Quiz 6

CS 373: Theory of Computation

Date: December 7, 2010. Lecture Section AL1. Time limit: 15 minutes.

Name	
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Discussion	Tu 2-2:50 Tu 3-3:50 Tu 4-4:50 W 4-4:50 W 5-5:50

Pick the correct alternative from among the choices (A), (B), and (C) provided for each question below. Each question is worth **1 point**.

1. Let G be a grammar in Chomsky Normal Form. Let $w_1, w_2 \in L(G)$ such that $|w_1| < |w_2|$.
   (A) Any derivation of w_1 has exactly the same number of steps as any derivation of w_2.
   (B) Some derivations of w_2 maybe shorter than some derivations of w_1.
   (C) All derivation of w_1 will be shorter than any derivation of w_2.

2. Consider the language $L = \{a^i b^j a^i b^j \mid i, j \geq 0\}$. Consider the following "proof" that L does not satisfy the pumping lemma. Let p be the pumping length. Choose $z = a^p b a^p b$. Consider a division of z, where $u = a^i$, $v = a^j$, $w = a^k$, $x = a^\ell b$ and $y = a^p b$. Clearly uv^0wx^0y is not in L.
   (A) This is an incorrect proof because all divisions of z have not been considered.
   (B) This is an incorrect proof because all possible z have not been considered.
   (C) This is a correct proof.

3. Consider the language $L = \{a^i b^j c^k d^\ell \mid \text{either } i = 0 \text{ or } j = k = \ell\}$. Consider the following "proof" that L satisfies the pumping lemma. Take $p = 1$. Let $z \in L$ be a string of length at least p. Take $u = \epsilon$, v to be the first symbol of z, $w = x = \epsilon$, and take y to be the rest of the string. Now $uv^iwx^iy \in L$ for every $i \geq 0$.
   (A) This is an incorrect proof because you cannot pick the pumping length p.
   (B) This is an incorrect proof because you cannot choose the division of z.
   (C) This is a correct proof.

4. Here is a faulty proof showing that $L = \{a^n b^n c^n \mid n \geq 0\}$ is context-free. Consider the grammar $G = (\{S\}, \{a, b, c\}, R, S)$ whose rules R are given as

$$S \to SaSbScS \mid SaScSbS \mid SbSaScS \mid SbScSaS \mid ScSaSbS \mid ScSbSaS \mid SS \mid \epsilon$$

The proof consists of the following sequence of assertions. Which of them is flawed?

(A) $L(G) = L_{eq}$ where $L_{eq} = \{w \mid w$ has an equal number of as, bs, and cs$\}$

(B) $L_{eq} \cap L(a^*b^*c^*) = \{a^n b^n c^n \mid n \geq 0\} = L$

(C) $L(G)$ is context-free, $L(a^*b^*c^*)$ is regular. Therefore $L(G) \cap L(a^*b^*c^*)$ is context-free.

5. For language L_1 and L_2, let $L_1 \oplus L_2 = \{w \mid w$ belongs to exactly one out of L_1 and $L_2\}$. Suppose L_1 is regular and L_2 is context-free, then which of the following statements is true.

(A) $L_1 \oplus L_2$ is regular.

(B) $L_1 \oplus L_2$ is context-free but not necessarily regular.

(C) $L_1 \oplus L_2$ is decidable but not necessarily context-free.

6. Suppose L is a context-free language. Then $\overline{L}$ is

(A) Necessarily context-free

(B) Necessarily non-context-free

(C) May or may not be context-free