Problem Set 2

Fall 09

Due: Thursday Oct 1 at 11:00 AM in class (i.e., Room 103 Talbot Lab)
Please follow the homework format guidelines posted on the class web page:
`http://www.cs.uiuc.edu/class/fa09/cs373/`

1. Pumping Lemma. [**Points**: 10]
   Prove that these languages are not regular:

$$A = \{0^{n^3} \mid n \geq 0\}$$
$$B = \{0^n 1^m 0^k \mid 0 \leq n \leq m \leq k\}$$

2. Pumping Lemma. [**Points**: 10]
   Is the following language regular? Prove or disprove.

$$A = \{0^{\lfloor \lg^{100} n \rfloor} \mid n \geq 1\}$$

   where $\lfloor x \rfloor$ is the largest integer not greater than x; and $\lg x$ is logarithm of x to the base 2.

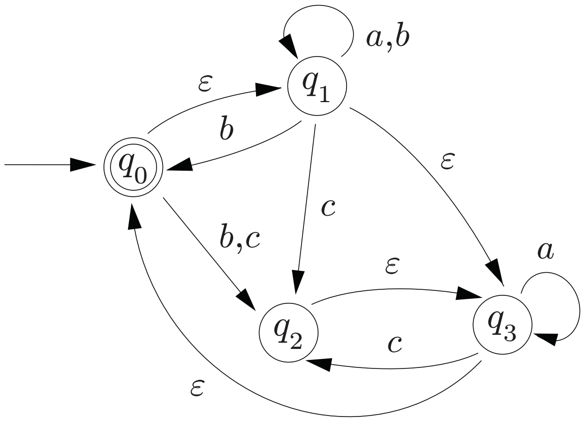
3. Regular Expression. [**Points**: 10]
   Write regular expressions generating the following languages:

$$A = \{w \in \{0,1\}^* \mid w \text{ has no substring } 011\}$$
$$B = \{w \in \{0,1\}^* \mid |w| \text{ is a multiple of } 3\}$$
$$C = \{w \in \{0,1\}^* \mid w \text{ has at least two 0 or at most one 1}\}$$

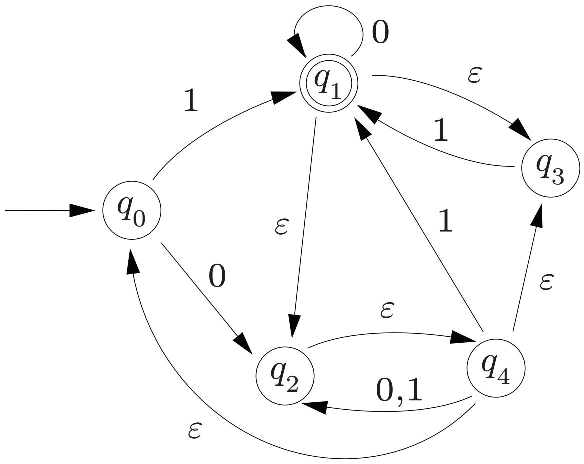
4. Regular Expression. [**Points**: 10]
   Prove or disprove: For languages A and B of the same alphabet Σ, if AB and B are regular, then A must be regular.

5. ϵ-closure [**Points**: 10]
Compute the ϵ-closure for the following NFAs

(a) $\Sigma = \{a, b, c\}$:



(b) $\Sigma = \{0, 1\}$:



6. DFA minimization [**Points**: 10]

(a) Determine the minimal-size DFA M such that $L(M) = L(M_1)$, $M_1 = (\Sigma, \delta, Q, q_{start}, F)$:
$\Sigma = \{0, 1\}$,
$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$,
$q_{start} = q_0$,
$F = \{q_3\}$,
and δ, the transition function

	0	1
q_0	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
q_3	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

(b) Determine the minimal-size DFA M such that $L(M) = L(M_2)$, $M_2 = (\Sigma, \delta, Q, q_{start}, F)$:
$\Sigma = \{0, 1\}$,
$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$,
$q_{start} = q_0$,
$F = \{q_5, q_6\}$,
and δ, the transition function

	0	1
q_0	q_7	q_1
q_1	q_7	q_0
q_2	q_4	q_5
q_3	q_4	q_5
q_4	q_5	q_6
q_5	q_5	q_5
q_6	q_6	q_5
q_7	q_2	q_2

7. NFA to DFA [**Points**: 10]
For each of the following NFAs: describe the language that it accepts, convert it to DFA and remove unreachable states.

(a) $M_1 = (\Sigma, Q, \delta, q_{start}, F), \Sigma = \{0, 1\}, Q = \{q_1, q_2, q_3, q_4\}, q_{start} = q_1, F = \{q_4\}$

δ	0	1	ϵ
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\{\}$
q_2	$\{q_3\}$	$\{q_3\}$	$\{\}$
q_3	$\{q_4\}$	$\{q_4\}$	$\{\}$
q_4	$\{\}$	$\{\}$	$\{\}$

(b) $M_2 = (\Sigma, Q, \delta, q_{start}, F), \Sigma = \{a, b, c\}, Q = \{q_1, q_2, q_3, q_4, q_5\}, q_{start} = q_1, F = \{q_4\}$

δ	a	b	c	ϵ
q_1	$\{q_1, q_2\}$	$\{q_1, q_3\}$	$\{q_1\}$	$\{\}$
q_2	$\{\}$	$\{q_3\}$	$\{q_4\}$	$\{\}$
q_3	$\{\}$	$\{\}$	$\{q_5\}$	$\{\}$
q_4	$\{\}$	$\{\}$	$\{\}$	$\{q_1\}$
q_5	$\{\}$	$\{\}$	$\{\}$	$\{q_4\}$

8. DFA to regular expression [**Points**: 10]
Give a regular expression for each of the following DFAs:

(a) $M_1 = (\Sigma, Q, \delta, q_{start}, F), \Sigma = \{0, 1\}, Q = \{q_1, q_2, q_3, q_4, q_5\}, q_{start} = q_1, F = \{q_4\}$

δ	0	1
q_1	q_3	q_2
q_2	q_4	q_3
q_3	q_3	q_4
q_4	q_5	q_5
q_5	q_5	q_5

(b) $M_2 = (\Sigma, Q, \delta, q_{start}, F), \Sigma = \{0,1\}, Q = \{q_1, q_2, q_3, q_4, q_5\}, q_{start} = q_1, F = \{q_4, q_5\}$

δ	0	1
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_5	q_4
q_4	q_1	q_4
q_5	q_1	q_2

Note: use algorithm described in the class

9. Irregularity [**Points**: 10]
Let h be a homomorphism $h : \Sigma \to \Gamma^*$, we define **inverse homomorphism** as the following function: For any $w \in \Gamma^*$, let $h^{-1}(w) = \{s \in \Sigma^* \mid h(s) = w\}$ and $h^{-1}(B) = \{s \in \Sigma^* \mid h(s) \in B\}$ for any $B \subseteq \Gamma^*$.

Theorem: The class of regular languages is closed under inverse homomorphism. That is, if B is regular then so is $h^{-1}(B)$.

Prove that the following languages are not regular using closure theorems, which were shown in the class:

(a) $B = \{0^n 1^m 0^k \mid 0 \leq n \leq m \leq k\}$

(b) $C = \{a^n b a^n \mid n \geq 0\}$

Note: You may use the fact that $\{0^n 1^n \mid n \geq 0\}$, $\Sigma = \{0,1\}$ is not regular without proving it. For part (b), you may find inverse homomorphism useful.