Algorithms and Data Structures for Data Science
Graph Traversals

CS 277
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Learning Objectives

Practice using NetworkX to build and explore graphs

Implement breadth and depth traversals on graphs

Extend NetworkX for weighted and directed graphs

NetworkX Graph ADT

Find

getVertices() —> list(G.nodes())

getEdges(v) —> G[v]

areAdjacent(u, v) —> G.has_edge(u, v)

Insert

insertVertex(v) —> G.add_node(v)

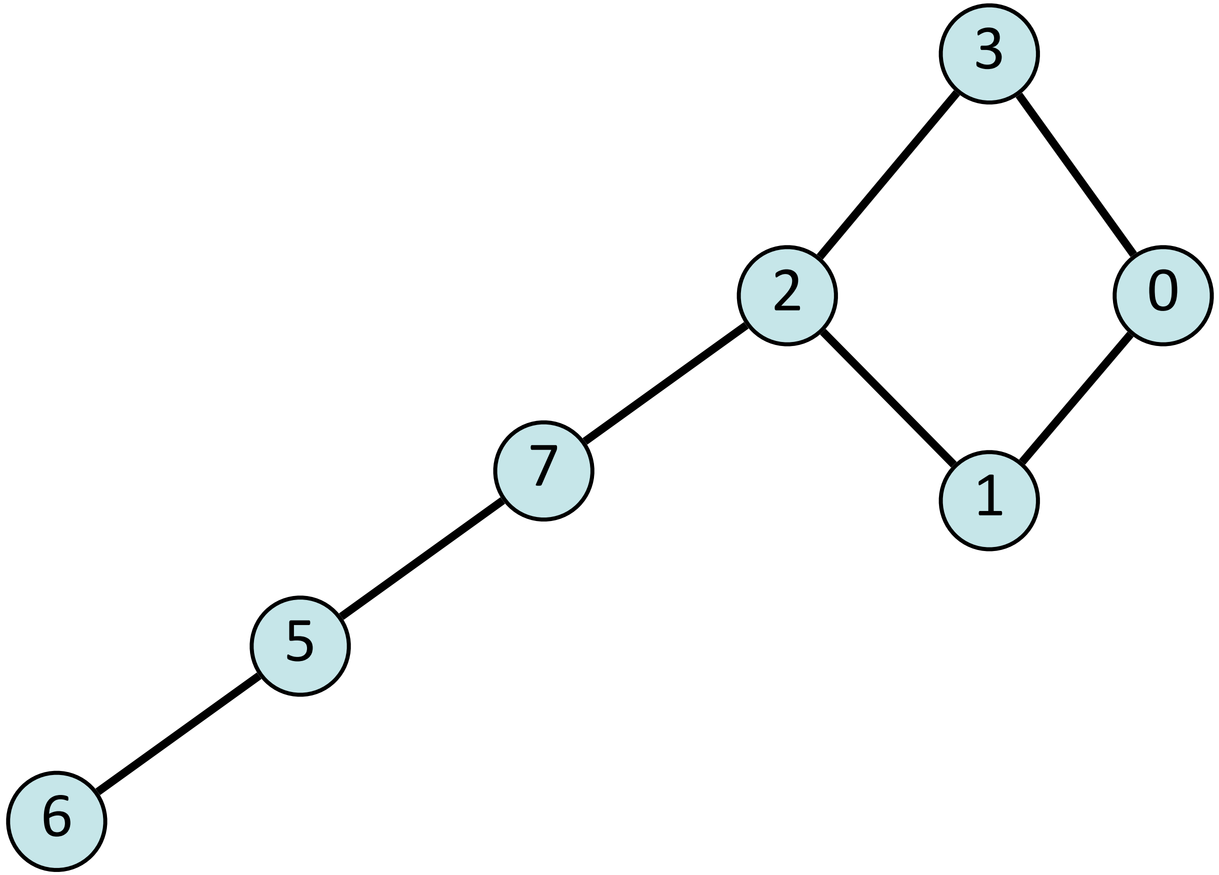
insertEdge(u, v) —> G.add_edge(u, v)

Remove

removeVertex(v) —> G.remove_node(v)

removeEdge(u, v) —> G.remove_edge(u, v)

Graph Practice 1: Build the following graph



Graph Practice 1: Build the following graph

We can build a graph in NetworkX by adding edges one at a time:

```
G = nx.Graph()

G.add_edge(0, 1)

G.add_edge(1, 2)

G.add_edge(2, 3)

G.add_edge(3, 0)

G.add_edge(5, 6)

G.add_edge(5, 7)

G.add_edge(7, 2)
```

Graph Practice 1: Build the following graph

Given a list of edges, we can build the graph all at once

```
G = nx.Graph([(0, 1), (0, 3), (1, 2), (2, 3), (2, 7), (5, 6), (5, 7)])
```

Given a NumPy matrix, we can build the graph all at once

```
G = nx.Graph(<NumPy Adjacency Matrix>)
```

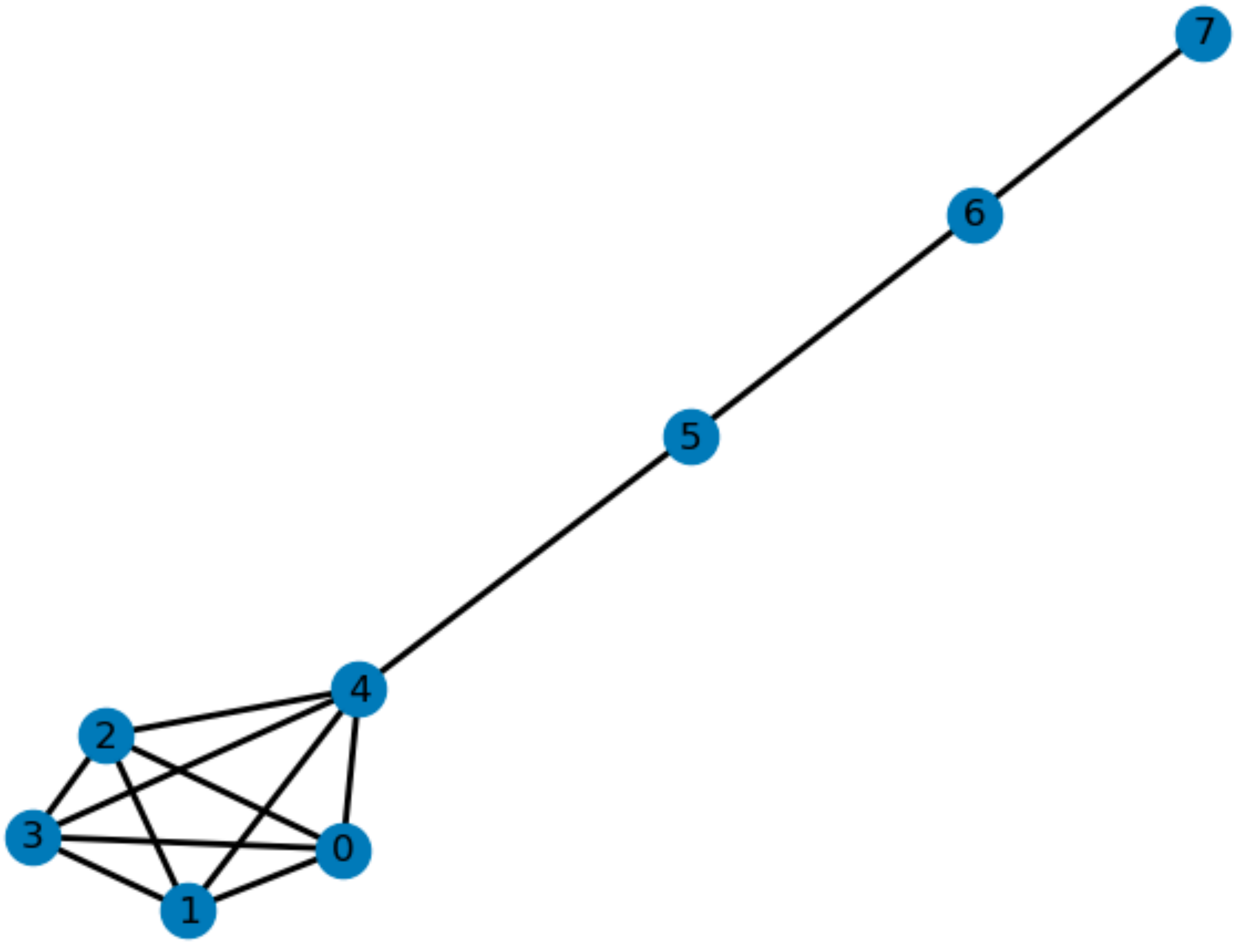
Given a file in format edge list or adjacency list

```
G = nx.read_edgelist(<edgeList file>)
```

```
G = nx. read_adjlist(<adjList file>)
```

Why not adjacency matrix?

Graph Practice 2: Remove all odd vertices

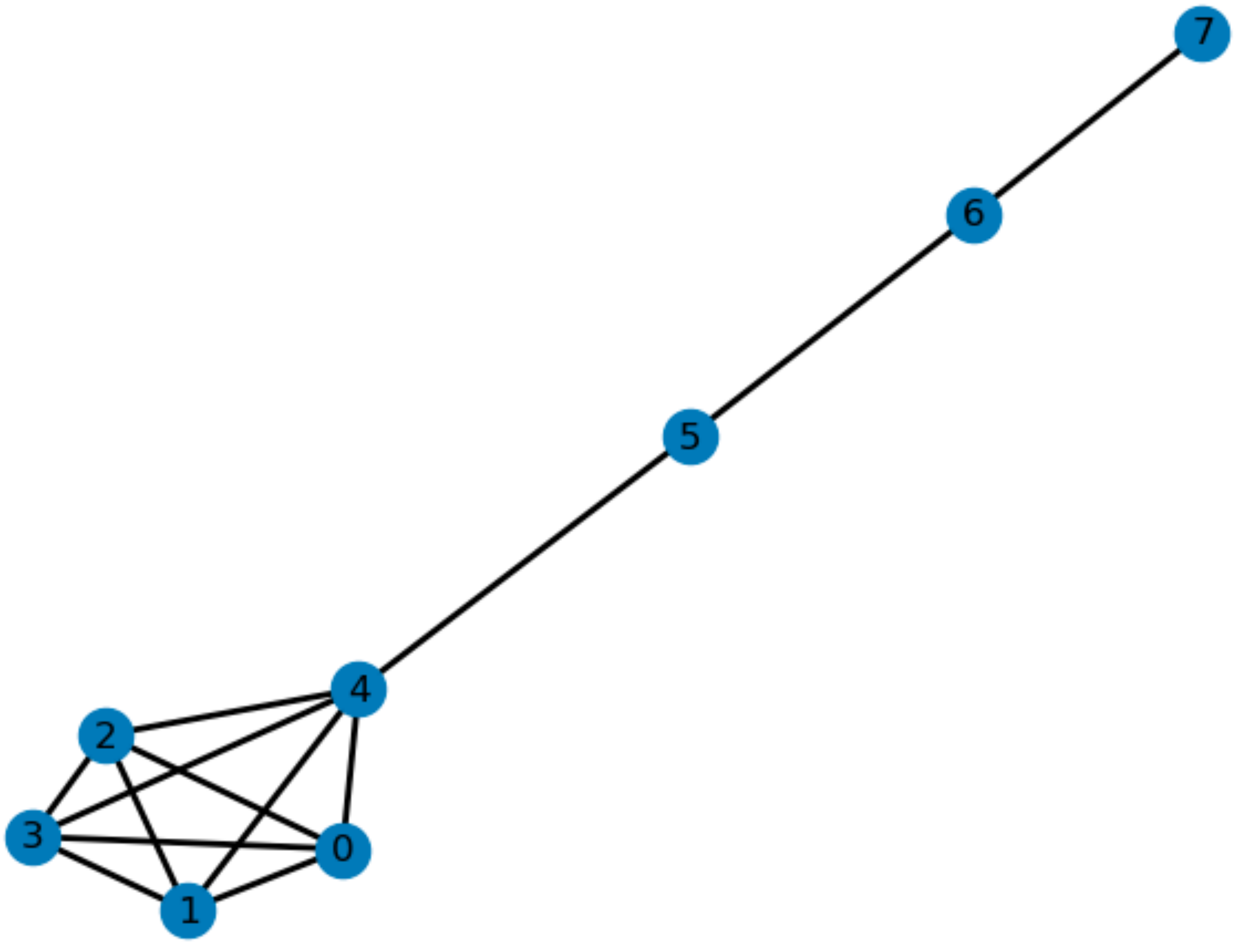


Graph Practice 2: Remove all odd vertices

G.nodes() by default returns a **dictionary**.

```
nodes = list(G.nodes())

for n in nodes:

    if n % 2 == 1:

        G.remove_node(n)
```

Graph Practice 3: Find the highest degree vertex



Graph Practice 3: Find the highest degree vertex

We can build a graph in NetworkX by adding edges one at a time:

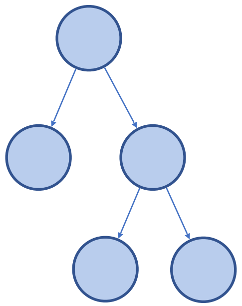
```
max = -1
v = None

for n in G.nodes():

    if len(G[n].keys()) > max:

        max = len(G[n].keys())

        v = n

print(v, max)
```

Graph Traversals

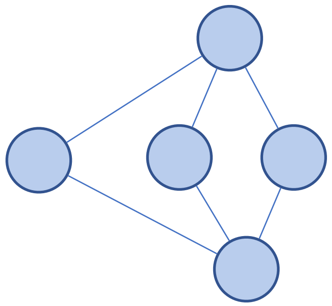
There is no clear order in a graph (even less than a tree!)

How can we systematically go through a complex graph in the fewest steps?

Tree traversals won't work — lets compare:



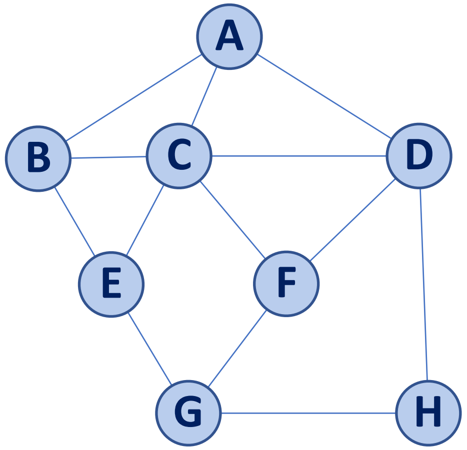
- Rooted
- Acyclic
- Clear base cases ('doneness')



- Arbitrary starting point
- Can have cycles
- Must track visited nodes directly

Simple BFS Traversal

1) Create a queue and a visit list
2) Initialize both to contain our start
3) While queue not empty:
   - Remove front vertex of queue
   - Check if each edge has been seen before
   - Add unvisited edges to queue (and list)



Queue

Visited

Simple BFS Traversal

1) Create a queue and a visit list
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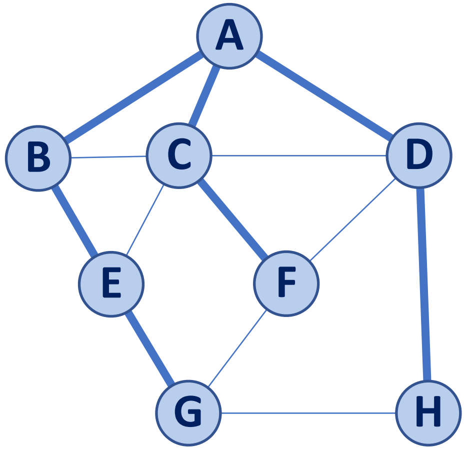
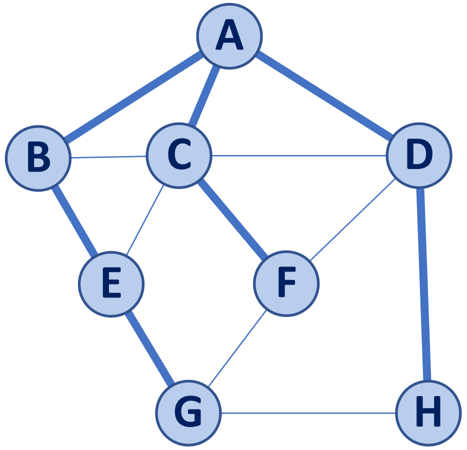


What is my runtime?

Queue ~~G~~ ~~H~~ ~~F~~ ~~E~~ ~~D~~ ~~C~~ ~~B~~ ~~A~~

Visited A B C D E F H G

Simple BFS Traversal



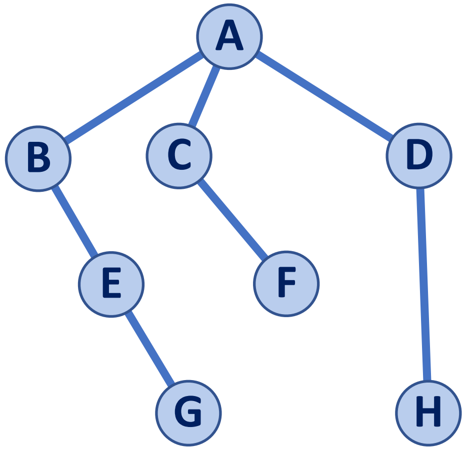
What is the shortest distance from **A** to **H**?

What is the shortest path from **A** to **H**?

What is the shortest path from **A** to **F**?

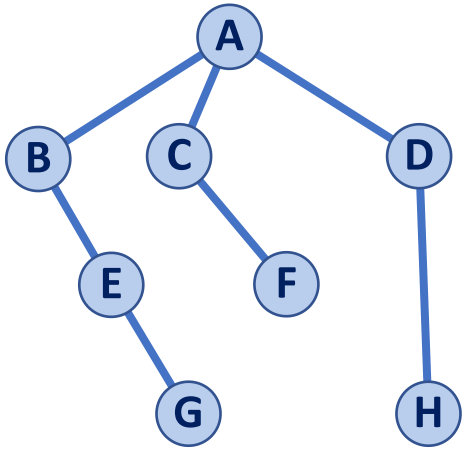
What is the shortest distance from **A** to **F**?

Simple BFS Traversal



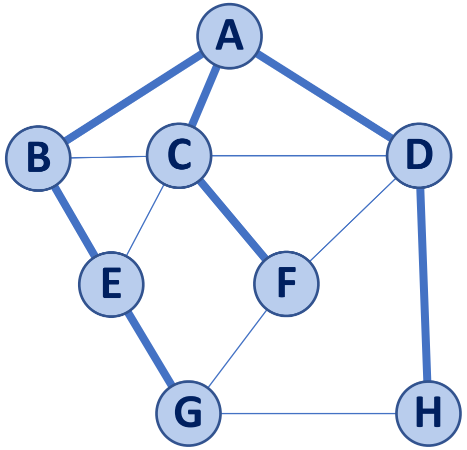
What data structure is this?

Simple BFS Traversal



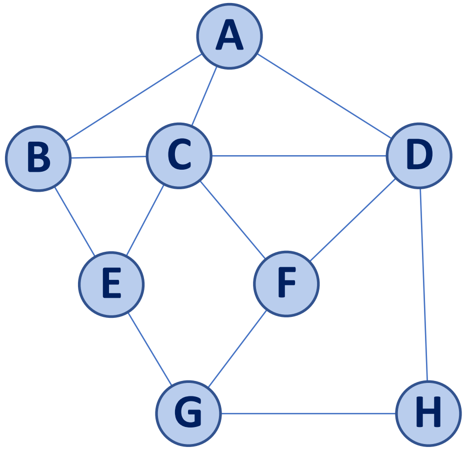
A **minimum spanning tree** is a tree formed by a subset of graph edges such that all vertices are connected with the smallest total possible edge weight

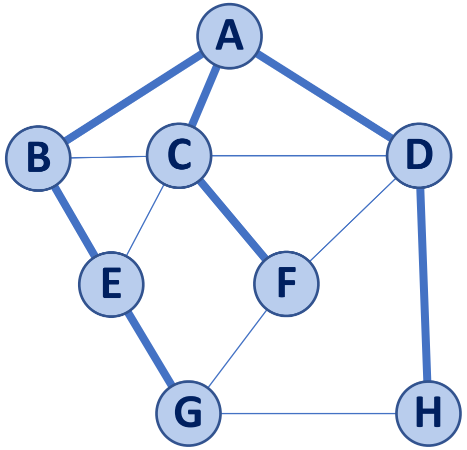
On an unweighted, undirected graph this MST can be built by tracking **discovery** edges during a BFS traversal



We call the remaining edges **cross** edges. What can I say about a graph with at least one **cross** edge?

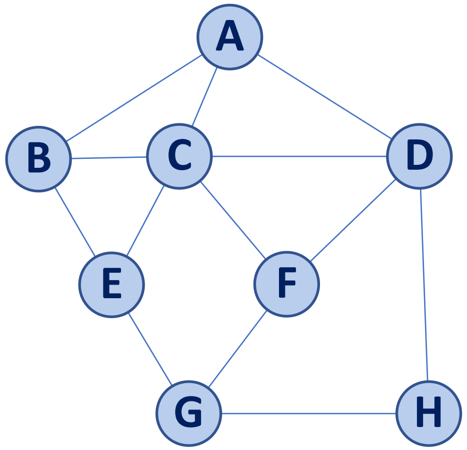
Traversal: BFS





If we modify our BFS traversal algorithm, we can track both **distances** and **discovery edges!**

Traversal: BFS



Replace 'visited' list with a **distance** and a **previous**

When we add to queue, record **previous**.

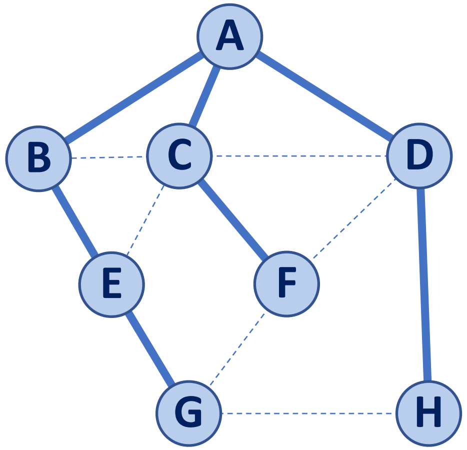
When we process vertex from queue, record **distance**.

"Unvisited" vertices have neither **distance** or **previous**

Vertex	Distance	Previous
A		
B		
C		
D		
E		
F		
G		
H		

Queue

Traversal: BFS



Replace ‘visited’ list with a **distance** and a **previous**

When we add to queue, record **previous**.

When we process vertex from queue, record **distance**.

“Unvisited” vertices have neither **distance** or **previous**

Vertex	Distance	Previo
A	0	-
B	1	**A**
C	1	**A**
D	1	**A**
E	2	**B**
F	2	**C**
G	3	**E**
H	2	**D**

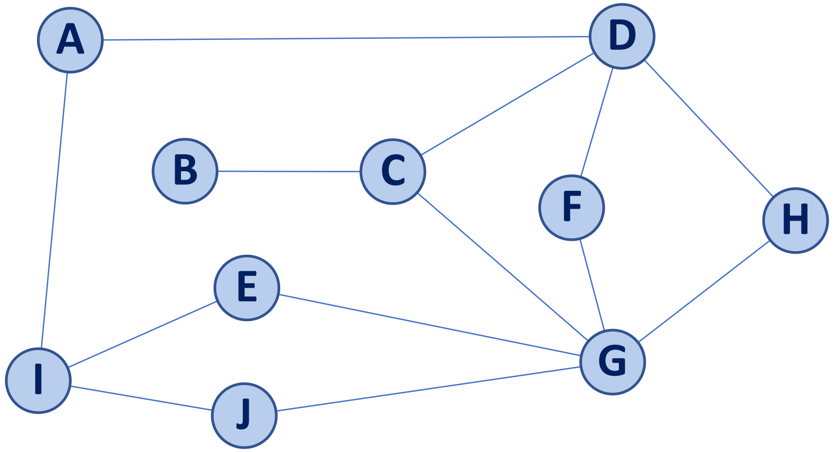
Queue ~~G~~ ~~H~~ ~~F~~ ~~E~~ ~~D~~ ~~C~~ ~~B~~ ~~A~~

BFS Traversal using NetworkX

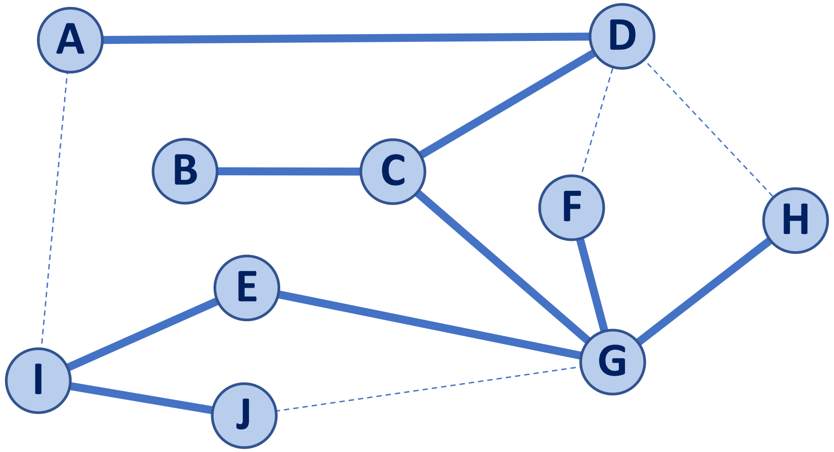
There are many different methods for running a BFS (different output):

```
G = nx.random_regular_graph(3, 6)

print(list(nx.bfs_edges(G, 0)))

print(list(nx.bfs_predecessors(G, 0)))

print(nx.descendants_at_distance(G, 0, 0))

print(nx.descendants_at_distance(G, 0, 1))

print(nx.descendants_at_distance(G, 0, 2))

print(nx.descendants_at_distance(G, 0, 3))

T = nx.bfs_tree(G, 0)
```

Traversal: DFS



Traversal: DFS



1) Create a stack and a visit list

2) Initialize both to contain our start

3) While stack not empty:

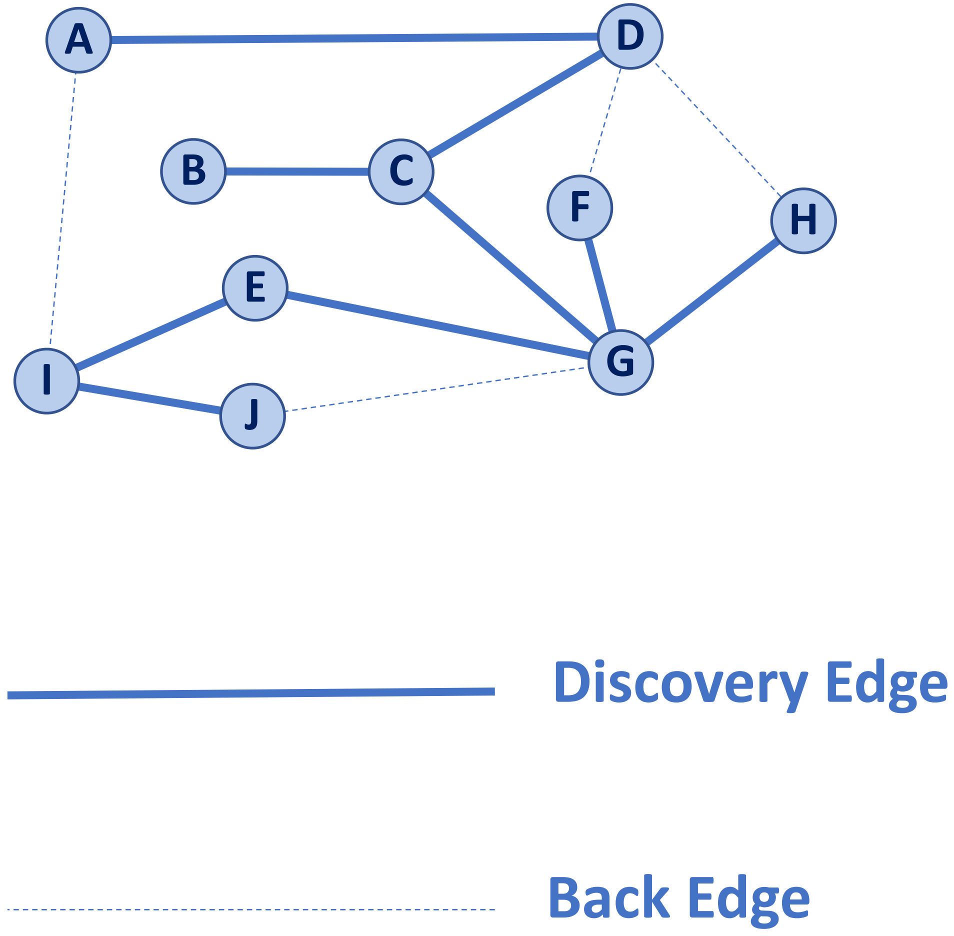
Use **top()** to look at current vertex

If no unvisited children, **pop()**

Otherwise, **push()** the first unvisited child

Traversal: DFS

Do we still make a spanning tree?



Does distance have meaning here?

Do our edge labels have meaning here?

DFS Traversal using NetworkX

What can the BFS do that the DFS cannot do?

```
G = nx.random_regular_graph(3, 6)


print(list(nx.dfs_edges(G, 0)))


print(list(nx.dfs_predecessors(G, 0)))


print(list(nx.find_cycle(G, 0)))


T = nx.dfs_tree(G, 0)
```

DFS vs BFS Runtime

DFS:

Use Cases:

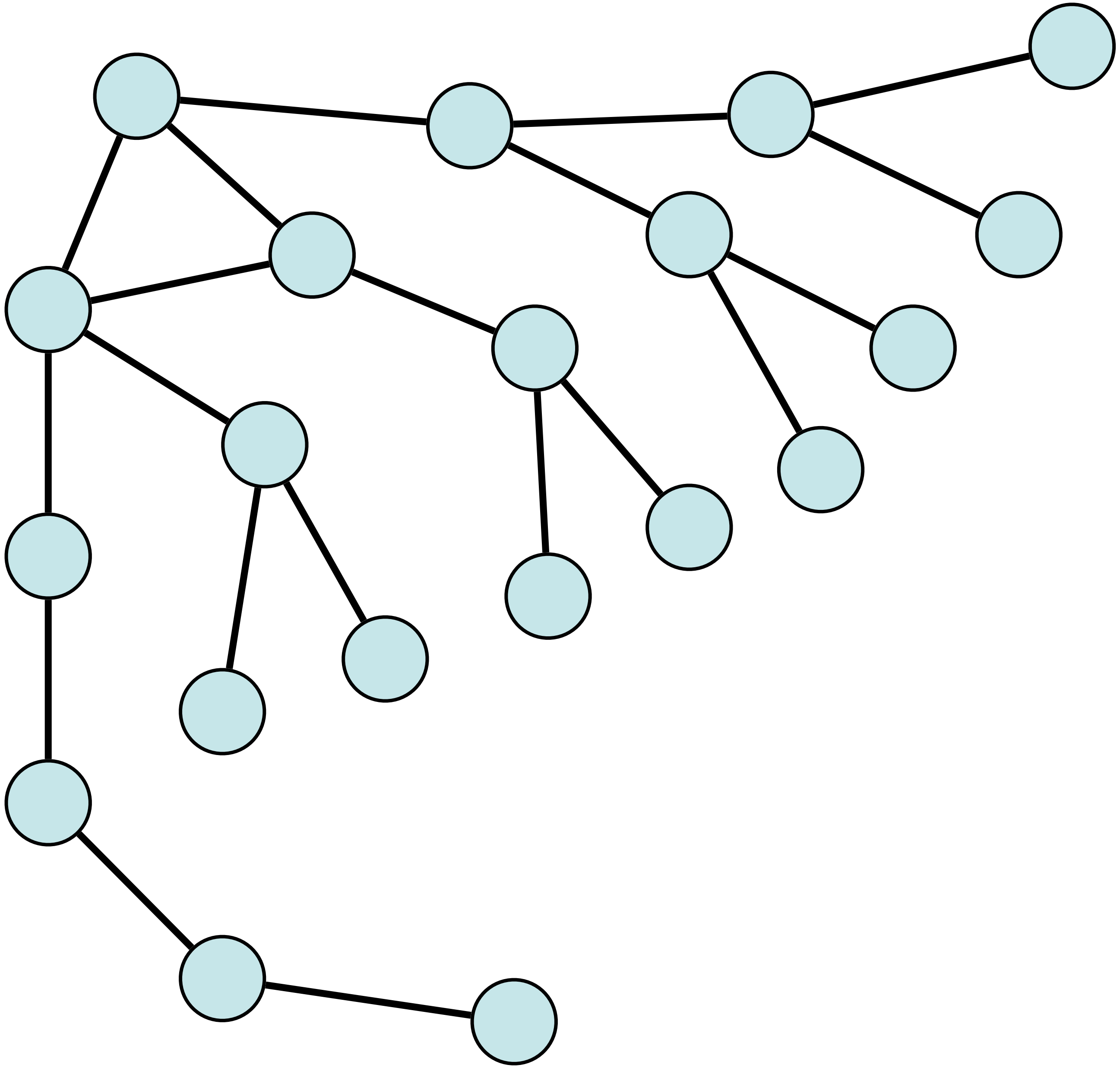
Peak Memory Cost:

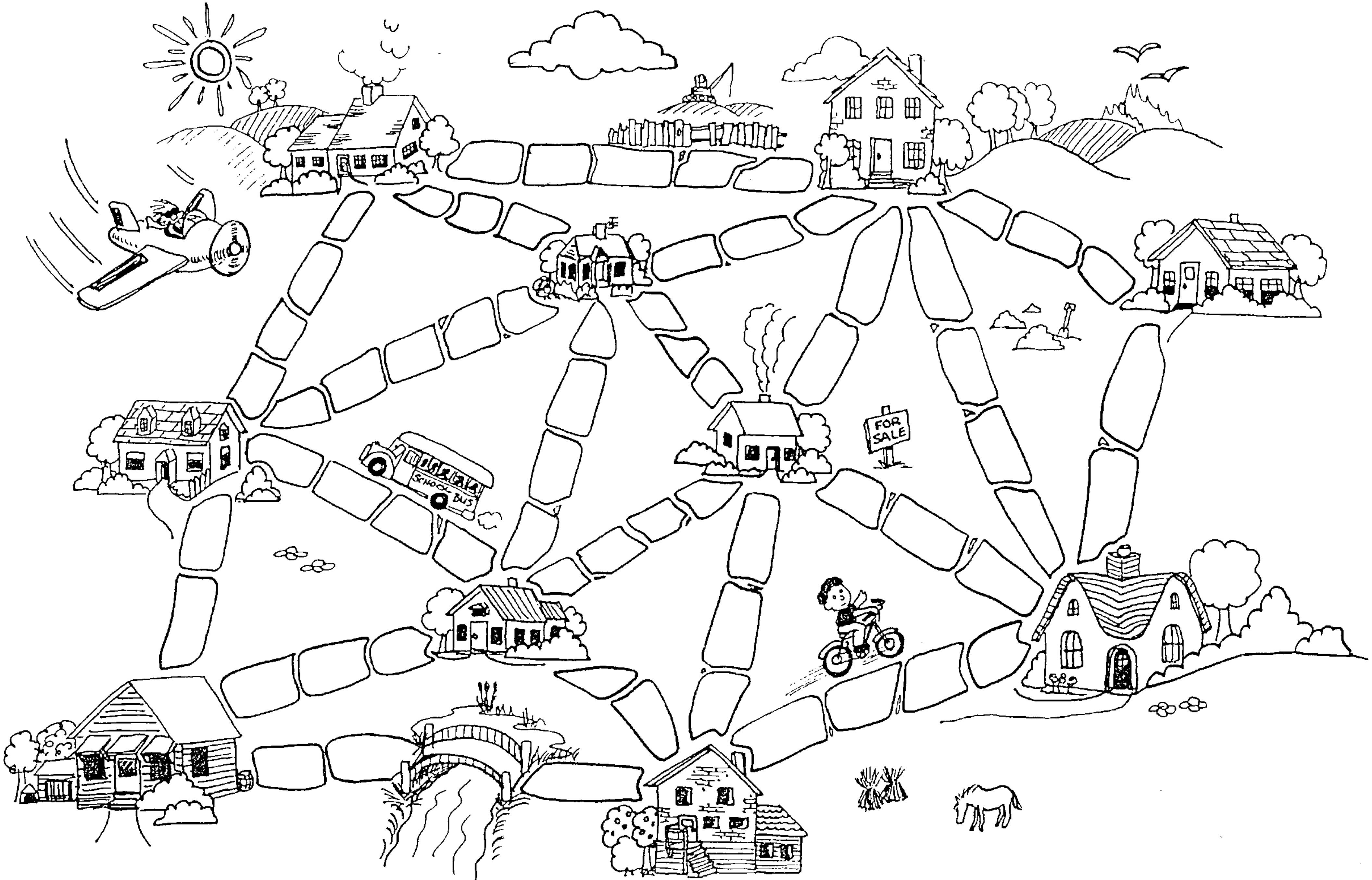
BFS:

Use Cases:

Peak Memory Cost:

DFS vs BFS

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