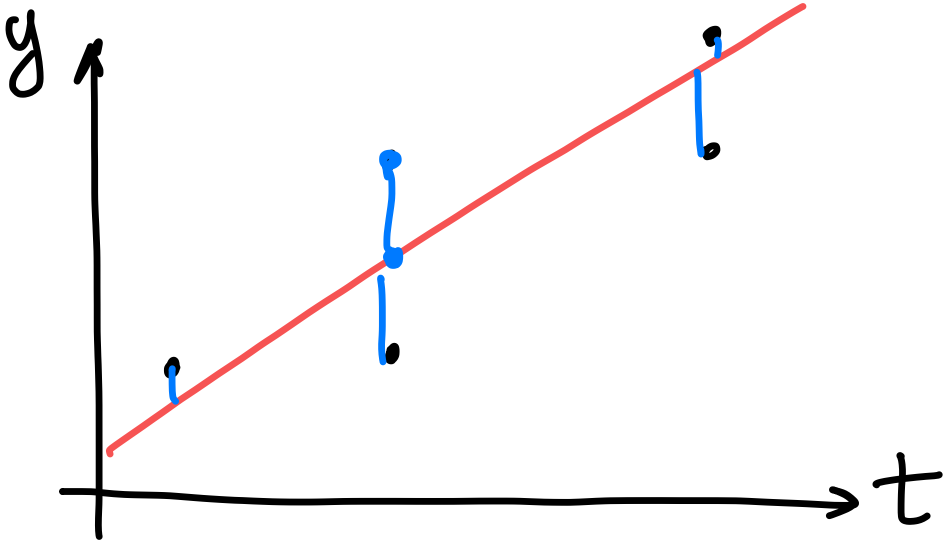

$$\begin{bmatrix}1 & t_1\\ 1 & t_2\\ \vdots & \\ 1 & t_m\end{bmatrix}\begin{bmatrix}x_0\\ x_1\end{bmatrix} = \begin{bmatrix}y_1\\ y_2\\ \vdots\\ y_m\end{bmatrix}$$

$m \times n$ $\qquad \underset{\sim}{x}_{n\times 1} \qquad \underset{\sim}{b}_{m\times 1}$

$\underset{=}{A}$

$$\boxed{y = x_0 + x_1 t}$$

m $\qquad$ n

$$b \notin \text{range}(A) \Rightarrow \boxed{\underset{=}{A}\,\underset{\sim}{x} \cong \underset{\sim}{b}}$$

$m \times n$ $\quad n \times 1$ $\quad n \times m$ $\quad m \times 1$

$$\min_{\underset{\sim}{x}} \|\underset{=}{A}\,\underset{\sim}{x} - \underset{\sim}{b}\|_2^2 \longrightarrow \text{Normal Equations}$$

$$\boxed{\text{A is full rank}} \qquad n \times m$$

$$\boxed{\underset{=}{A}^T \underset{=}{A}\,\underset{\sim}{x} = \underset{=}{A}^T \underset{\sim}{b}}$$

rank deficient $\longrightarrow$ SVD (no longer unique)

$$A = U_R \Sigma_R V^T$$

$\underset{=}{A}$: $m \times n$, U_R: $m \times n$, Σ_R: $n \times n$, V^T: $n \times n$

$\underset{\sim}{x}$ solution

$$\min_x \| \underset{=}{A} \underset{\sim}{x} - b \|_2^2$$

$+$

$$\min_x \|x\|_2$$

$$\underset{\sim}{x} = \underset{=}{V} \, \underset{=}{\Sigma_R^+} \, \underset{=}{U_R^T} \, b$$

x: $n \times 1$, V: $n \times n$, Σ_R^+: $n \times n$, U_R^T: $n \times m$, b: $m \times 1$

$\longrightarrow$ full rank

$$\Sigma_R^+ = \Sigma_R^{-1}$$

Example:

Consider solving the least squares problem $\boldsymbol{A}\,\boldsymbol{x} \cong \boldsymbol{b}$, where the singular value decomposition of the matrix $\boldsymbol{A} = \boldsymbol{U}\,\boldsymbol{\Sigma}\,\boldsymbol{V}^{T}\boldsymbol{x}$ is:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{x} \cong \begin{bmatrix} 12 \\ 9 \\ 9 \\ 10 \end{bmatrix}$$

Determine $\|\boldsymbol{b} - \boldsymbol{A}\,\boldsymbol{x}\|_2$

$$r = \begin{bmatrix} 0 \\ 0 \\ \boxed{} \to 10 \\ \boxed{} \to 9 \end{bmatrix}$$

$$\|r\| = \sqrt{10^2 + 9^2} = \sqrt{181}$$

$$\underset{\sim}{r} = \underset{=}{A}\,\underset{-}{x} - \underset{-}{b} = (U\Sigma V^T \underbrace{x}_{n\times 1} - \underbrace{b}_{m\times 1})$$

$$\|\underset{\sim}{r}\| \rightarrow \|\underbrace{U^T}_{m\times m}(\underbrace{U\Sigma}_{m\times n} \underbrace{V^T}_{n\times n} x - b)\|$$

$$\|\Sigma \underbrace{V^T x}_{y} - U^T b\| = \|\Sigma y - \underbrace{U^T}_{n\times m}\underbrace{b}_{m\times 1}\| \rightarrow n\times 1$$

$$y_i = \frac{u_i \cdot b}{\sigma_i}$$

$$\sigma_i \neq 0$$

$$\underbrace{\begin{bmatrix} \cancel{\boxed{\sigma_1}} & & & & & \\ & \cancel{\sigma_2} & & & & \\ & & \ddots & & & \\ & & & \cancel{\sigma_r} & & \\ & & & & 0 & \\ & & & & & \ddots \; 0 \\ & & \emptyset & & & \end{bmatrix}}_{\substack{m\times n \\ m>n}} \underbrace{\begin{bmatrix} \cancel{y_1} \\ \cancel{y_2} \\ \vdots \\ \cancel{y_r} \\ y_{r+1} \\ \vdots \\ y_n \end{bmatrix}}_{n\times 1} - \begin{bmatrix} \cancel{u_1 \cdot b} \\ \cancel{u_2 \cdot b} \\ \vdots \\ \cancel{u_r \cdot b} \\ u_{r+1} \cdot b \\ \vdots \\ u_n \cdot b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -u_{r+1}\cdot b \\ \vdots \\ -u_n \cdot b \end{bmatrix} \begin{matrix} \rightarrow 0 \\ \\ \\ \rightarrow r \\ \\ \\ \\ \end{matrix}$$

$\|r\| =$ la.norm( )

$$(U^T b)[r+1:]$$

not

python not.

$$\text{score} = w_1\,\text{rough} + w_2\,\text{size} + w_3\,t$$

feature

$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_m \end{array} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \text{score}_1 \\ \text{score}_2 \\ \vdots \\ \text{score}_m \end{bmatrix} \to \begin{bmatrix} B \\ M \\ \\ \end{bmatrix}$$

row

number → label