CS 373 Fall 2010
Quiz 4 Solutions

Lecture 1 - Mahesh

1. C. For any string w, either w is in A, w is in B, or w is in neither (and it cannot be in both). The machines that decide A and B simulate the recognizers for A, B, and $\overline{A \cup B}$ in parallel on any input w. One of them will accept in finite time, which identifies where w is.

2. C. Say $L' = A_{TM}$. (a) is incorrect if $L = \Sigma^* \times \Sigma^*$, and (b) is incorrect if $L = \emptyset$. We can reduce L as follows: on input w, we actually compute if $w \in L$ (which can be done in finite time since L is decidable). If yes, $f(w) = 0011$. Else $f(w) = 011$. Thus $w \in L$ iff $f(w) \in L_{0n1n}$.

3. C. The reduction tells us that there is a function f such that $w \in A$ iff $f(w) \in B$. Thus $w \in \bar{A}$ iff $w \notin A$ iff $f(w) \notin B$ iff $w \in \bar{B}$ for the same function f.

4. C. Since L_d is not RE, we learn this about L.

5. C. (c) is a feature of a machine, not a language. Notice that (b) is actually a feature of a language, since a TM with an odd number of states can be in that set.

6. B. We can build a recognizer for L as follows: on input M, dovetail all possible strings until 312929 are accepted. There is no decider though, since we cannot tell which strings are not in $L(M)$ without running M on all of them.

Lecture 2 - Gul

1. C. For any string w, either w is in A, w is in B, or w is in both (and it cannot be in neither). The machines that decide A and B work as follows: on input w, put w in the $(A \cap \bar{B}) \cup (\bar{A} \cap B)$ decider. If it rejects, w is in $A \cap B$ so accept w. If it accepts, the A and B recognizers can be run in parallel to find out which set w is in.

2. C. Say $L' = L_d$. (a) is incorrect if $L = \Sigma^*$, and (b) is incorrect if $L = \emptyset$. We can reduce L as follows: on input w, pass $f(w) = \langle M_L, w \rangle$ to a machine for A_{TM}, where M_L is a machine for L. Thus $w \in L$ iff $\langle M_L, w \rangle \in A_{TM}$.

3. C. The reduction tells us that there is a function f such that $w \in A$ iff $f(w) \in B$. Thus $w \in \bar{A}$ iff $w \notin A$ iff $f(w) \notin B$ iff $w \in \bar{B}$ for the same function f.

4. B. Since A_{TM} is not decidable, we learn that L is also not decidable. We do not know whether L is recognizable or not.

5. A. This is a feature of M's language, where the other two are features of the machine.

6. C. $\bar{L}$ is RE, so option (b) is impossible. Intuitively, we can see there is no machine for L. On input M, such a machine would have to check all strings to see that only 312929 are accepted by M. Thus L is not RE, though its complement is.