Discussion 13: Reductions

16 April 2008

Questions on homework 12?

Any questions? Complaints, etc?

1 Easy Reductions

Question: Reduce A_{TM} to L:

$$L = \left\{ M \;\middle|\; M \text{ accepts 00 and doesn't accept 11} \right\}.$$

Solution: Let N be a decider for L. To decide $\langle M, w \rangle \in A_{\text{TM}}$, build a new TM M' (with input x) that simulates M on w and if M accepts, and if x is 00, accepts (how can it verify this?), otherwise rejects. Now observe that:

$$\langle M, w \rangle \in \mathrm{A_{TM}} \iff M' \in L$$

Lefthand side can be decided using N.

Question: Reduce A_{TM} to L:

$$L = \{M : L(M), \text{ and } \overline{L(M)} \text{ are both infinite}\}$$

Solution: Let N be a decider for L. To decide $\langle M, w \rangle \in A_{\text{TM}}$, build a new TM M' (with input x) that simulates M on w and if M accepts, and if $|x|$ is even, accepts (how can it verify this?), otherwise rejects. Now observe that:

$$\langle M, w \rangle \in \mathrm{A_{TM}} \iff M' \in L$$

Lefthand side can be decided using N.