University of Illinois at Urbana-Champaign
Department of Computer Science

Mock Midterm 2

CS 373: Formal Models of Computation
Fall 2009

Name:
Netid:

- Print your name and netid, *neatly* in the space provided above; print your name at the upper right corner of *every* page. Please print legibly.
- This is a *closed book* exam. No notes, books, dictionaries, calculators, or laptops are permitted.
- You are free to cite and use any theorems from class or homeworks without having to prove them again.
- Write your answers in the space provided for the corresponding problem. Let us know if you need more paper.
- Suggestions: Read through the entire exam first before starting work. Do not spend too much time on any single problem. If you get stuck, move on to something else and come back later.
- If you run short on time, remember that partial credit will be given.
- If any question is unclear, ask us for clarification.

Question	Points	Score
Problem 1	20	
Problem 2	15	
Problem 3	15	
Problem 4	15	
Problem 5	15	
Problem 6	20	
Total	100	

1. Short Problems (20 points)

   Give answers to each of the following questions, including a short justification. For each, you will get two points for the correct answer and two points for the correct justification.

   (a) If B is a finite set of strings and A is not a context-free language, then could $C = A - B$ possibly be a context-free language? (4 points)

   (b) Is the language $\{w \mid w$ has an equal number of as, bs and cs$\}$ over the alphabet $\Sigma = \{a, b, c\}$ context free? (4 points)

   (c) If A is a context-free language, then is $B = \{w^R \mid w \in A\}$ a context-free language (recall that w^R is the reverse of w)? (4 points)

   (d) For a grammar in Chomsky normal form, is it possible for it to produce the same string using different, alternative derivation trees? (4 points)

   (e) Let G be a grammar in Chomsky normal form and consider any $x, y \in L(G)$ for which $|x| = |y|$. Does there always exist a derivation for x and a derivation for y such that each uses the same number of steps (the same number of rules get applied)? (4 points)

2. Grammar Design (15 points)

   Consider the following language over the alphabet $\Sigma = \{a, b, \#\}$.

   $$A = \{w_1 \# w_2 \# \ldots \# w_n \mid n \geq 2, \text{ for all } i,\ w_i \in \{a,b\}^* \text{ and for at least one } i,\ |w_i| = |w_{i+1}|\}.$$

   Design a grammar G such that $L(G) = A$.

3. Closure of Some Kind (15 points)

   Prove that if A is a regular language and B is a context-free language, both over some alphabet Σ, then $A \cap B$ is a context-free language.

4. Pumping Lemma (15 points)

   Prove that A is not a CFL, where

$$A = \{a^n b^{n^2+n} \mid n \geq 0\}.$$

5. Grammars (15 points)

   A context-free language is called ϵ-free if ϵ is not a member of it. Prove or disprove the following statement:

   For any ϵ-free context-free language, there exists a context-free grammar that describes it in which all rules are of the form $A \rightarrow BCD$, $A \rightarrow ab$, or $A \rightarrow a$. Furthermore, there are no rules of the form $A \rightarrow \epsilon$.

6. Null-stack deterministic PDA (20 points)

   A *null-stack deterministic pushdown automaton (NSDPDA)* is a deterministic pushdown automaton (DPDA) with some special characteristics. Rather than reaching an accept state in Q, it accepts an input string if and only if the stack becomes empty at any point during execution. It starts by pushing a special symbol # onto its stack.

   Prove the following two statements:

   (a) For any language A, if there are two strings $x, y \in A$ for which x is a prefix of y and $x \neq y$, then there does not exist any NSDPDA that recognizes A. (10 points)

   (b) For any language A over an alphabet Σ, if there exists a DPDA that recognizes it, and if λ is a symbol for which $\lambda \notin \Sigma$, then there is an NSDPDA that accepts the language $A \circ \{\lambda\}$ (recall that $\circ$ means *concatenation*). (10 points)