Problem Set 1

Fall 09

Due: Thursday Sep 17 at 11:00 AM in class (i.e., Room 103 Talbot Lab)
Please follow the homework format guidelines posted on the class web page:
`http://www.cs.uiuc.edu/class/fa09/cs373/`

1. DFA/NFA construction.
   [**Category**: Design, **Points**: 25]

   (a) For each of the following sets A, construct a DFA or NFA (as indicated) such that its language is A.

      i. DFA for $A = \{x \in \Sigma^* \mid \text{for all substring } w \text{ of } x \text{ with } |w| = 4, w \text{ has at least two 0's}\}$, in which $\Sigma = \{0, 1\}$.

      ii. NFA for $A = \{x \in \Sigma^* \mid x \text{ contains } ab, bc \text{ or } ca \text{ as a substring}\}$, in which $\Sigma = \{a, b, c\}$.

      iii. DFA for $A = \{x \in \Sigma^* \mid 111 \text{ happens exactly once as a substring in } x\}$, in which $\Sigma = \{0, 1\}$. Note that $1111 \notin A$.

   (b) Describe the language accepted by the DFA shown in the figure.

2. Set theory.
   [**Category**: Proof, **Points**: 15]

   For any two languages A and B prove that $(A^*B^*)^* = (A \cup B)^*$.

3. Regularity.
   [**Category**: Proof, **Points**: 20]

   Prove that these languages are regular:

   (a) $\{x \in \Sigma^* \mid x \text{ represents a binary number divisible by 3}\}$, in which $\Sigma = \{0, 1\}$.

   (b) $\{x \in \Sigma^* \mid x \text{ represents a binary number divisible by } k\}$, in which $\Sigma = \{0, 1\}$ and k is any fixed positive integer.

4. Closure properties.
[**Category**: Proof, **Points**: 20]

Let A be a regular language and k a positive integer. Define $A_{1/k}$ as follows:

$$A_{1/k} = \{x \mid \text{there exists strings } w \in A \text{ and } y \text{ with } |x| = |w|/k, \text{ such that } w = xy\}.$$

Prove that $A_{1/k}$ is regular. (Hint: you may first want to prove this just for $k = 2$ as a warm-up)

5. State explosion.
[**Category**: Proof, **Points**: 30]

Assume $\Sigma = \{0, 1\}$. Consider the following language:

$$A_k = \{x \in \Sigma^* \mid \text{the } k\text{-th symbol from the end of } x \text{ is } 0\}.$$

We want to prove that the minimum number of states for a DFA that recognizes the language A_k is 2^k states.
For any $k > 0$:

(a) Prove that any DFA with language A_k has at least 2^k states.

(b) Construct a DFA with 2^k states for A_k.

Now construct an NFA for the language A_k that has $k + 1$ states.

6. Regularity.
[**Category**: Proof, **Points**: 20]

Let Σ be any alphabet that contains only a single symbol. Prove that for every $A \subseteq \Sigma^*$, A^* is a regular language.

7. Regularity.
[**Category**: Proof, **Points**: 20]

Assume $\Sigma = \{0, 1\}$. Let p and t be two fixed positive integers. Consider:

$$A = \{w \in \Sigma^* \mid w \text{ represents a binary number with value } 2^n p + t \text{ for some non-negative integer } n\}.$$

Prove that A is a regular language.