# Quiz 6

## CS 373: Theory of Computation

Date: December 7, 2010. Lecture Section AL2. Time limit: 15 minutes.

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| **Name** | |
| **netid** | |
| **Discussion** | Tu 2-2:50 Tu 3-3:50 Tu 4-4:50 W 4-4:50 W 5-5:50 |

Pick the correct alternative from among the choices (A), (B), and (C) provided for each question below. Each question is worth **1 point**.

1. Let $G$ be a grammar in Chomsky Normal Form. Let $w_1, w_2 \in L(G)$ such that $|w_1| = |w_2|$.
   (A) Any derivation of $w_1$ has exactly the same number of steps as any derivation of $w_2$.
   (B) Some derivations of $w_1$ maybe shorter than some derivations of $w_2$.
   (C) Different derivations of $w_1$ have different lengths.

2. Consider the language $L = \{a^i b^j a^i b^j \mid i, j \geq 0\}$. Consider the following "proof" that $L$ satisfies the pumping lemma. Let $p$ be the pumping length. Choose $z = a^p b a^p b$. Consider a division of $z$, where $u = a^{p-1}$, $v = a$, $w = b$, $x = a$ and $y = a^{p-1}b$. Clearly $uv^iwx^iy$ is in $L$ for every $i$.
   (A) This is an incorrect proof because all divisions of $z$ have not been considered.
   (B) This is an incorrect proof because all possible $z$ have not been considered.
   (C) This is a correct proof.

3. Consider the language $L = \{a^n b^n c^n \mid n \geq 0\}$. Consider the following "proof" that $L$ does not satisfy the pumping lemma. Let $p \geq 1$ be the pumping length. Choose $z = a^p b^p c^p$. Consider the division of $z$, where $u = \epsilon$, $v = a$, $w = \epsilon$, $x = \epsilon$, and $y = a^{p-1}b^p c^p$. Clearly $uv^0wx^0y$ is not in $L$.
   (A) This is an incorrect proof because all divisions of $z$ have not been considered.
   (B) This is an incorrect proof because all possible $z$ have not been considered.
   (C) This is a correct proof.

4. Consider the language $L = \{a^i b^j c^k d^\ell \mid \text{either } i = 0 \text{ or } j = k = \ell\}$. Consider the following "proof" that $L$ is not context-free. Let $L_1 = L \cap L(aa^*b^*c^*d^*)$. Now define a homomorphism $h$ such that $h(a) = \epsilon$, $h(b) = b$, $h(c) = c$, and $h(d) = d$. $L_2 = h(L_1) = \{b^n c^n d^n \mid n \geq 0\}$. Since $L_2$ is not context-free, $L$ is not context-free.

   (A) $L$ is context-free and hence this is an incorrect proof.

   (B) $L$ is not context-free, but this is an incorrect proof because context-free languages are not closed under intersection.

   (C) $L$ is not context-free and this is a correct proof.

5. Let $L$ be a context-free language. Let $L^R = \{w^R \mid w \in L\}$ where $w^R$ denotes the reverse of the string $w$.

   (A) $L^R$ is definitely not context-free.

   (B) $L^R$ may not be context-free, depending on $L$.

   (C) $L^R$ is definitely context-free.

6. Suppose $L_1$ is a context-free language and $L_2$ is a non-context-free language, such that $L_1 \cap L_2 = \emptyset$. Then $L_1 \cup L_2$ is

   (A) Necessarily context-free

   (B) Necessarily non-context-free

   (C) May or may not be context-free