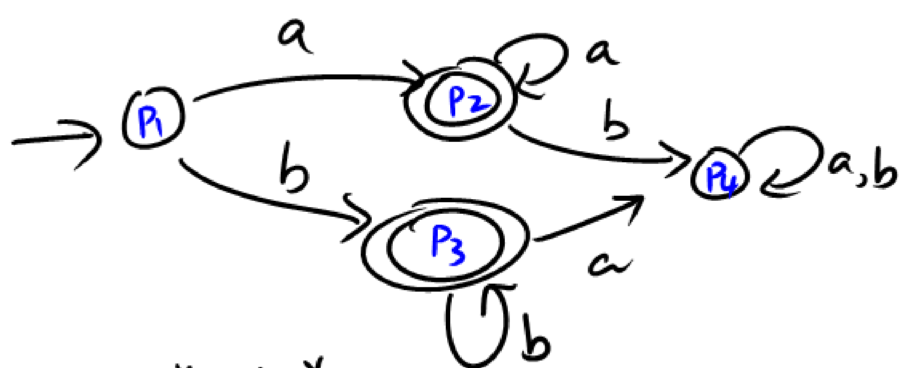
Lecture 10

Suffix languages and DFA minimization



$L = aa^* + bb^*$

$L_{p_1} = aa^* + bb^*$; $L_{p_2} = a^*$; $L_{p_3} = b^*$

$L_{p_4} = \phi$

A <u>state</u> of a DFA is responsible for accepting some language.

Lemma: Let $L \subseteq \Sigma^*$.
If L has a finite number of suffix languages, then L is regular.

$$\mathcal{C}(L) = \{L_1, L_2, \ldots L_n\}$$

$$A = (Q, \Sigma, \delta, q_0, F)$$

$$Q = \{L_1, \ldots L_n\}$$

$$q_0 = L/\varepsilon = L$$

$$F = \{L/w \mid \varepsilon \in L/w,\ w \in \Sigma^*\}$$
$$= \{L_i \mid \varepsilon \in L_i\}$$

$$\delta(L/w, a) = L/wa$$

This is well-defined.

Whatif $L/w = L/w'$ $(w \neq w')$

If $L/w = L/w'$

then $L/wa = L/w'a$.

<u>Claim</u>. $\mathcal{L}(A) = L$.

Claim. A, after reading w, will be in state L/w

Induction on $|w|$.

$w = \varepsilon$; A's initial state is L/ε.

$w = w'a$; A's state after reading w' is L/w' (by IH)

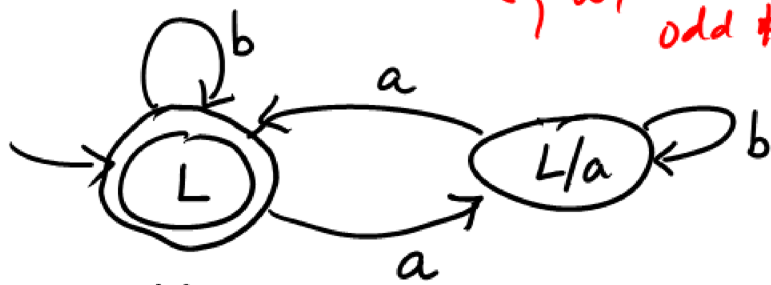
So A on reading $w'a$ is in $L/w'a$.

So $w \in L(A)$ iff L/w contains ϵ iff $w \in L$ — $\square$

Eg. $L = \{ w \in \{a,b\}^* / \ w$ has an even # of a's $\}$

$\ell(L) = \{ L, \ L/a)$

$\{ w \mid w$ has an odd # of a's $\}$



Initial state: $L/\varepsilon = L$

$L \xrightarrow{a} ?$ $\quad L = L/\varepsilon \xrightarrow{a} L/\varepsilon a = L_a$

$L = L/aa \xrightarrow{a} L/aaa = L_a$

$L/a \xrightarrow{a} L/aa = L \ ; \ L/\varepsilon \xrightarrow{b} L/b = L$

Theorem. (Myhill-Nerode theorem)

L is regular iff the number of suffix languages for L is finite

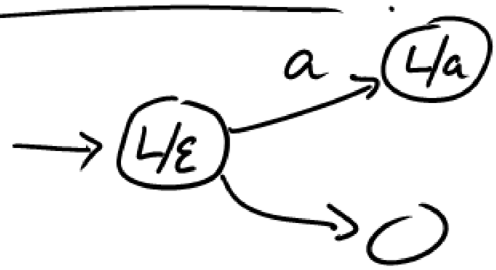
Moreover, if L has k suffix languages then there is a DFA for L that has k states.

Also, any DFA for L must have at least k states.

Also, any DFA with k states accepting L is isomorphic to A (constructed in proof)

Let B accept L.

Intuitive proof idea



All suffix languages have to feature in any DFA for L.

$$L = \{a^n b^n \mid n \in \mathbb{N}\}$$

$$L/\varepsilon = L$$

$$L/a = \{a^{n-1} b^n \mid n \in \mathbb{N}\}$$

$$L/aa = \{a^{n-2} b^n \mid n \in \mathbb{N}\}$$

$$\vdots$$

$$L/a^i = \{a^{n-i} b^n \mid n \in \mathbb{N}\}$$

Minimization algorithm

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Given a regular L (given A^{acc}_{L})
can we decide whether

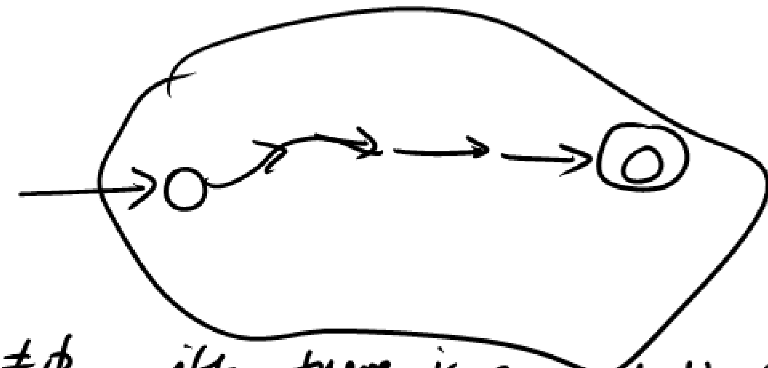
$$L/w = L/w'$$

Let A^q be A with q as initial state

$A^{q'}$ be A with q' as initial state.

$$L/w = L/w'$$
$$\iff L(A^q) = L(A^{q'})$$

How do I check whether $L(A) = \phi$?



$L(A) \neq \phi$ iff there is a reachable final state.

Solvable in linear time.

$$\mathcal{L}(A_1) \subseteq \mathcal{L}(A_2)$$

$$\mathcal{L}(A_1) \cap \overline{\mathcal{L}(A_2)} = \emptyset$$

$$\mathcal{L}(A_1 \cap \bar{A}_2) = \emptyset$$

Hence we can decide if

$$\mathcal{L}(A_1) \subseteq \mathcal{L}(A_2)$$

$$\mathcal{L}(A_1) = \mathcal{L}(A_2)$$

$\Leftrightarrow$ $L(A_1) \subseteq \mathcal{L}(A_2)$ and $\mathcal{L}(A_2) \subseteq \mathcal{L}(A_1)$

Hence decidable.