Problem Set 0

Fall 09

Due: Tuesday Sep 01 at 11:00 AM in class (i.e., Room 103 Talbot Lab)
Please follow the homework format guidelines posted on the class web page:
`http://www.cs.uiuc.edu/class/fa09/cs373/`

1. True/false/meaningless.
   [**Category**: Notation, **Points**: 20]

   Answer each of the following with true, false or meaningless. Follow the notations in Sipser. We use $\{\dots\}$ to represent sets (not, for example, multisets).

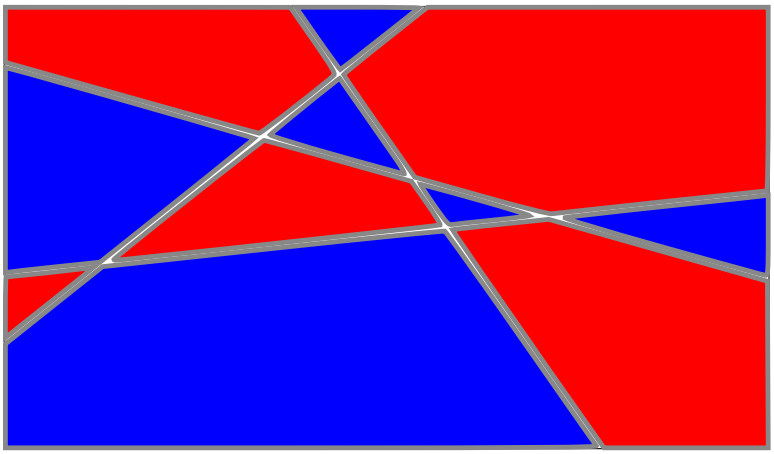
   D1) $\varnothing \in \{\varnothing, 1, 8\}$
   D2) $\varnothing \subseteq \{\varnothing, 1, 8\}$
   D3) $1 \subseteq \{\varnothing, 1, 8\}$
   D4) $\{1\} + \{1, 2\} = \{1, 2\}$
   D5) $\{1, 2\} \setminus \{1\} = \{2\}$
   D6) $\{1, 2\} \setminus \{3\} = \{0\}$
   D7) $\{1, 2\} \cap \{3, 4\} = \{\}$
   D8) $\{1, 2\} \cap \{3, 4\} = \{\{\}\}$
   D9) $\{1, 2\} \cap \{3, 4\} = \{\varnothing\}$
   D10) $\{1, 2\} \cup \{1, 3\} = \{1, 1, 2, 3\}$
   D11) $\{1, 2\} \cup \{1, 3\} = \{1, 2, 3\}$
   D12) $\{1, \{1\}, \{\{1\}\}\} = \{1\}$
   D13) $\{1, \{1\}, \{\{1\}\}\} = \{1, 1, 1\}$
   D14) $\{1\} \in \{1, \{1\}, \{\{1\}\}\}$
   D15) $\{1\} \subseteq \{1, \{1\}, \{\{1\}\}\}$
   D16) $\{\{1\}\} \in \{1, \{1\}, \{\{1\}\}\}$
   D17) $\{a, b\} \times \{c, d\} = \{(a, b), (c, d)\}$
   D18) $\Big(\{a, b\} \times \{c, d\}\Big) \cap \Big(\{c, d\} \times \{a, b\}\Big) = \{\}$
   D19) $|\{a, b, c\} \times \{d, e\}| = 6$
   D20) $\{a, b\} \times \{\} = \{a, b\}$
   D21) $\{a, b\} \setminus \{b, a\} = \{\}$

2. Divisible by 9.
   [**Category**: Proof, **Points**: 20]

   Prove that for any positive integer n, the quantity $4^n + 15n - 1$ is divisible by 9. We recommend to use induction. Note that you may need to apply induction more than once.

3. Countably Infinite.
[**Category**: Proof, **Points**: 20]

Prove that the set of all valid Java programs is countably infinite.
Hint: Try to build an injection from this set into the natural numbers.

4. Coloring.
[**Category**: Proof, **Points**: 20] There are n lines on the plain such that no three of them pass through a common point. They partition the plain into a number of cells (each cell is a connected piece of plain bounded to some of these lines). Prove that we can color these cells "Red" and "Blue" so that no two cells whose boundary share at least two points are in the same color (Again we recommend using induction).

In the following situation with $n = 4$, we have 11 cells, and as you see it is possible to color them such that every two cells which have more than one point in common, get different colors (The plain is infinite of course and the following figure just shows some interesting part of the colored plain).