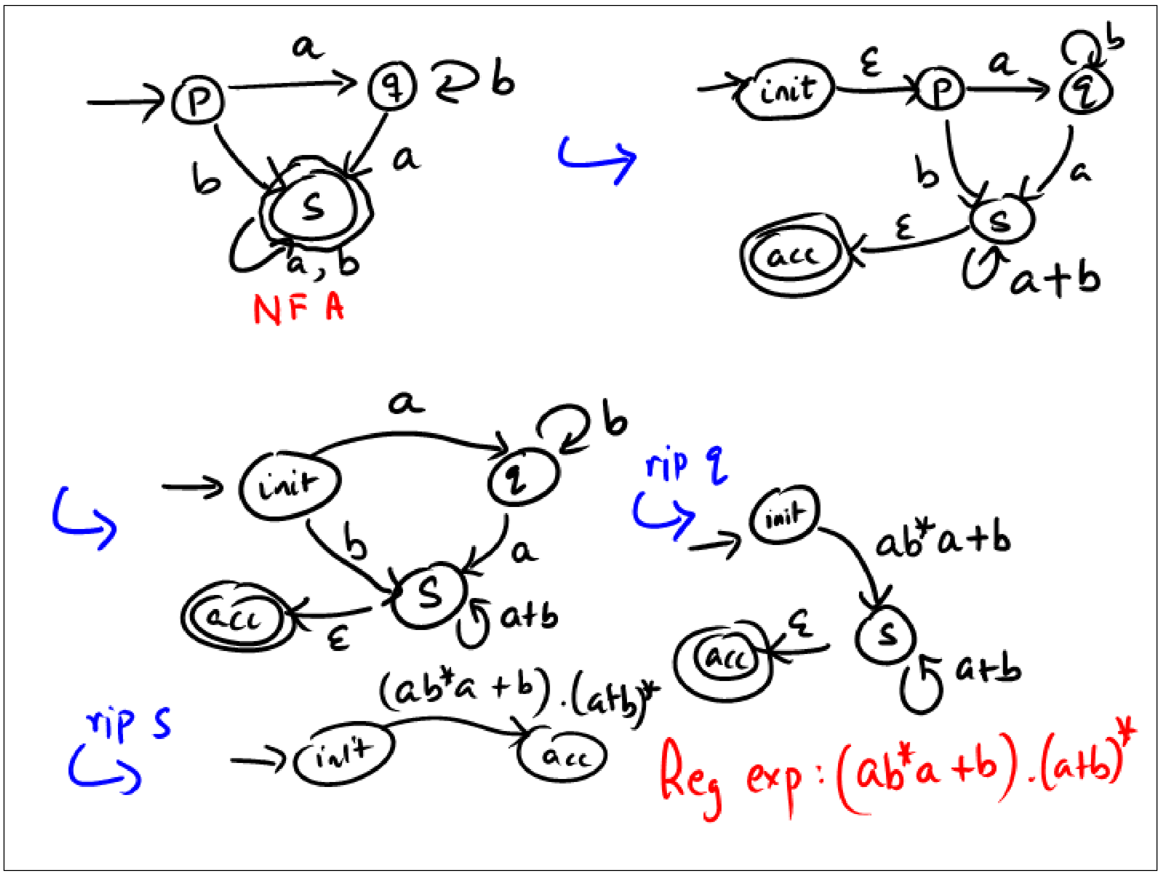
Regexp for $L(\mathcal{A})$: r


n-state
DFA/NFA
n+2 - state
GNFA
rip states
one by one
n+1 GNFA
n GNFA
2-state
GNFA
r

Ripping states




r_4
q_{rip}
q_1
r_1
q_2
r_3
r_2
q_1
$r_1(r_2^*)\,r_3 \cup r_4$
q_2


a
b
a, b
NFA
init
ε
acc
a + b
rip q
ab*a + b
rip s
(ab*a + b).(a+b)*
Reg exp: (ab*a + b).(a+b)*

Formally, a GNFA $= (Q, \Sigma, \delta, q_{init}, q_{acc})$

$\delta : (Q \setminus \{q_{acc}\}) \times (Q \setminus \{q_{init}\}) \to \text{Regexp}_\Sigma$

Lemma. For every NFA with n states, there is an $(n+2)$-state GNFA accepting the same language.

Ripping lemma

Let $A = (Q, \Sigma, \delta, q_{init}, q_{acc})$ be a GNFA.

$q \neq q_{init}$, $q \neq q_{acc}$: $A_{rip\,q} = (Q \setminus \{q\}, \Sigma, \delta', q_{init}, q_{acc})$

$q_1 \neq q_{acc}$ & $q_2 \neq q_{init}$: $\delta'((q_1, q_2)) = r_1 r_2^* r_3 \cup r_4$

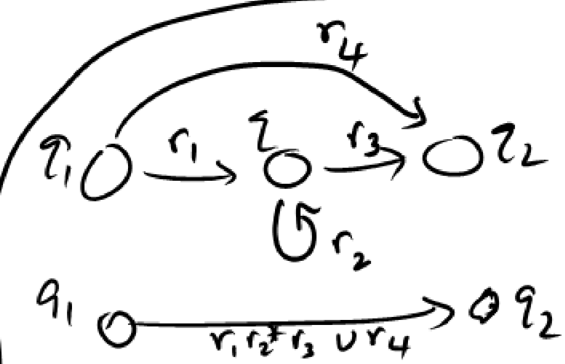
~~where~~

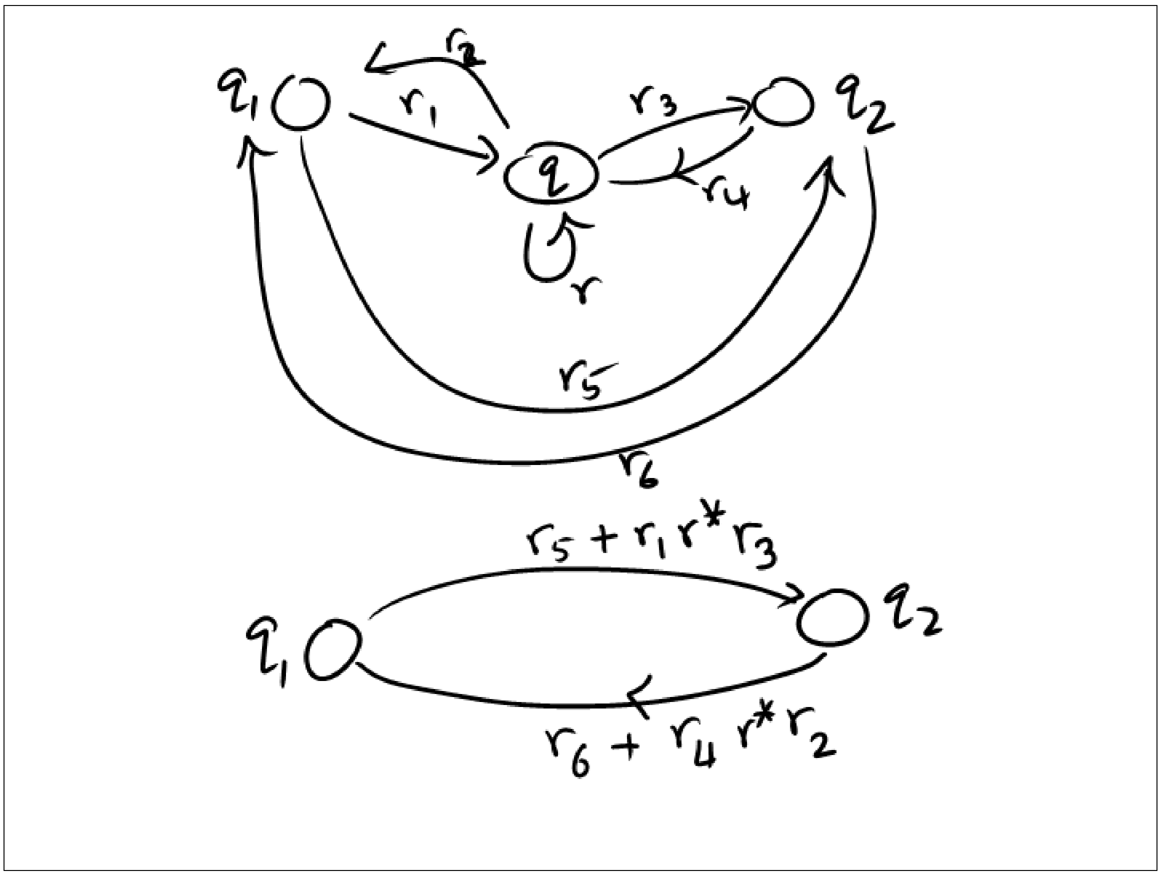
$r_1 = \delta(q_1, q)$

$r_2 = \delta(q, q)$

$r_3 = \delta(q, q_2)$

$r_4 = \delta(q_1, q_2)$





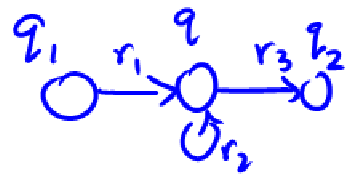
q1
r2
r1
r3
q2
q
r4
r
r5
r6
r5 + r1 r* r3
q1
q2
r6 + r4 r* r2

Proof. A

$A_{rip\,q}$

$L(A) = L(A_{rip\,q})$

$L(A) \subseteq L(A_{rip\,q})$



$x \in L(A)$ Run on x

Run in A $\quad q_{init} \; \overset{q_1}{\bullet} \; \overset{q}{\bullet} \; \overset{q}{\bullet} \; \overset{q}{\bullet} \; \overset{q_2}{\bullet} \; \overset{q}{\bullet} \; \overset{q}{\bullet} \to q_{acc}$

$w_1 \quad y_1 \quad y_2 \quad w_2$

$w_1 \in r_1 \quad y_1, y_2 \ldots \in r_2$

$w_2 \in r_3$

Run in $A_{rip\,q}$ $\quad q_{init} \; \overset{q_1}{\bullet} \; \overset{q_2}{\bullet} \to q_{acc}$

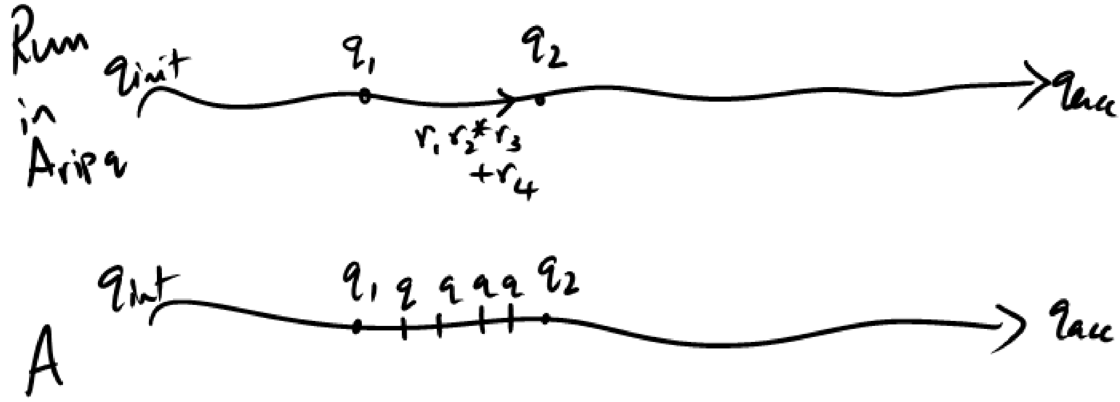
$w_1 y_1 y_2 w_2$

$w_1 y_1 y_2 w_2 \in r_1 r_2^* r_3$

So $x \in L(A_{rip\,q})$

Must show $L(A_{rip\,q}) \subseteq L(A)$

$x \in L(A_{rip\,q})$



Closure under homomorphism

$$h : \Sigma \longrightarrow \Pi^*$$

$\Sigma = \{a, b\}$ $\qquad$ $h: a \mapsto 1112$

$\qquad\qquad\qquad\qquad b \mapsto 23$

$\Pi = \{1, 2, 3\}$

$h(abba) = 1112\ 23\ 23\ 1112$

$h(a_1 \ldots a_n) = h(a_1) \ldots h(a_n)$

$$h(L) = \{h(x) \mid x \in L\}$$

Lemma. If L is regular, $h(L)$ is regular.

$$h: a \mapsto 112, \quad b \mapsto 23, \quad c \mapsto \varepsilon$$

$L : (ab)^*.(b+c)$

$h(L) : (112.23)^*.(23+\varepsilon)$