Algorithms and Data Structures for Data Science

Lists Implementation

CS 277

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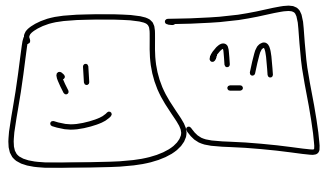
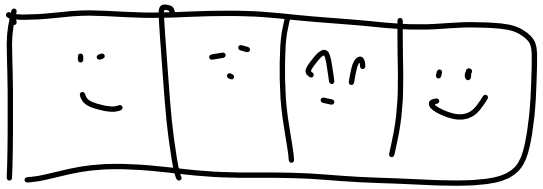
February 5, 2024





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Department of Computer Science

Learning Objectives

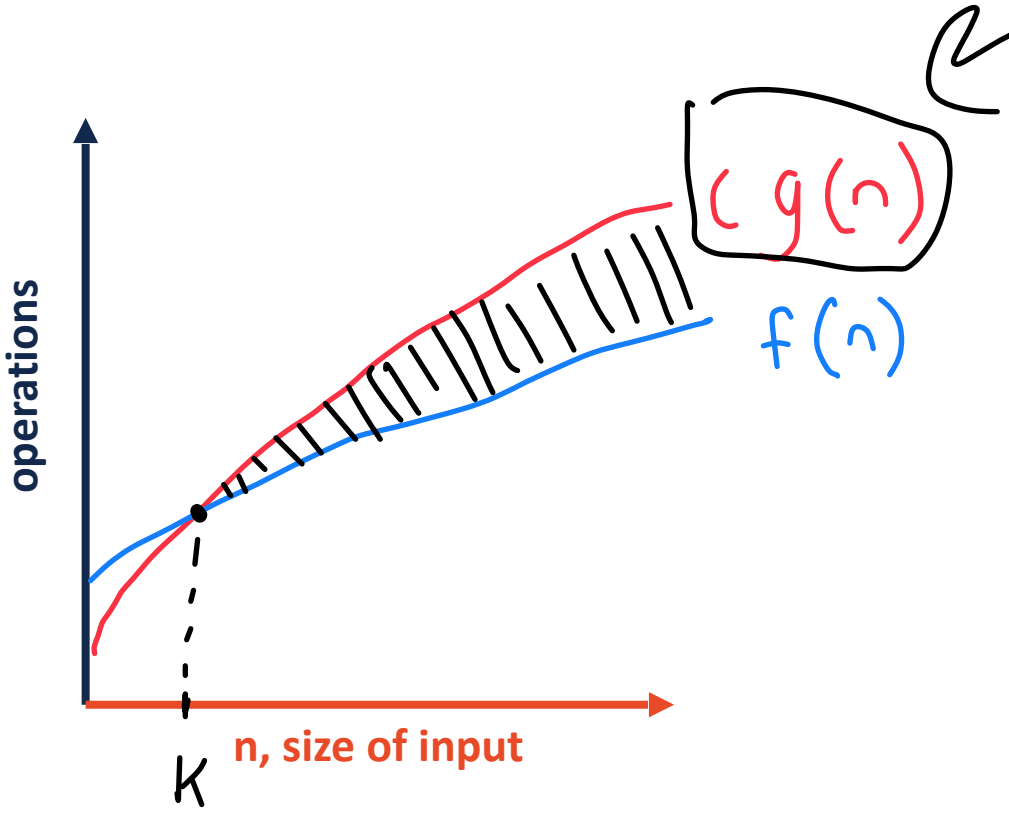
List Implementations

Re-introduce and review Big O

Discuss list implementation strategies in the context of Big O

Big-O notation

$f(n)$ is $O(g(n))$ iff $\exists c, k$ such that $f(n) \leq cg(n)\ \forall n > k$



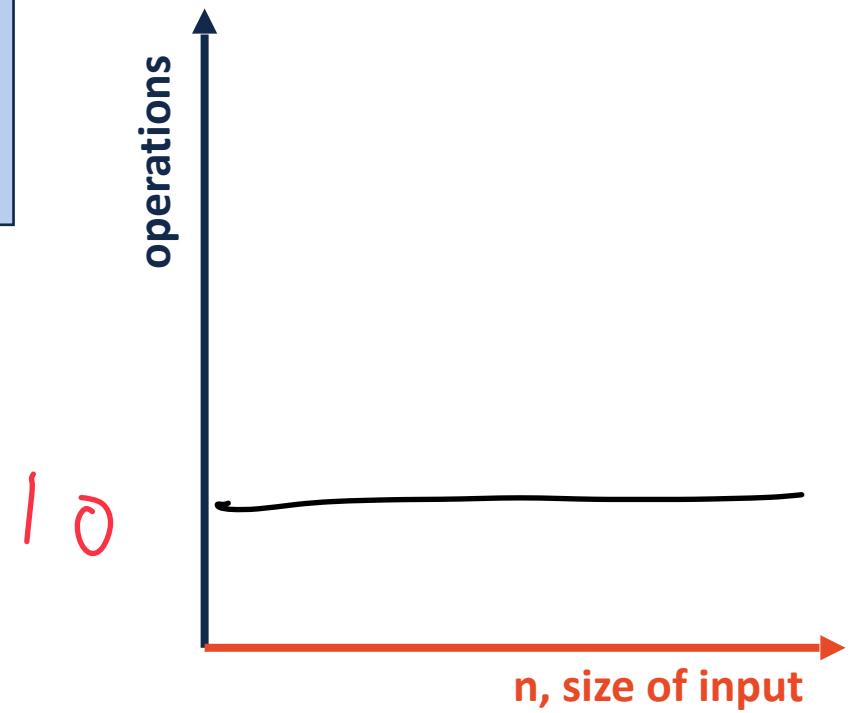
1) $cg(n)$ is an upper bound on $f(n)$

2) This is true for all input values larger than some arbitrary k

↳ as $n \to \infty$

Constant Time, O(1)

```
1  def constant(n):
2      ops = 0
3      for i in range(10):
4          ops+=n
5      return ops
6
7  print(constant(5))
8  print(constant(9001))
```



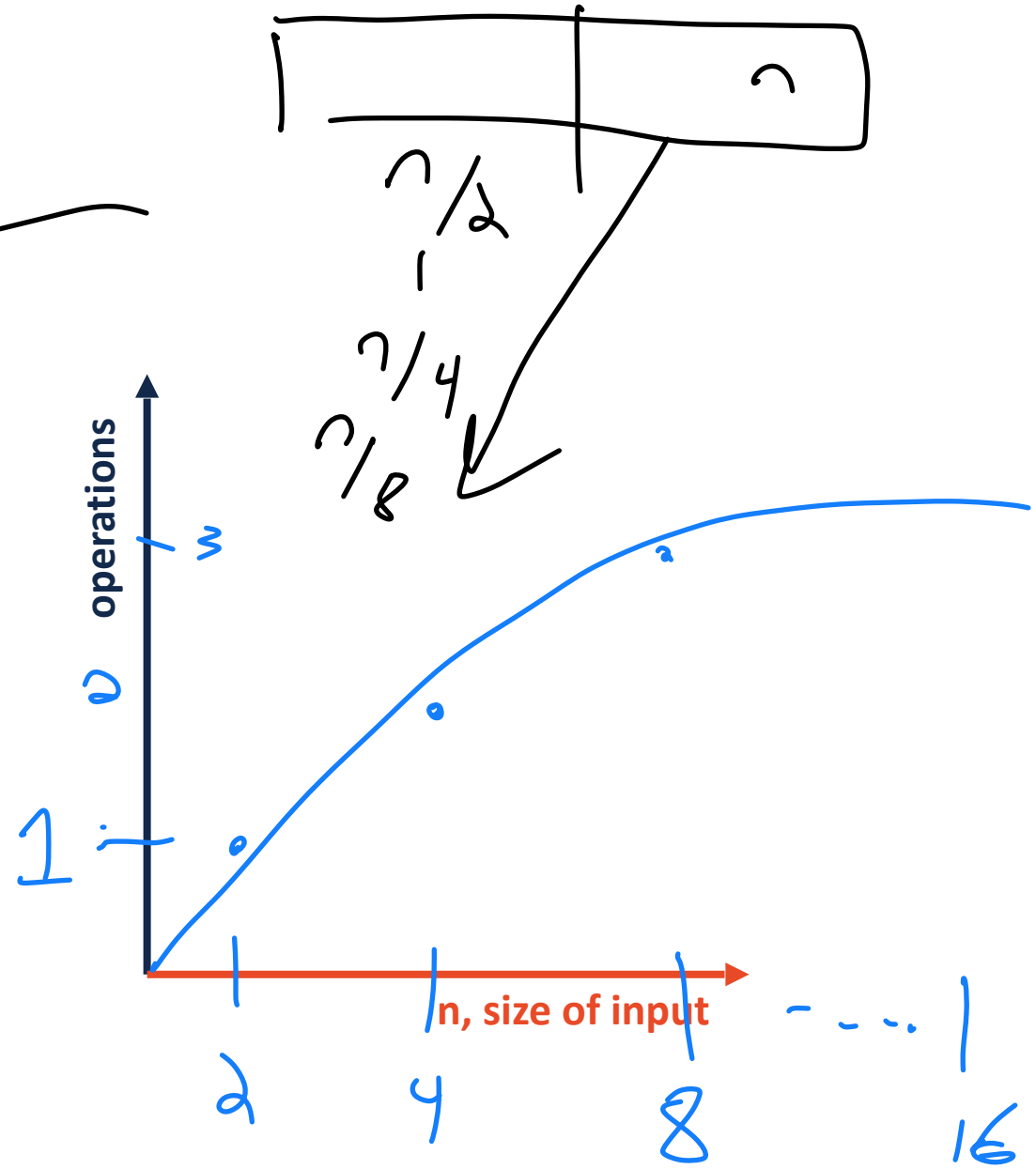
Logarithmic Time, O(log n)

```
1 import math
2 def logarithmic(n):
3     ops = 0
4     for i in range(int(math.log2(n))):
5         ops+=1
6     return ops
7
8 print(logarithmic(5))
9 print(logarithmic(9001))
```

$\log_{10} 10 = 1$

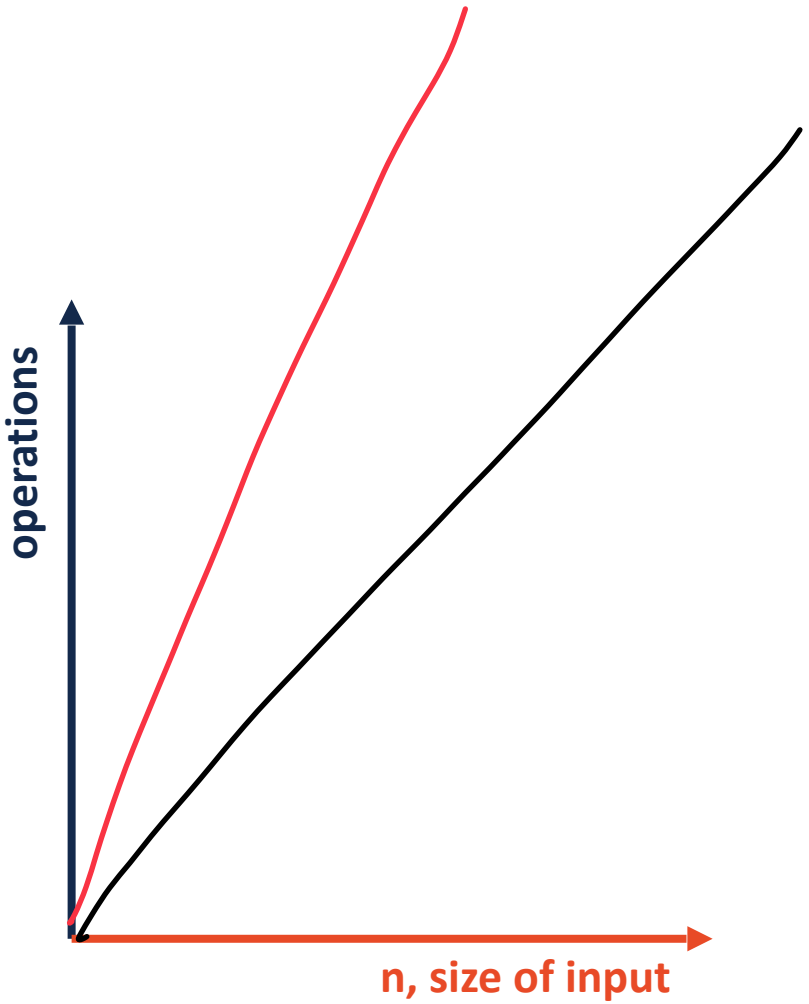
$100 = 2$

$1000 = 3$



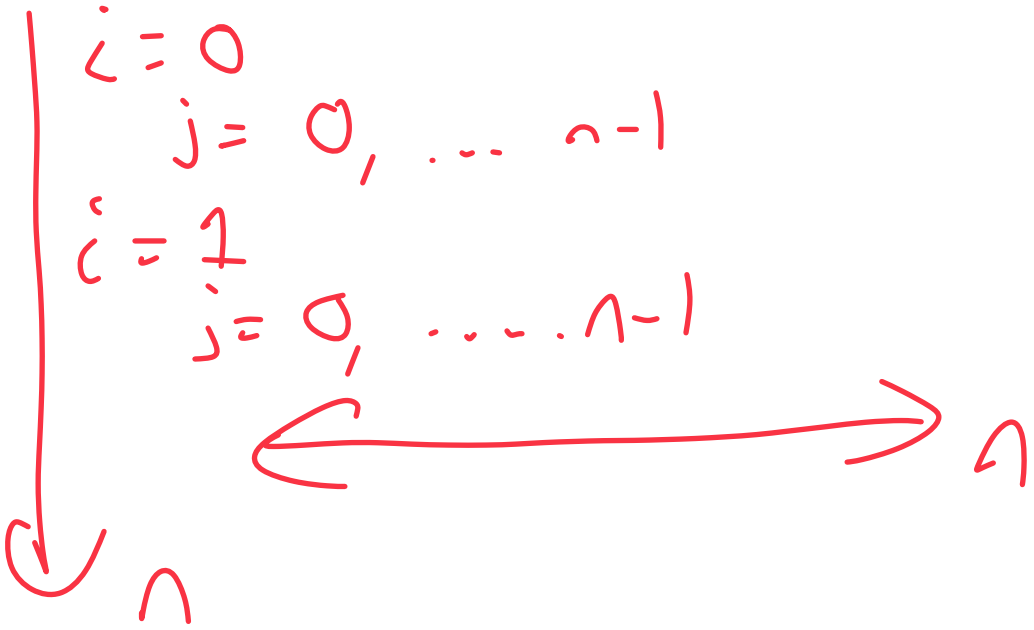
Linear Time, O(n)

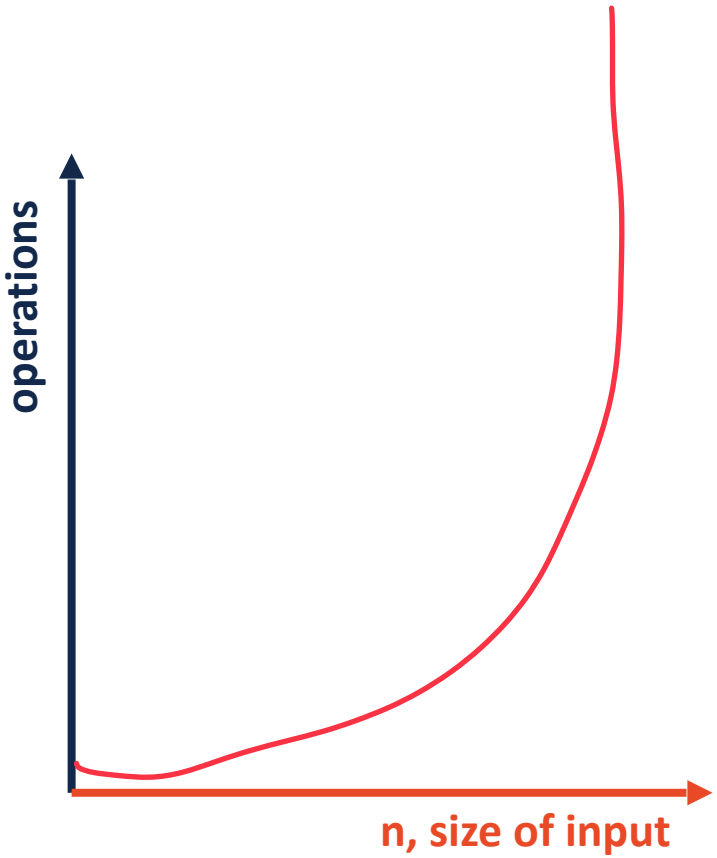
```
1  def linear(n):
2      ops = 0
3      for i in range(n):
4          ops+=1
5      return ops
6
7  print(linear(5))
8  print(linear(9001))
```



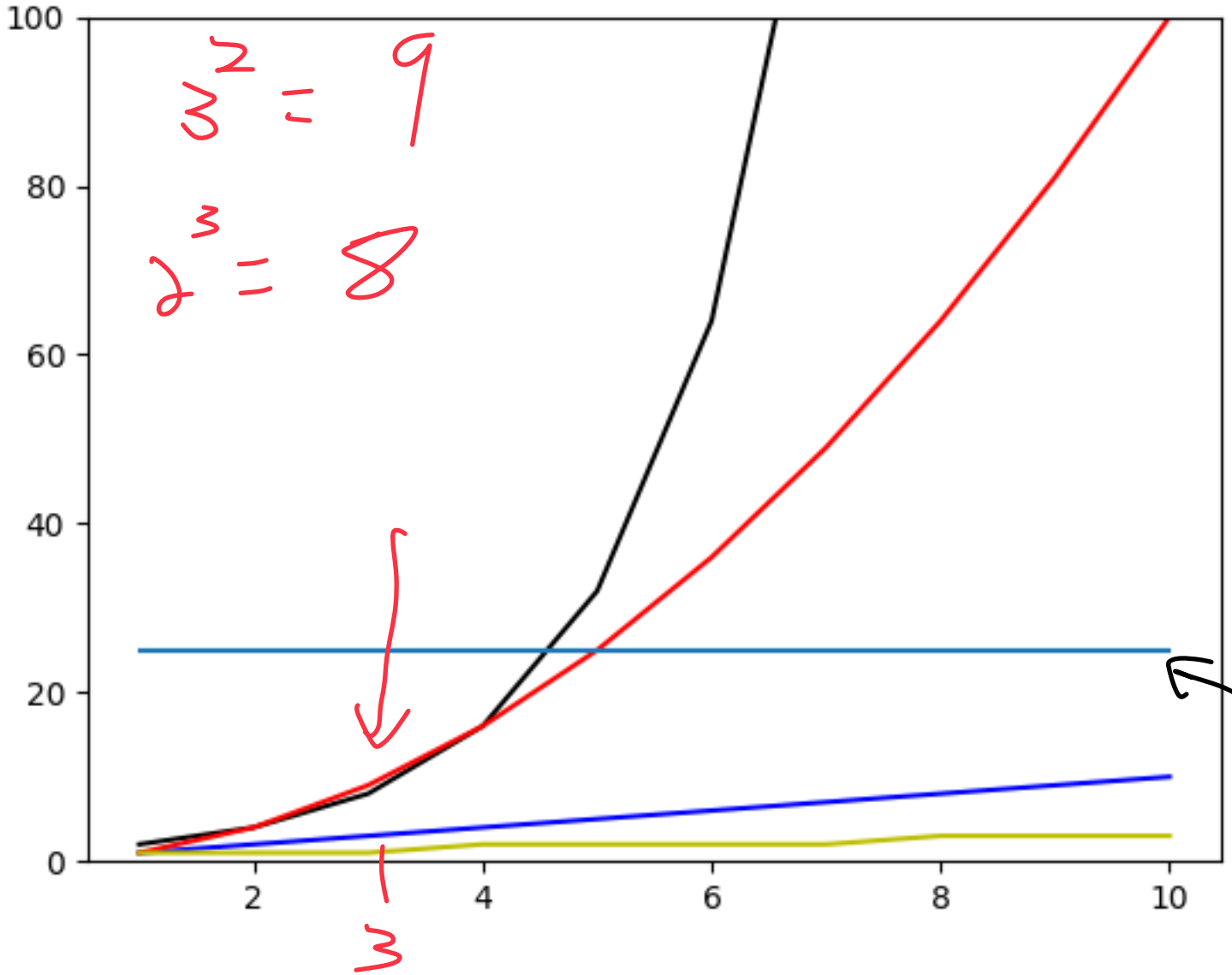
Quadratic Time, $O(n^2)$

```
1  # Quadratic Time
2  def quadratic(n):
3      ops = 0
4      for i in range(n):
5          for j in range(n):
6              ops+=1
7      return ops
8
9  print(quadratic(5))
10 print(quadratic(9001))
```





Big-O Complexity Classes



$O(2^n)$

$O(n^2)$

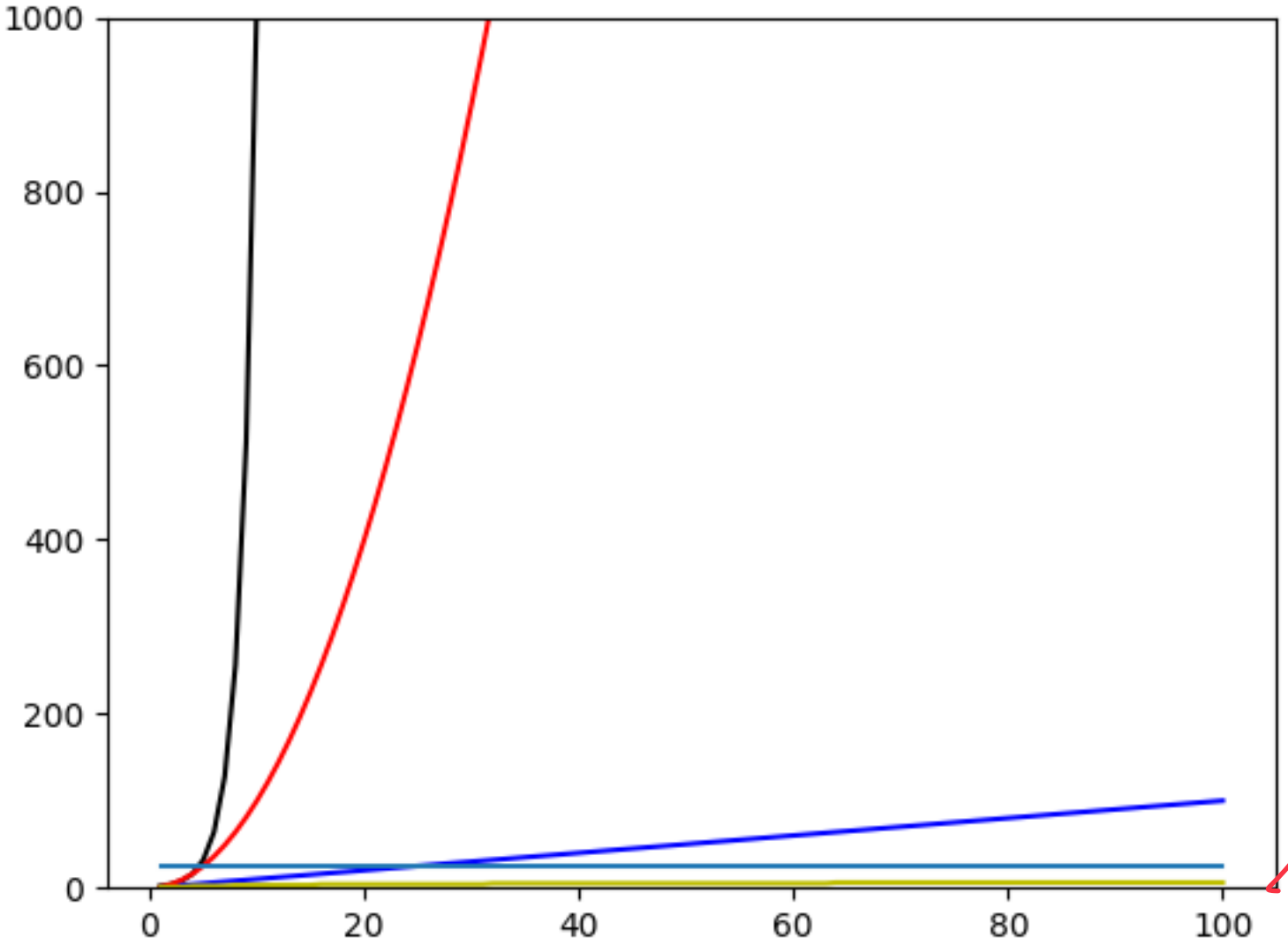
$O(n)$

$O(\log n)$

$O(1)$

3rd best

Big-O Complexity Classes



$O(2^n)$

$O(n^2)$

$O(n)$

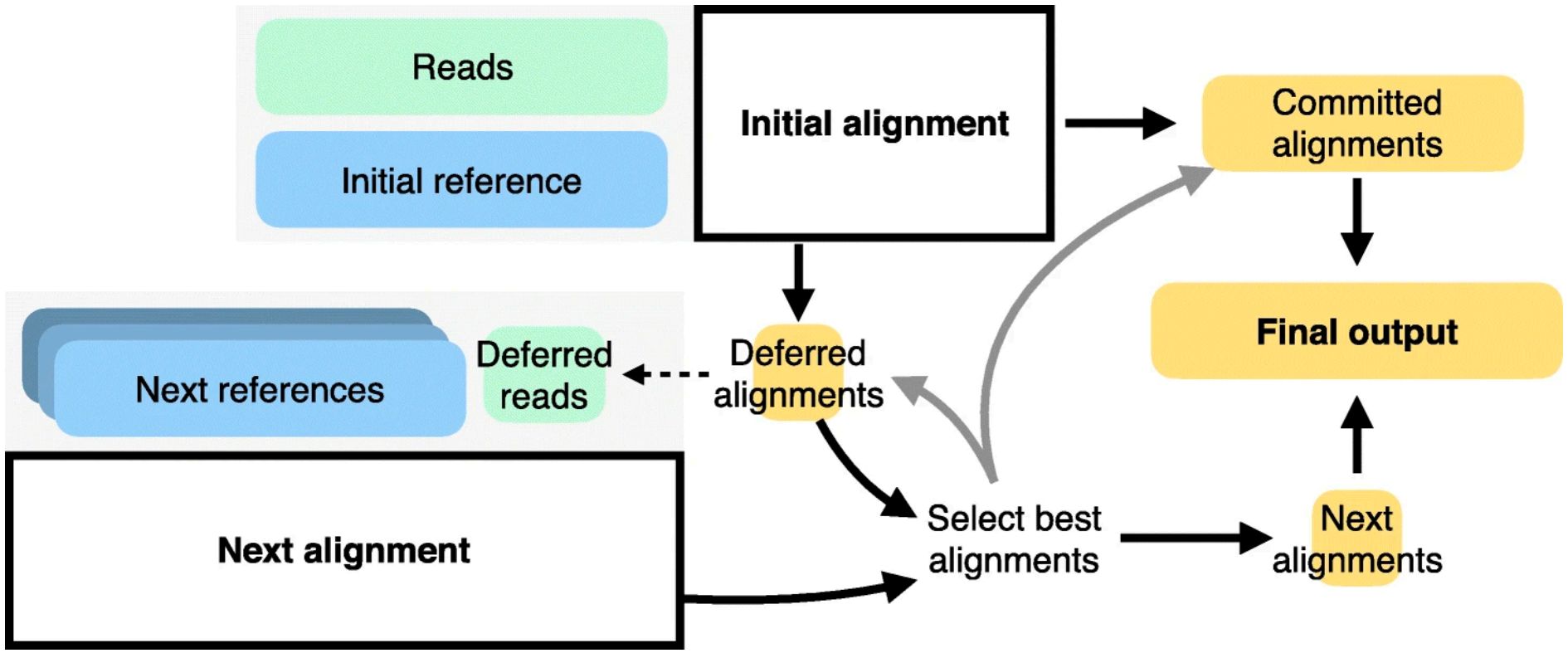
$O(\log n)$

$O(1)$

Big O: Communicating Efficiency

I want to find the most efficient data processing pipeline

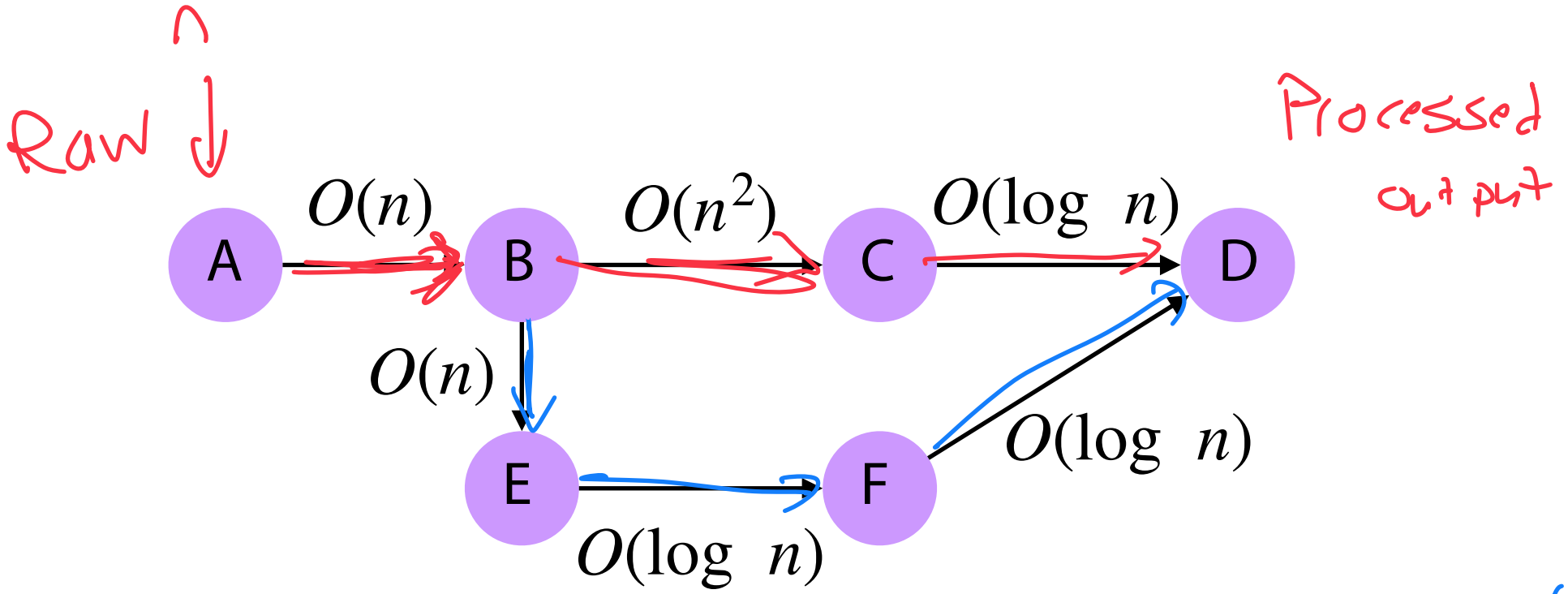
Bottleneck?



Reference flow: reducing reference bias using multiple population genomes. Chen et al 2021

Big O: Communicating Efficiency

I want to find the most efficient data processing pipeline

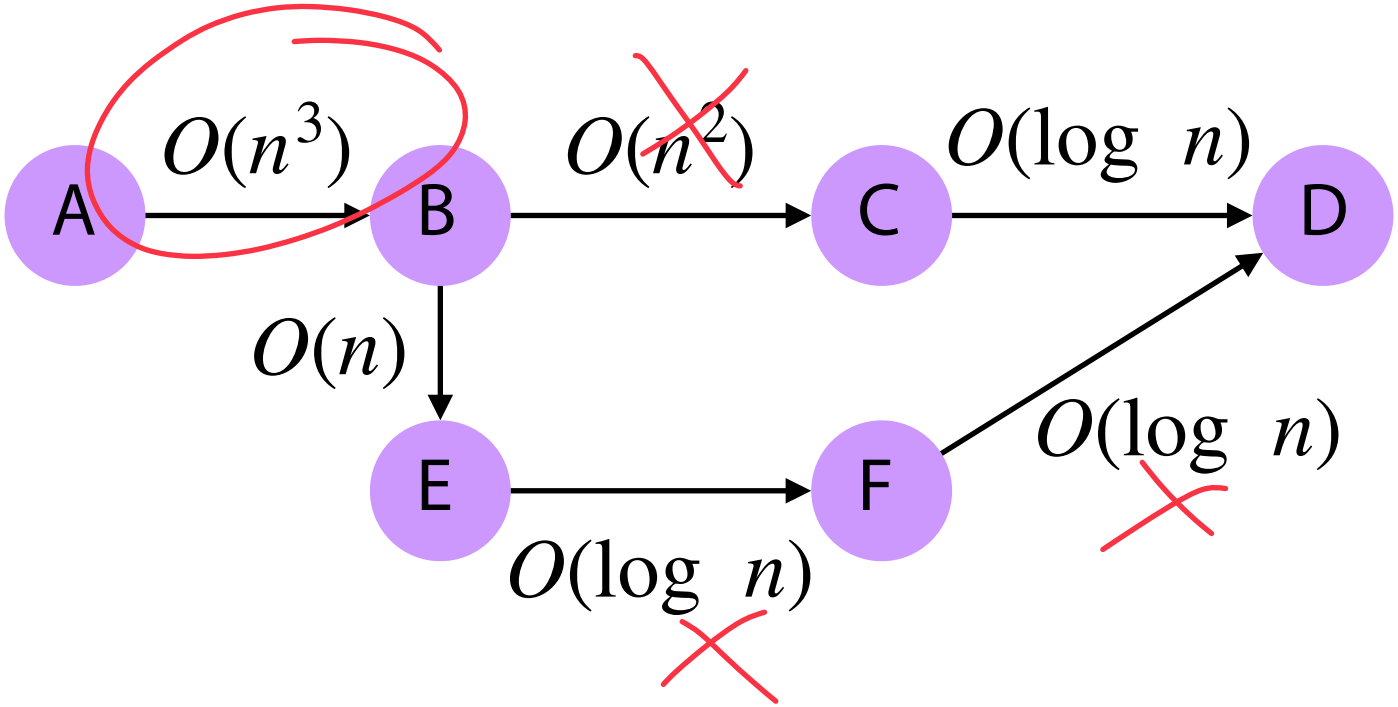


p1: $O(n + n^2 + \log n) \simeq O(n^2)$

p2: $O(n + \log n + \log n) \simeq O(n)$

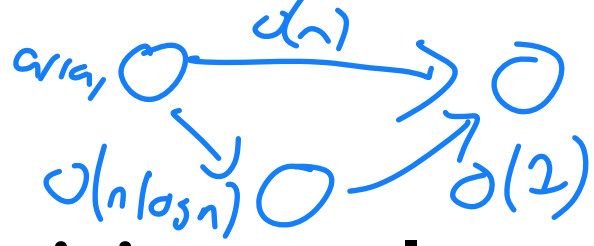
Big O: Communicating Efficiency

I want to find the most efficient data processing pipeline



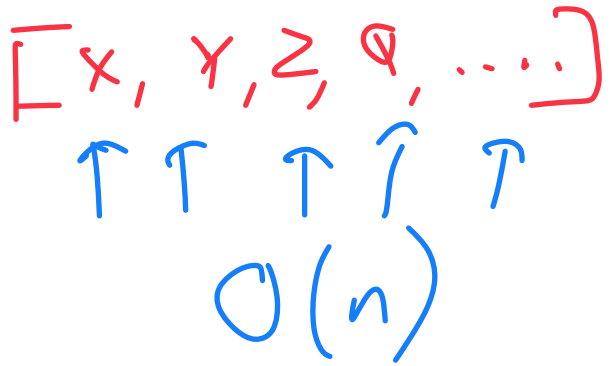
$O(n^3)$

Big O: Communicating Efficiency



I want to design the best approach to finding the minimum value.

Given n items, what is my Big O to find the min?



Given n items which are sorted, what is my Big O to find the min?

[min] or [... min]

O(1)

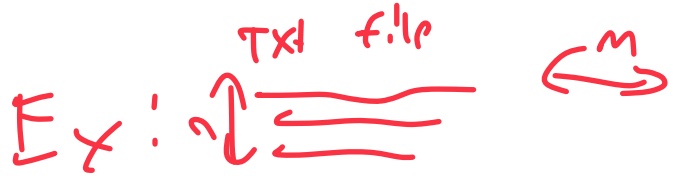
What is the runtime to sort an unsorted dataset? ~ O(n log n)

Identifying the Big O of an algorithm

1) Label the key factors that drive algorithm performance

↳ What is my input?

↳ What inputs change or grow in size?

Ex: txt file 

2) Write out the worst-case performance for each step

↳ In terms of key factors

3) Identify (or reduce to) the largest terms for each factor

↳ Identify largest term "Dominant term"

↳ Ignore constants

Big O: Identifying key factors (variables)

Imagine I have a PNG and want to count the number of blue pixels

What are the variables I need to keep track of?

If I look at each pixel individually, what is my Big O?

Big O Practice: Simplifying to highest order

What is the big O for the following functions?

$$a(n) = n^4 + 50n + 10$$ → $O(n^4)$

$$b(n) = 500nlog\ n + 50n + log(n)$$ → $O(n \log n)$

$$c(n) = n^3 + 3n! + 12$$ → $O(n!)$

$n > \log n$

$$d(n) = n^2 + nlog\ n$$ → $O(n^2)$

$n \cdot n > n \log n$

Big O Practice: Reading code

n = 10, 100

```
1  def doStuff(inList1, inList2):
2
3      c1 = 0
4      for i in inList1:
5          c1+=1
6
7
8
9      c2 = 0
10     for v1 in inList1:
11         for v2 in inList2:
12             c2+=1
13
14
15
16     return c1, c2
17
18
19
20
21
22
23
```

O(n)

O(n * m)

O(n * m + n)

1) What are my variables?

n = size of inlist 1

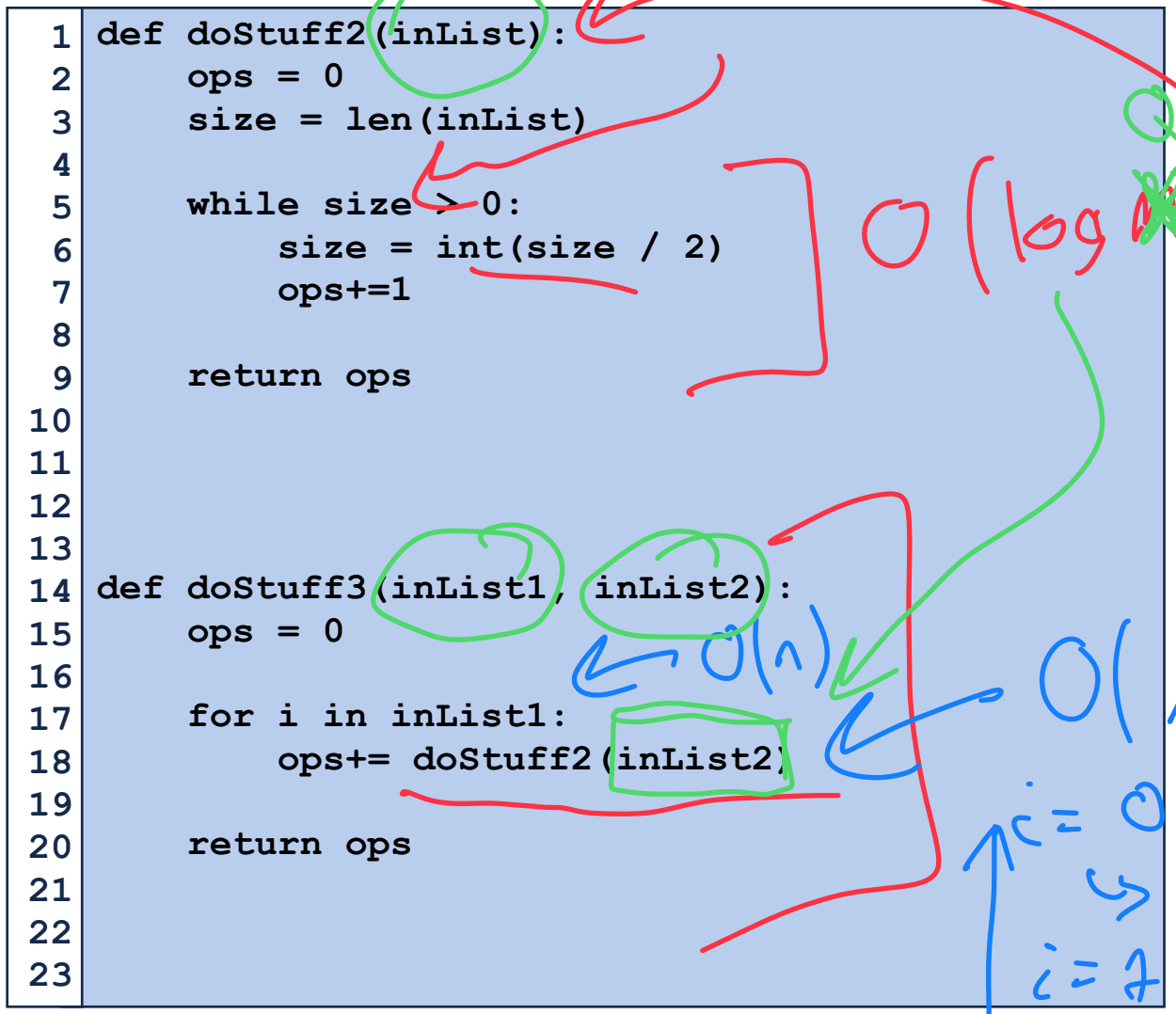
m = size of inlist 2

2) Define our code by giving runtime to each "function", "snippet"

3) do stuff is O(n m)

n vs m in size?

Big O Practice: Reading code



```
1  def doStuff2(inList):
2      ops = 0
3      size = len(inList)
4
5      while size > 0:
6          size = int(size / 2)
7          ops+=1
8
9      return ops
10
11
12
13
14 def doStuff3(inList1, inList2):
15     ops = 0
16
17     for i in inList1:
18         ops+= doStuff2(inList2)
19
20     return ops
21
22
23
```

Big O of the List ADT

A minimally functional list must have the following functions:

Constructor: `__init__()`

Insert: `append(x)` `insert(i, x)`

Delete: `remove(x)` `pop()`

Index `__getitem__()` `[]` `index(x)`

Size() `len(list)`

Python List

There are many implementations of lists in Python. Here are three*:

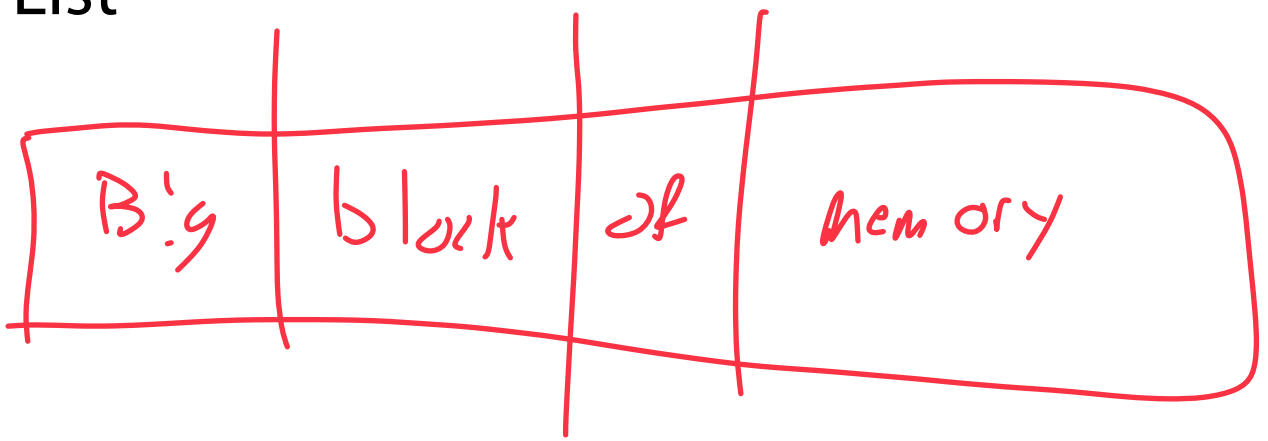
```
1  myList = [1, 2, 3, 4, 5]
2
3  print(myList)
4
5  print(len(myList))
6
7  print(myList[2])
8
9
```

```
1  myTuple = (1, 2, 3, 4,
2  5)
3
4  print(myTuple)
5
6  print(len(myTuple))
7
8  print(myTuple[2])
9
```

```
1  import numpy as np
2  myNP =
3  np.array([1,2,3,4,5])
4
5  print(myNP)
6
7  print(len(myNP))
8
9  print(myNP[2])
```

(Theoretical) List Implementations

1. Array List



2. Linked List

Creating an array list

```
1  l1 = [None]*5
2
3  l2 = [0, "1", 2.0, "W", (1, 2,)]
4
5  np1 = np.zeros(5)
6
7  np2 = np.empty(5)
```

heterogeneous

homogeneous (numbers)

Costs more Memory ??

Memory ??? Variable

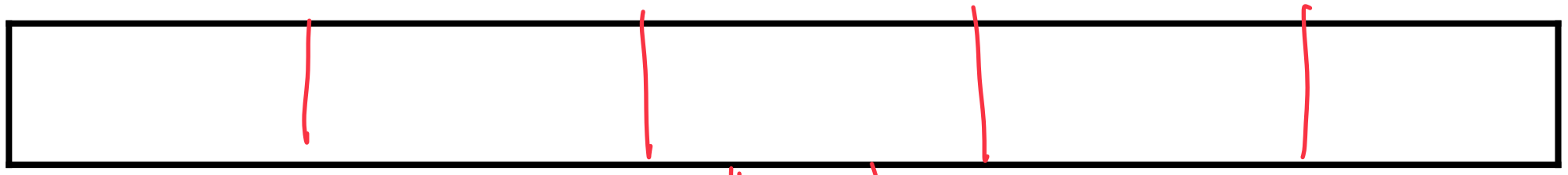
??!!

Conceptual Idea 1:

Much easier to search

Conceptual Idea 2:

Fixed size



equal size on all blocks

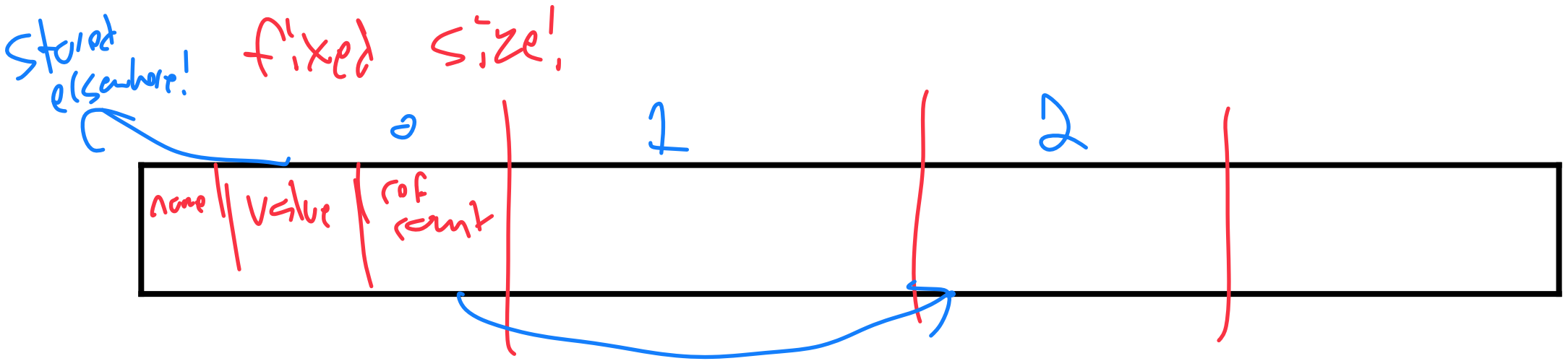
Python lists are stored as PyObjects

```
1  l1 = [None]*5
2
3  l2 = [0, "1", 2.0, "W", (1, 2,)]
4
5  m1 = np.zeros(5)
6
7  m2 = np.empty(5)
8
9
10
11
12
13
14
```

Best of both worlds!

↳ Index List[] O(1)

↳ store mixed data

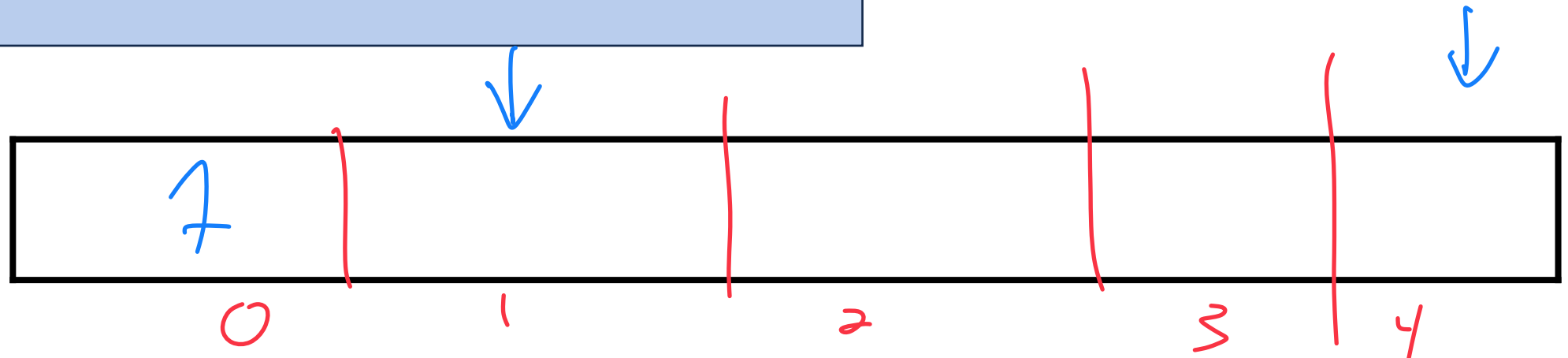


Python List Memory Allocation

```
1  l1 = [1, 2, 3]
2  print(sys.getsizeof(l1))
3
4  l2 = [(1, 2, 3,), (4, 5, 6), (7, 8, 9)]
5  print(sys.getsizeof(l2))
6
7  bigString=""
8  for i in range(10**4):
9      bigString+="A"
10
11 print(sys.getsizeof(bigString))
12
13 l3 = [bigString]
14
15 print(sys.getsizeof(l3))
16
17
```

Not annotation for size but in-class question

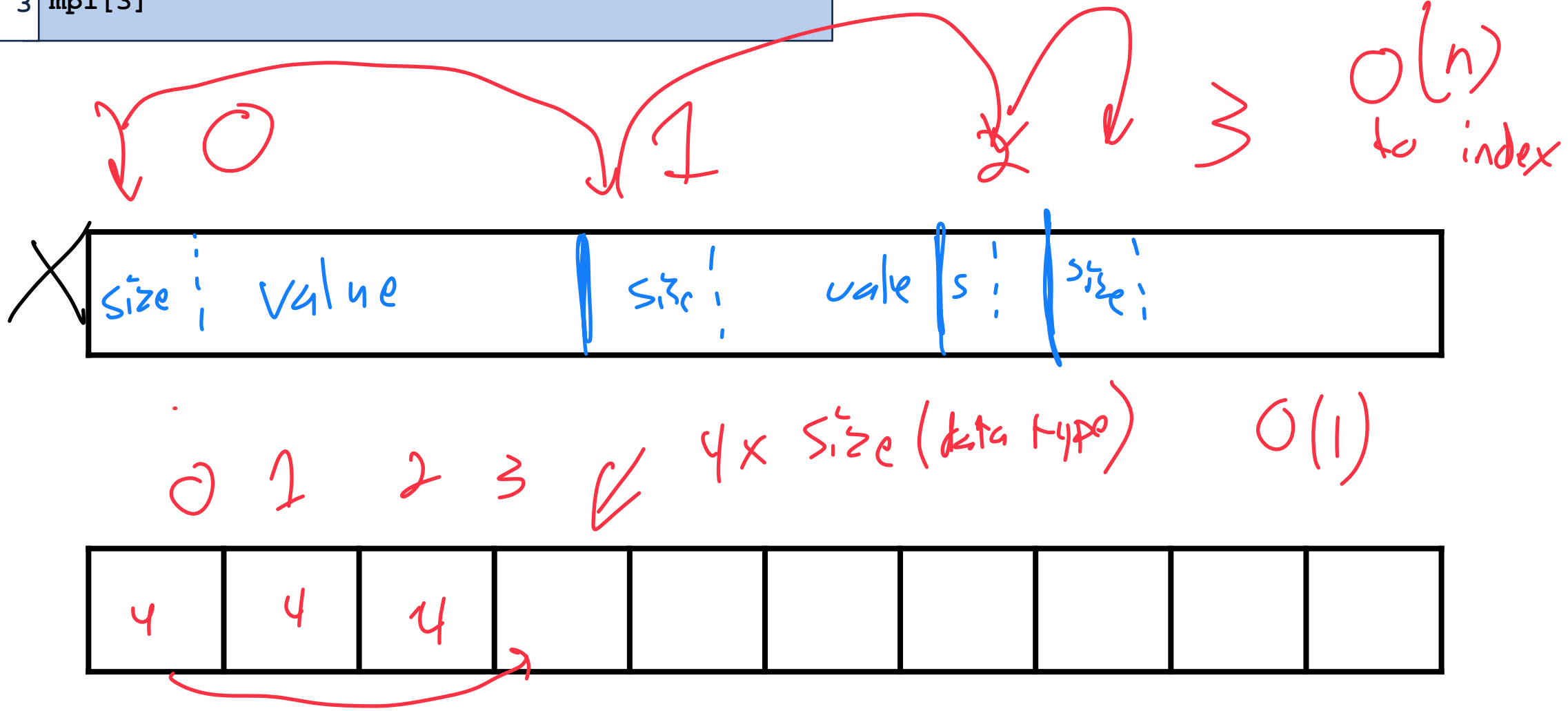
1) size – # of items
   ↳ 1
2) capacity – max # items
   ↳ 5



Array __getitem__()

Top line not used in Python?

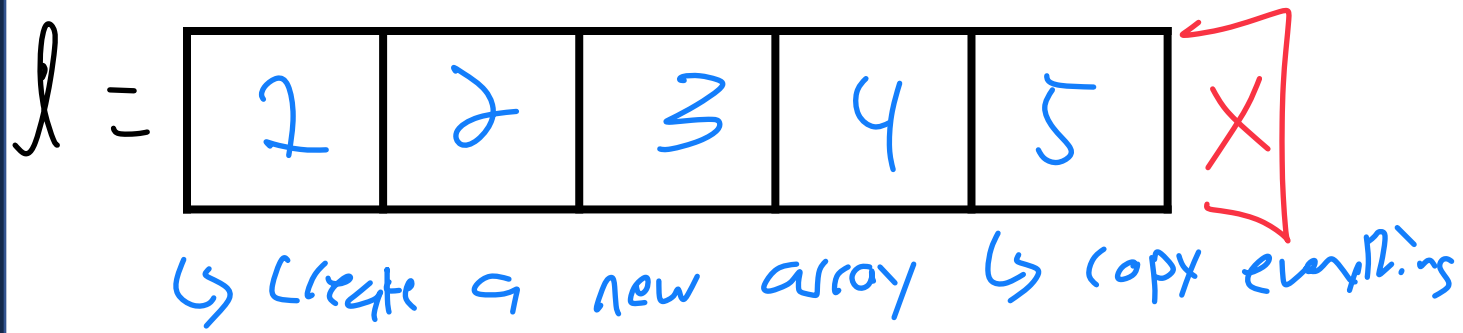
```
1 l2[3]
2
3 mp1[3]
```



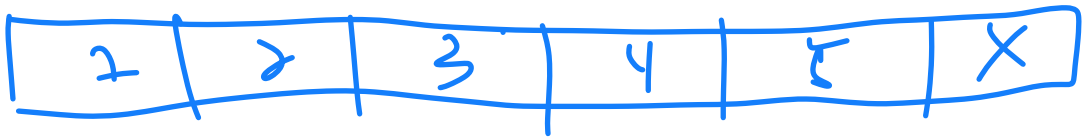
Array 'Add'

Python command to add to back of list:

situation 1 O(n)

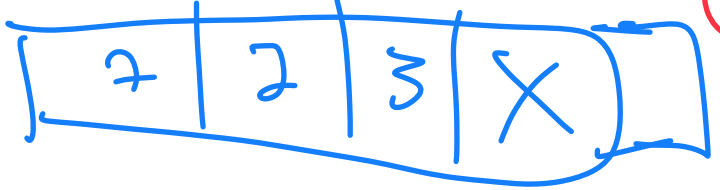


↳ Create a new array ↳ copy everything



l.append(x) → ???

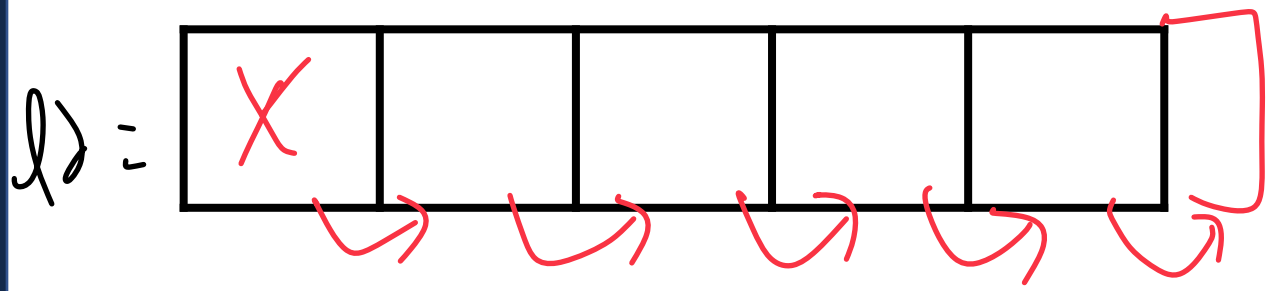
situation 2 O(1)



Python command to add to front of list:

l.insert(0, x)

↳ O(n)



In-Class Brainstorm

We don't want to have to remake a new array every time we 'add'.

What sorts of things can we do?

Need to make new array?

1) Allocate new memory O(1)
2) Copy every value O(n) :(

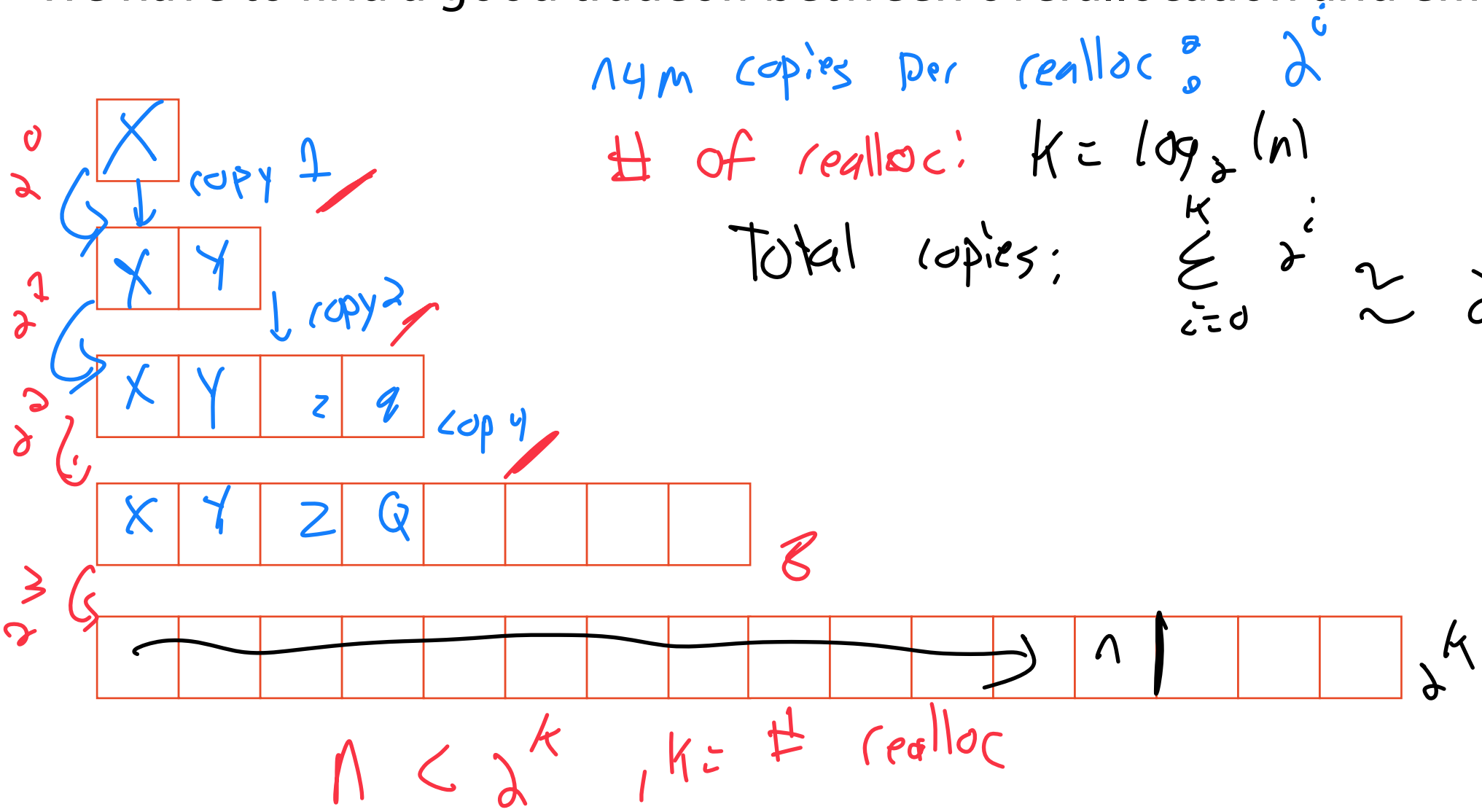
Need to overallocate space!

✓ too much space allocated is inefficient!

\ too little and our runtime is bad

Resize Strategy: x2 elements every resize

We have to find a good tradeoff between overallocation and efficiency!

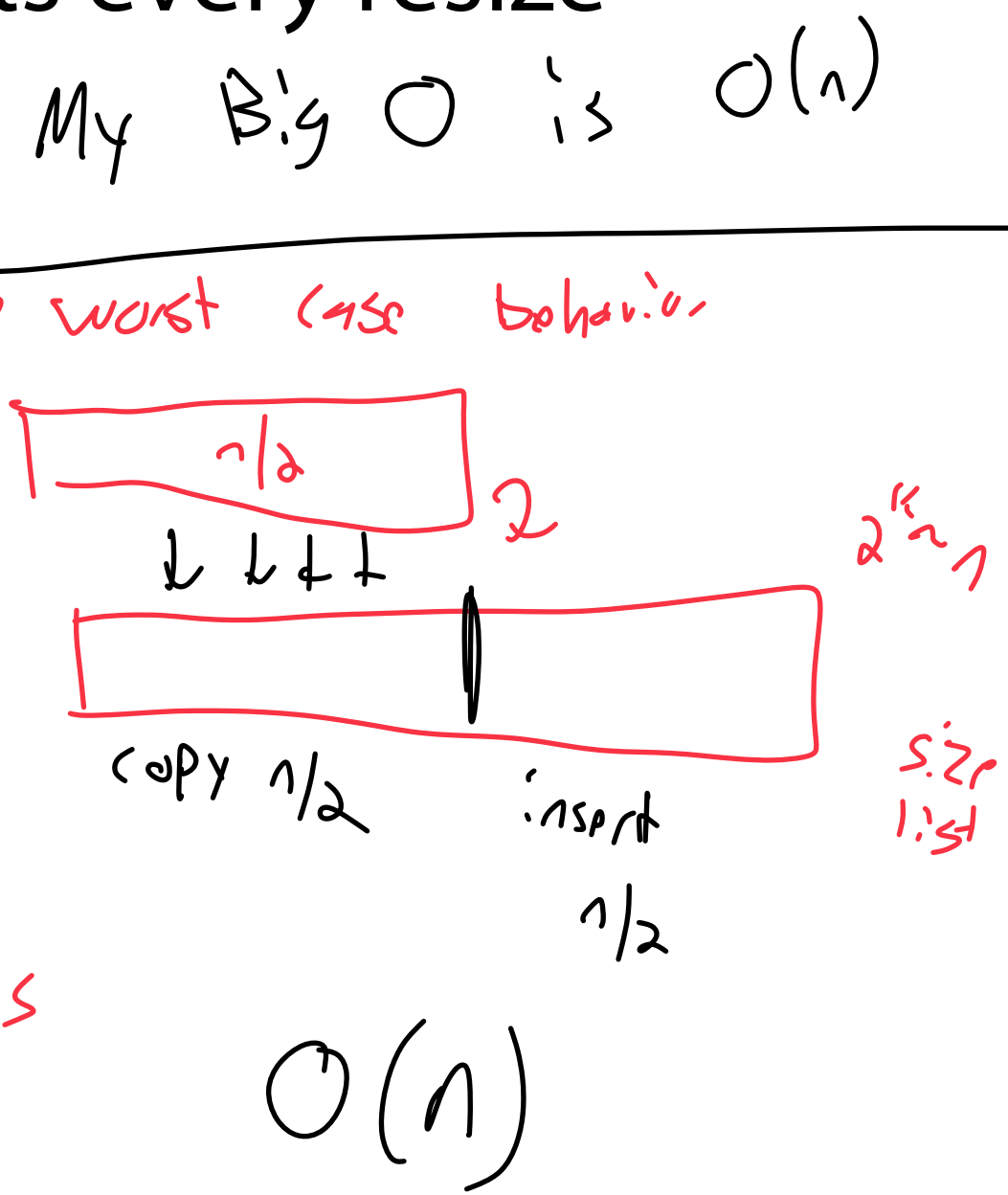


Resize Strategy: x2 elements every resize

Total copies for n inserts: $2n - 1$

Total work for 1 insert: $\frac{2n-1}{n}$

2 "work" copies



Python List Size Implementation

```
 1 memory_size = {}
 2
 3 for length in range(50):
 4     lst = []
 5     for length_loop in range(length):
 6         lst.append(length_loop)
 7     memory_size[length] = sys.getsizeof(lst)
 8
 9 print(memory_size)
10
```

```
{0: 56, 1: 88, 2: 88, 3: 88, 4: 88, 5: 120, 6: 120, 7: 120, 8: 120, 9: 184, 10: 184, 11:
184, 12: 184, 13: 184, 14: 184, 15: 184, 16: 184, 17: 248, 18: 248, 19: 248, 20: 248, 21:
248, 22: 248, 23: 248, 24: 248, 25: 312, 26: 312, 27: 312, 28: 312, 29: 312, 30: 312, 31:
312, 32: 312, 33: 376, 34: 376, 35: 376, 36: 376, 37: 376, 38: 376, 39: 376, 40: 376, 41:
472, 42: 472, 43: 472, 44: 472, 45: 472, 46: 472, 47: 472, 48: 472, 49: 472}
```

Python List Size Implementation

```
 1 memory_size = {}
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 3 for length in range(50):
 4     lst = []
 5     for length_loop in range(length):
 6         lst.append(length_loop)
 7     memory_size[length] = sys.getsizeof(lst)
 8
 9 print(memory_size)
10
```

Python will hardcode small cases

```
{0: 56, 1: 88, 2: 88, 3: 88, 4: 88, 5: 120, 6: 120, 7: 120, 8: 120, 9: 184, 10: 184, 11:
184, 12: 184, 13: 184, 14: 184, 15: 184, 16: 184, 17: 248, 18: 248, 19: 248, 20: 248, 21:
248, 22: 248, 23: 248, 24: 248, 25: 312, 26: 312, 27: 312, 28: 312, 29: 312, 30: 312, 31:
312, 32: 312, 33: 376, 34: 376, 35: 376, 36: 376, 37: 376, 38: 376, 39: 376, 40: 376, 41:
472, 42: 472, 43: 472, 44: 472, 45: 472, 46: 472, 47: 472, 48: 472, 49: 472}
```

Numpy List Implementation

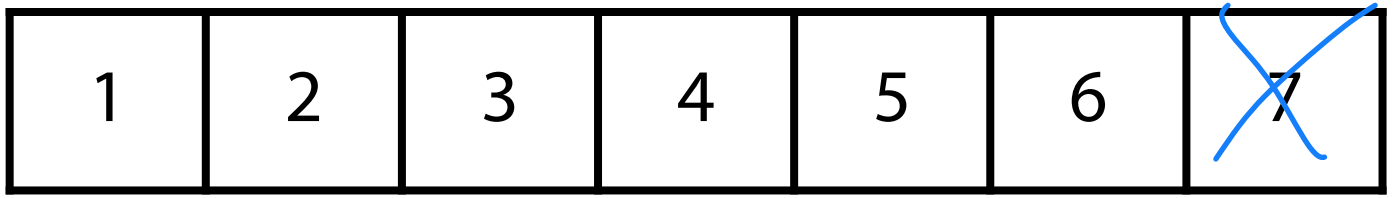
```
1  nms = {}
2  for length in range(50):
3      npa = np.array([])
4      for length_loop in range(length):
5          npa = np.append(npa, length)
6      nms[length] = sys.getsizeof(npa)
7
8  print(nms)
9
10
```

Numpy likes fixed data sets!

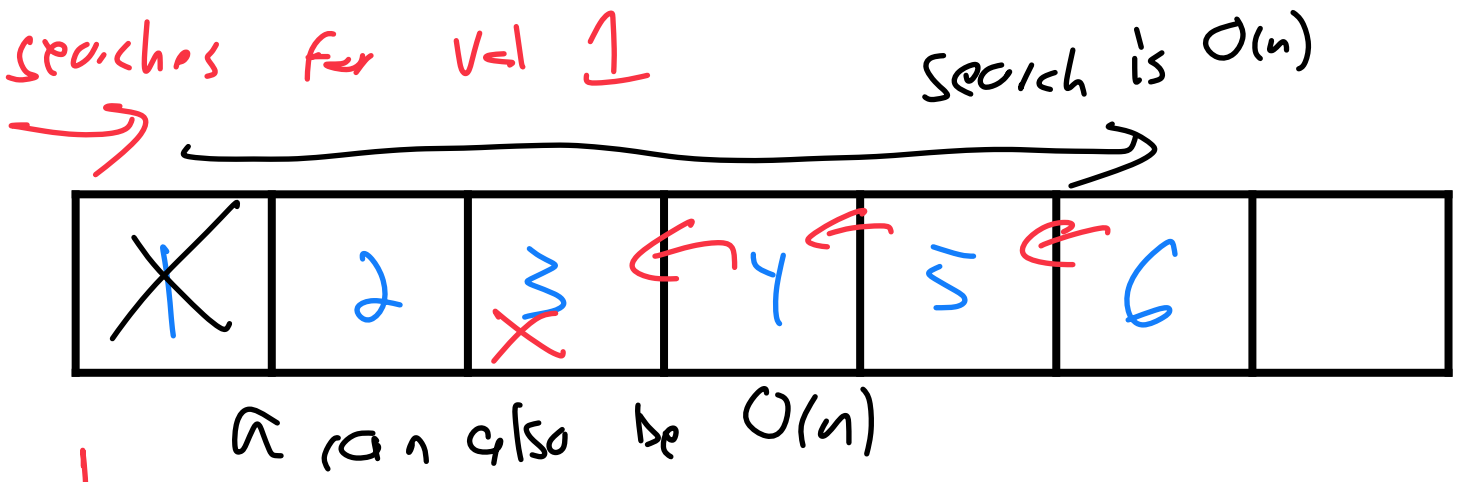
```
{0: 112, 1: 120, 2: 128, 3: 136, 4: 144, 5: 152, 6: 160, 7: 168, 8: 176, 9: 184, 10: 192,
11: 200, 12: 208, 13: 216, 14: 224, 15: 232, 16: 240, 17: 248, 18: 256, 19: 264, 20: 272,
21: 280, 22: 288, 23: 296, 24: 304, 25: 312, 26: 320, 27: 328, 28: 336, 29: 344, 30: 352,
31: 360, 32: 368, 33: 376, 34: 384, 35: 392, 36: 400, 37: 408, 38: 416, 39: 424, 40: 432,
41: 440, 42: 448, 43: 456, 44: 464, 45: 472, 46: 480, 47: 488, 48: 496, 49: 504}
```

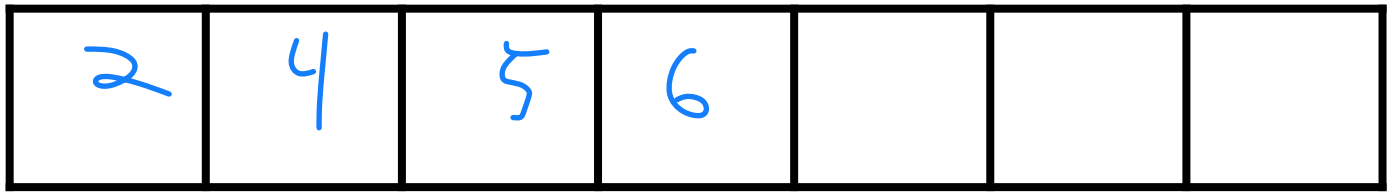
Array remove()

```
1  l = [1, 2, 3, 4, 5, 6, 7]
2  l.pop()
3
4  l.remove(1)
5
6  l.pop(3)
7
8
```



↳ O(n) is correct!
↳ I have to sometimes reallocate my list
↳ If not removing at end have to move all items over





??? when to reduce in size

Experiment on your own: Python remove

Practice your Python programming and confirm your hypothesis

How does Python's list size change as you remove objects? Numpy's?

↳ about half in size?

↓

Immediately

Array Implementation

	Array
Look up given an input **position**	O(1)
Search given an input **value**	O(n)
Insert/Remove at **front**	O(n)
Insert/Remove at **arbitrary** location	O(n)

Usually is sometimes fast