- NFAs continued
- Closure constructions on NFAs
  ($\cup$, $\cdot$, $*$, reversal)
- Regexps $\hookrightarrow$ NFAs

Formal notion of acceptance for NFA

Let $A = (Q, \Sigma, \delta, q_0, F)$ be an NFA.

A accepts w if

$$w = y_1 y_2 \ldots y_m \quad \forall i \; y_i \in \Sigma_\epsilon$$

m could be larger than $|w|$

and there is sequence of states
$r_0, r_1, \ldots r_m$

- $r_0 = q_0$
- $r_m \in F$
- $r_{i+1} \in \delta(r_i, y_{i+1})$ where $i = 0, \ldots m-1$

y_{i+1} could be ϵ

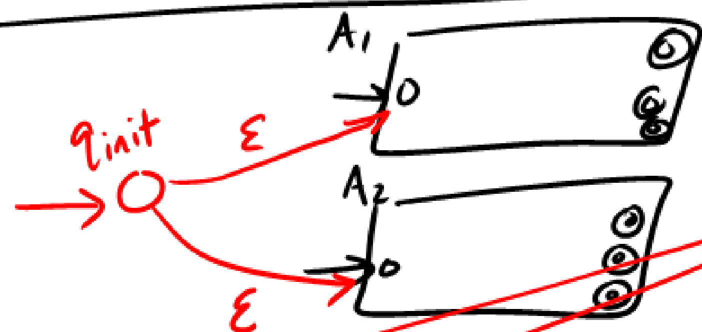
	$\cap$	$\cup$	$\neg$	$\cdot$	$*$
DFA	Easy (product)	Easy (product)	Easy	Hard	Hard
NFA	Double (HW)	Easy*	Hard	Easy*	Easy*
Regexp	Hard	Easy	Hard	Easy	Easy

Regular languages.

* - Today's class

2

NFA closure under union.



Assume
$Q_1 \cap Q = \emptyset$
$q_{init} \notin Q_1 \cup Q_2$

$$A_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$$

$$A_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$$

$$A = (Q_1 \cup Q_2 \cup \{q_{init}\}, \Sigma, \delta, q_{init}, F_1 \cup F_2)$$

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \in Q_1, a \in \Sigma_\varepsilon \\ \delta_2(q,a) & \text{if } q \in Q_2, a \in \Sigma_2 \\ \{q_1, q_2\} & \text{if } q = q_{init}, a = \varepsilon \\ \emptyset & \text{if } q = q_{init}, a \neq \varepsilon \end{cases}$$

NFAs : Concatenation

Make final states of A_1 non-final.

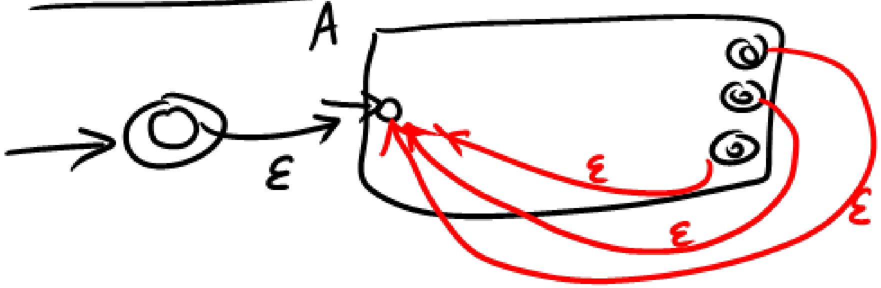
$$A_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$$

$$A_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$$

$$A = (Q_1 \cup Q_2, \Sigma, \delta, q_1, F_2)$$

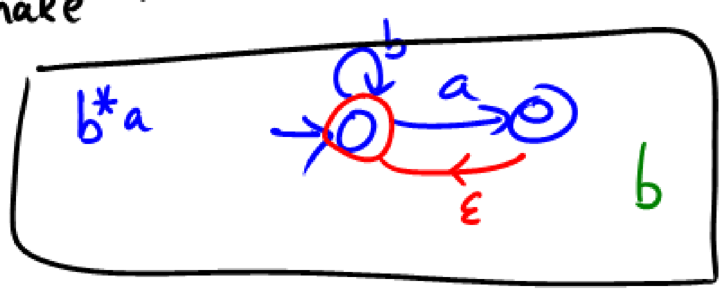
$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \setminus F_1,\ a \in \Sigma_\varepsilon \\ \delta_1(q,a) & q \in F_1,\ a \neq \varepsilon \\ \delta_1(q,\epsilon) \cup \{q_2\} & q \in F_1,\ a = \varepsilon \\ \delta_2(q,a) & q \in Q_2,\ a \in \Sigma_\varepsilon \end{cases}$$

NFA : Kleene-*



Tempting to make initial state final ...

but that's wrong



$$A = (Q, \Sigma, \delta, q_0, F)$$

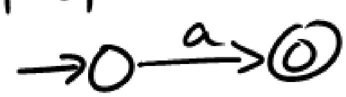
$$A^* = (Q \cup \{q_{init}\}, \Sigma, \delta^*, q_{init}, F \cup \{q_{init}\})$$

$$\delta^*(q,a) = \begin{cases} \delta(q,a) & q \in Q \setminus F,\ a \in \Sigma_\varepsilon \\ \delta(q,a) & q \in F,\ a \in \Sigma \\ \delta(q,\varepsilon) \cup \{q_0\} & q \in F,\ a = \varepsilon \\ \{q_0\} & q = q_{init},\ a = \varepsilon \\ \emptyset & q = q_{init},\ a \neq \varepsilon \end{cases}$$

Lemma: For every regular expression R over Σ, there is an NFA A_R such that $\mathcal{L}(R) = \mathcal{L}(A_R)$

Proof. Induction on structure of R
(induction on number of operators in R)

Base cases : # operators = 0.

$R = a$: 

$R = \varepsilon$: 

$R = \emptyset$: 

Induction step..

$R = R_1 + R_2$ By ind hypothesis, there is an NFA acc. $L(R_1)$ and an NFA acc $L(R_2)$. Do union construction on NFAs to get an NFA accepting $L(R_1) \cup L(R_2)$.

$R = R_1 \cdot R_2$ Take NFA A_1 for R_1 and A_2 for R_2 and do concatenation construction

$R = R_1^*$ Take automaton for R_1 and do Kleene-* construction.

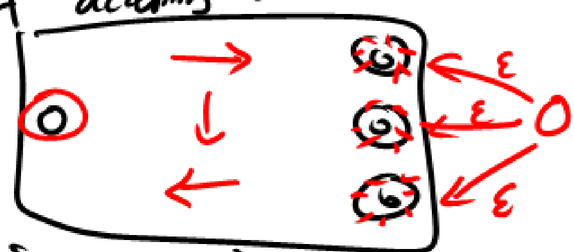
String reversal

$$w = a_1 a_2 \ldots a_{n-1} a_n$$

$$w^R = a_n a_{n-1} \ldots a_2 a_1$$

$$L \subseteq \Sigma^*, \quad L^R = \{ w^R \mid w \in L \}$$

A accepting L



$$A = (Q, \Sigma, \delta, q_0, F)$$

$$A^r = (Q \cup \{q_{init}\}, \Sigma, \delta, q_{init}, \{q_0\}$$

$$\delta^R(q,a) = \begin{cases} \{q' \mid q \in \delta(q',a)\}, & q \in Q,\ a \in \Sigma_\varepsilon \\ F & q = q_{init},\ a = \varepsilon \\ \phi & q = q_{init},\ a \neq \varepsilon \end{cases}$$

$$q' \xrightarrow{a} q$$

$$q \xrightarrow{a} q'$$