Algorithms and Data Structures for Data Science
Binary Search Tree

CS 277
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March 6, 2024

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
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Reminder: mp_automata due Friday

93% credit late day extension through Saturday

Additional extensions by request

Reminder: Spring Break next week

Lab on Friday will still happen, will be due after spring break

No office hours during spring break

Exam 2: 3/19 - 3/21

Yes its right after spring break. Sorry!

Covered material described on website

One coding question — likely similar to mp_automata

Practice exam (hopefully) later this week

Learning Objectives

Finish implementation of BST ADT

Introduce the Huffman Tree

Practice recursion in the context of trees

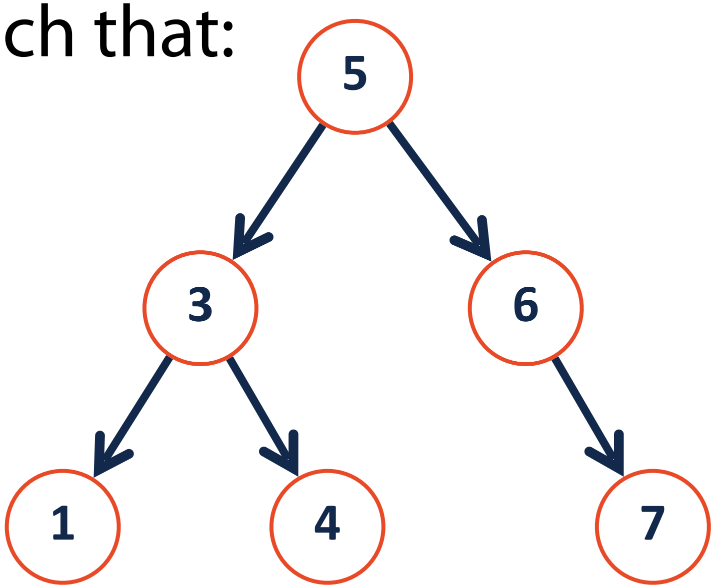
Binary Search Tree

```
class bstNode:
    def __init__(self, key, val, left=None, right=None):
        self.key = key
        self.val = val
        self.left = left
        self.right = right
```

A **BST** is a binary tree $T = treeNode(val, T_L, T_r)$ such that:

$\forall n \in T_L,\ n.val < T.val$

$\forall n \in T_R,\ n.val > T.val$



Key	5	3	6	7	1	4
Value	A	B	C	D	E	F

Binary **Search** Tree ADT — **what changed?**

Constructor: Build a new (empty) tree

Insert: Find the correct insert location based on BST structure

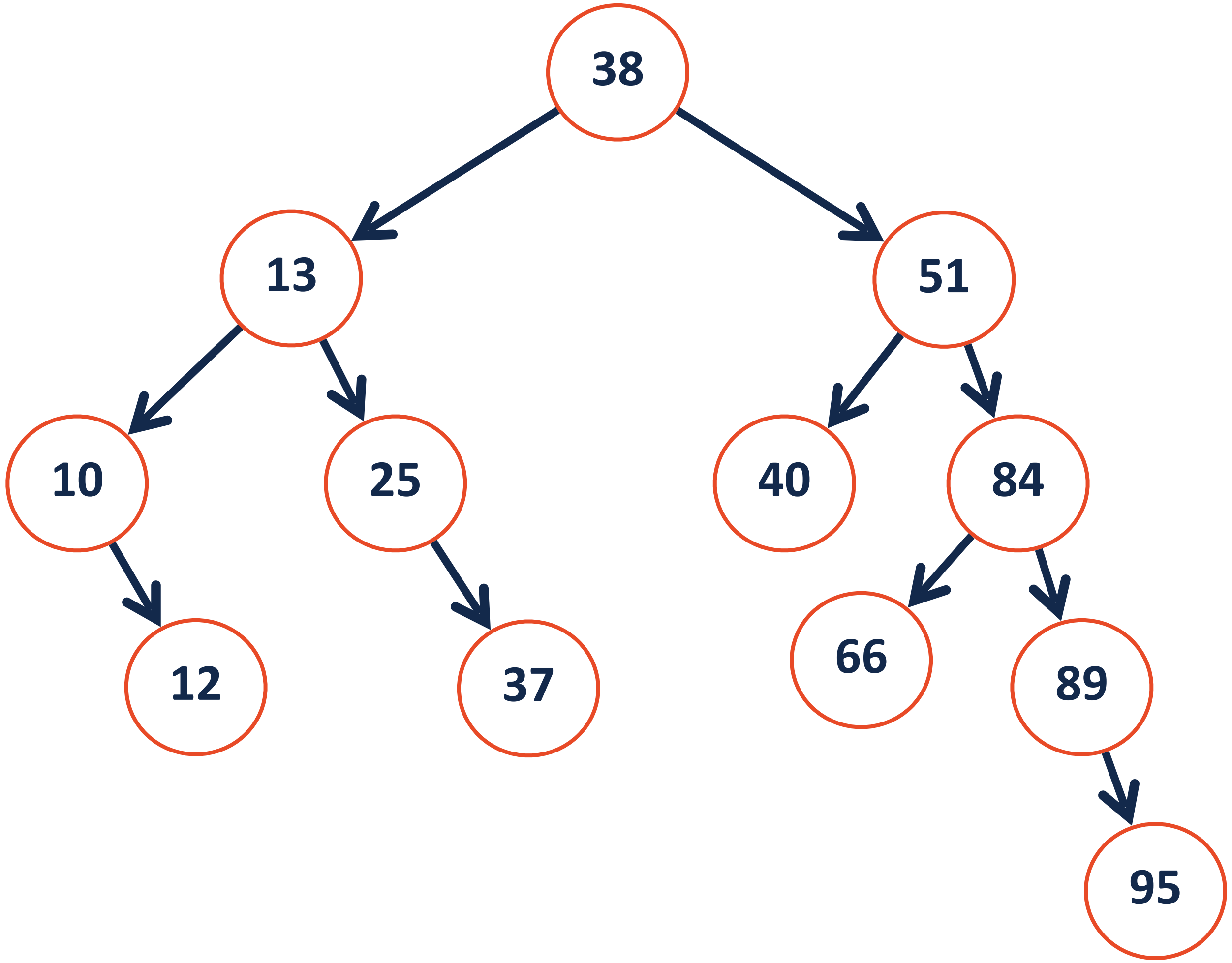
Remove: Find the node being removed and… ???

Traverse: Visit every node in tree (all objects)

Search: Find a specific node in the tree using the 'key' value

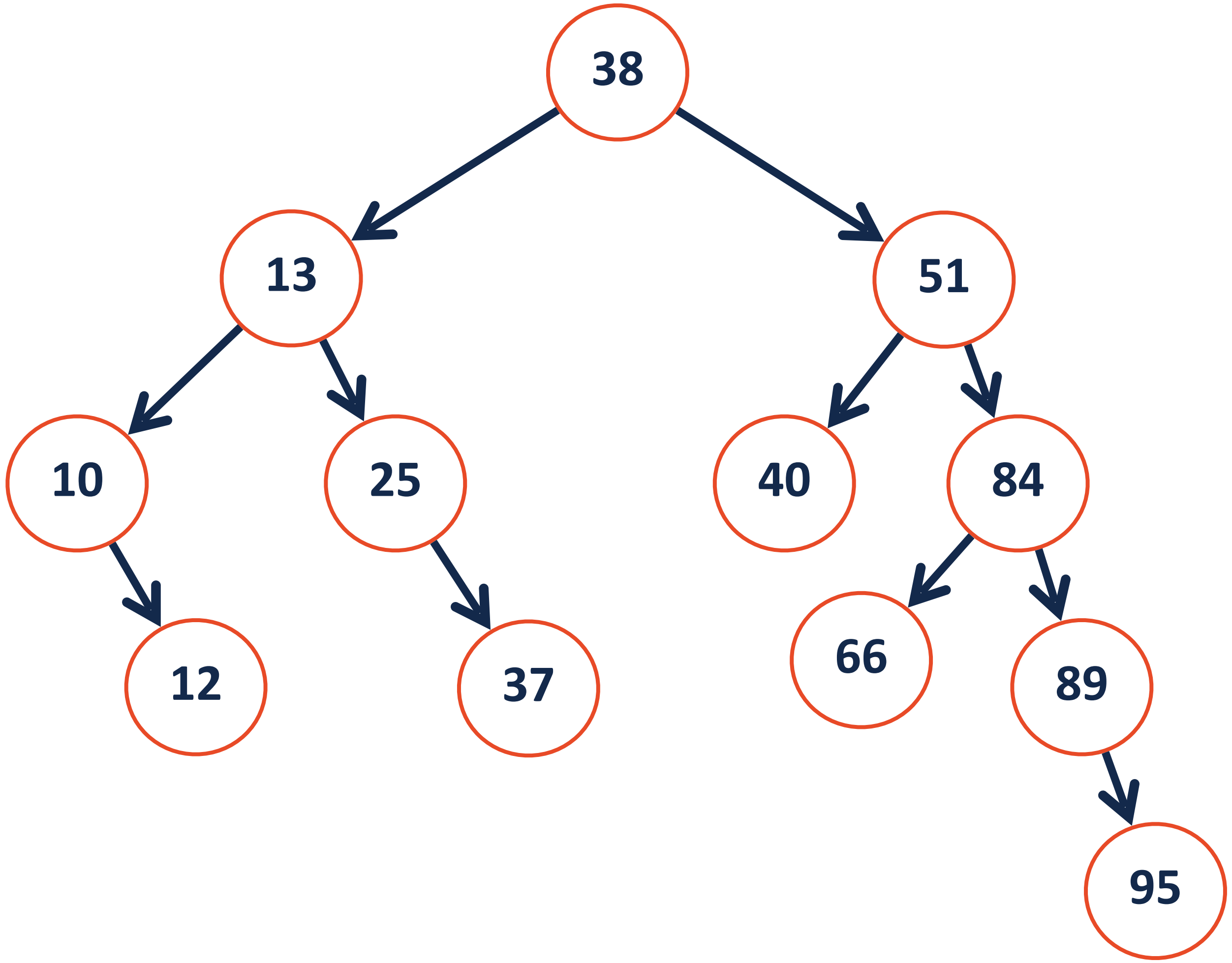
BST Remove

`remove(40)`



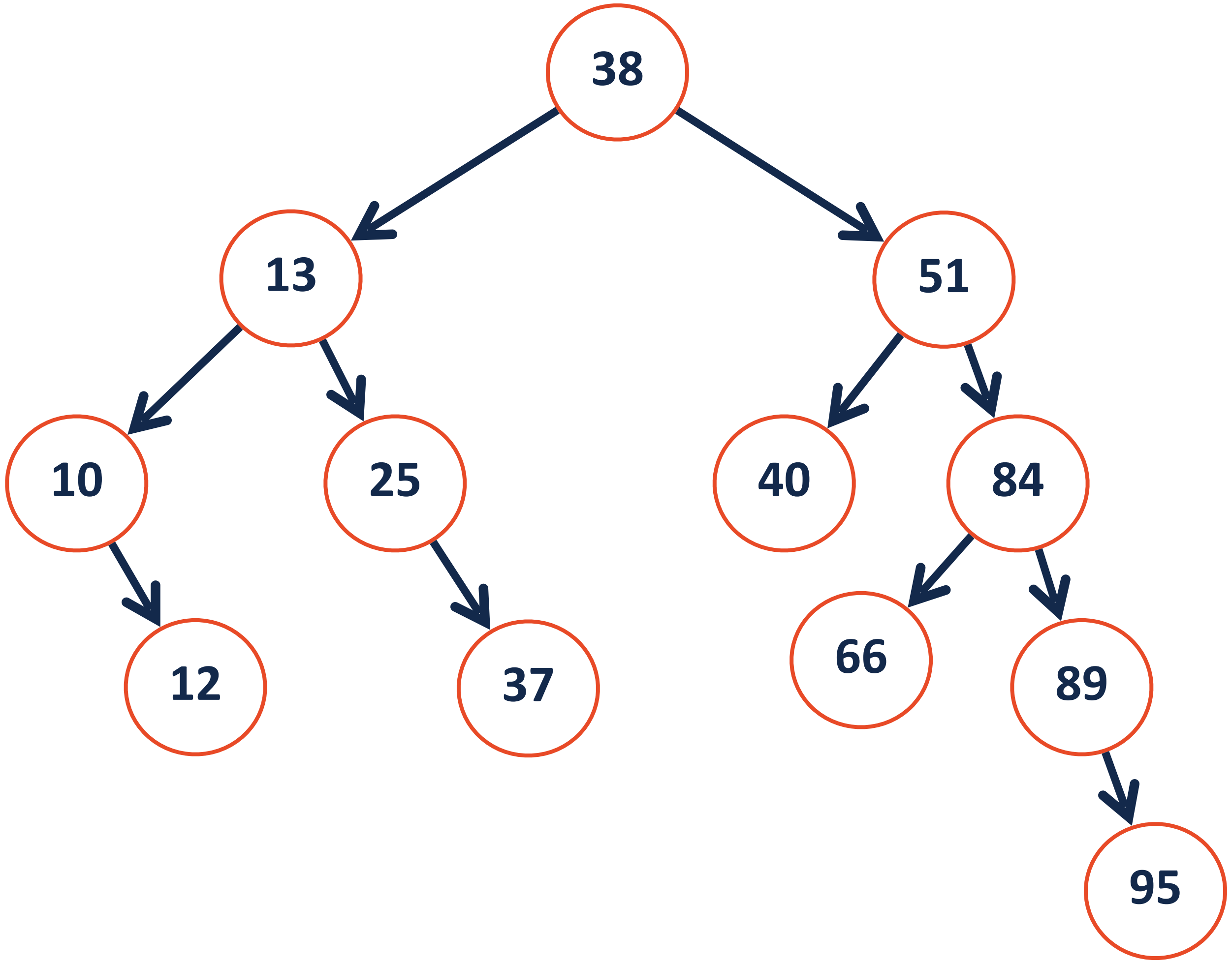
BST Remove

`remove(25)`



BST Remove

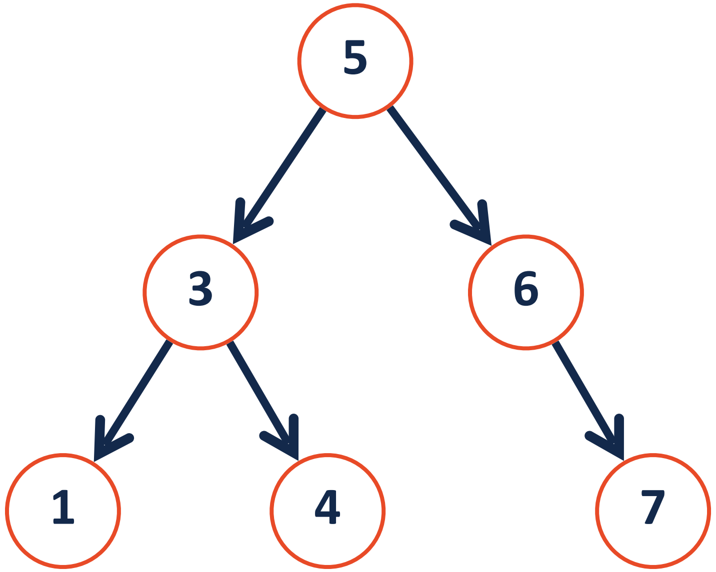
`remove(13)`



BST Remove

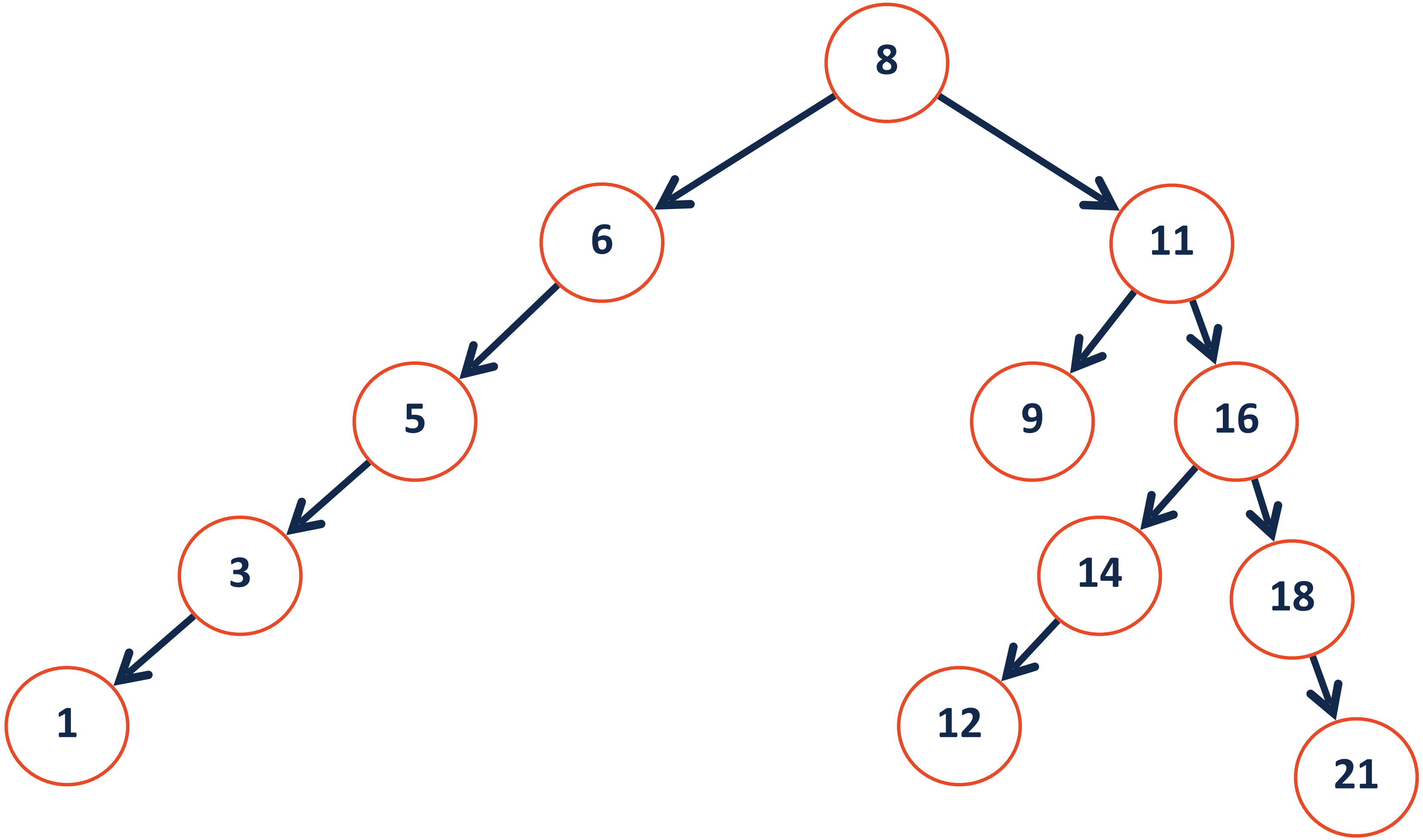
```
def remove(self, key):
    self.root = self.remove_helper(self.root, key)

def remove_helper(self, node, key):
```



BST Remove

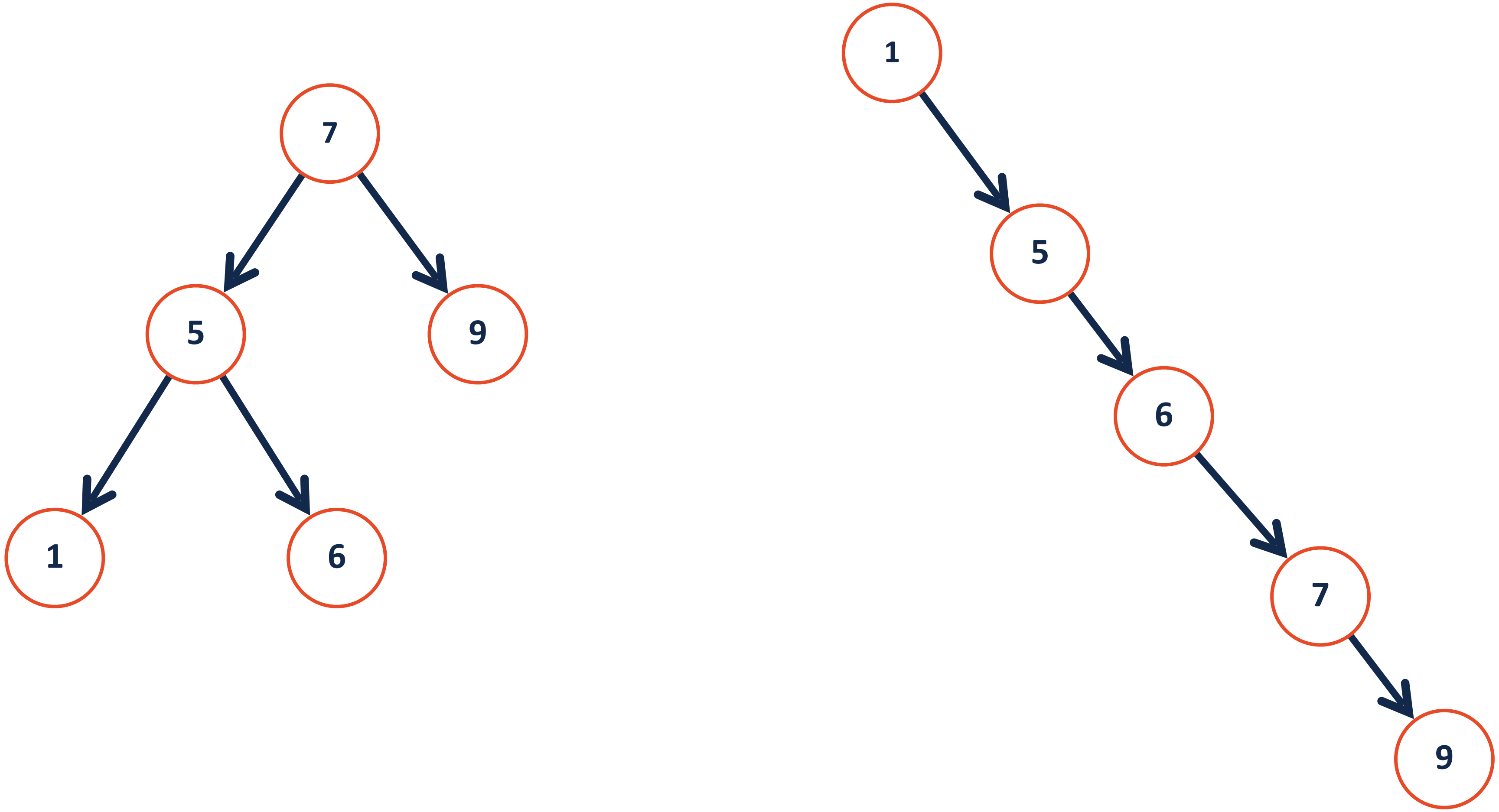
What will the tree structure look like if we remove node 16 using IOS?



BST Analysis – Running Time

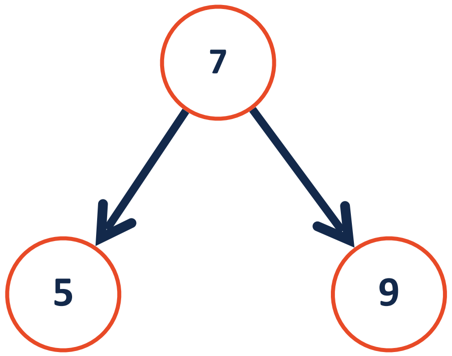
Operation	BST Worst Case
find	
insert	
delete	
traverse	

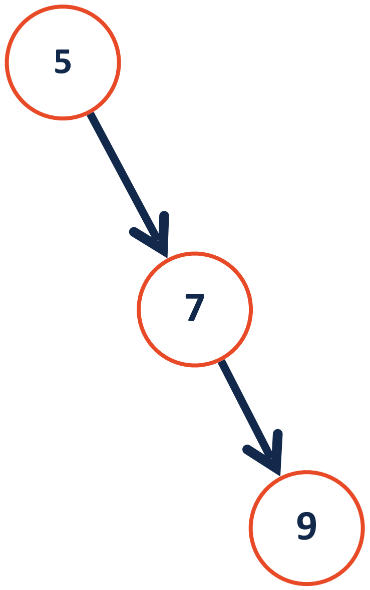
Limiting the height of a tree



Height-Balanced Tree

What tree is better?

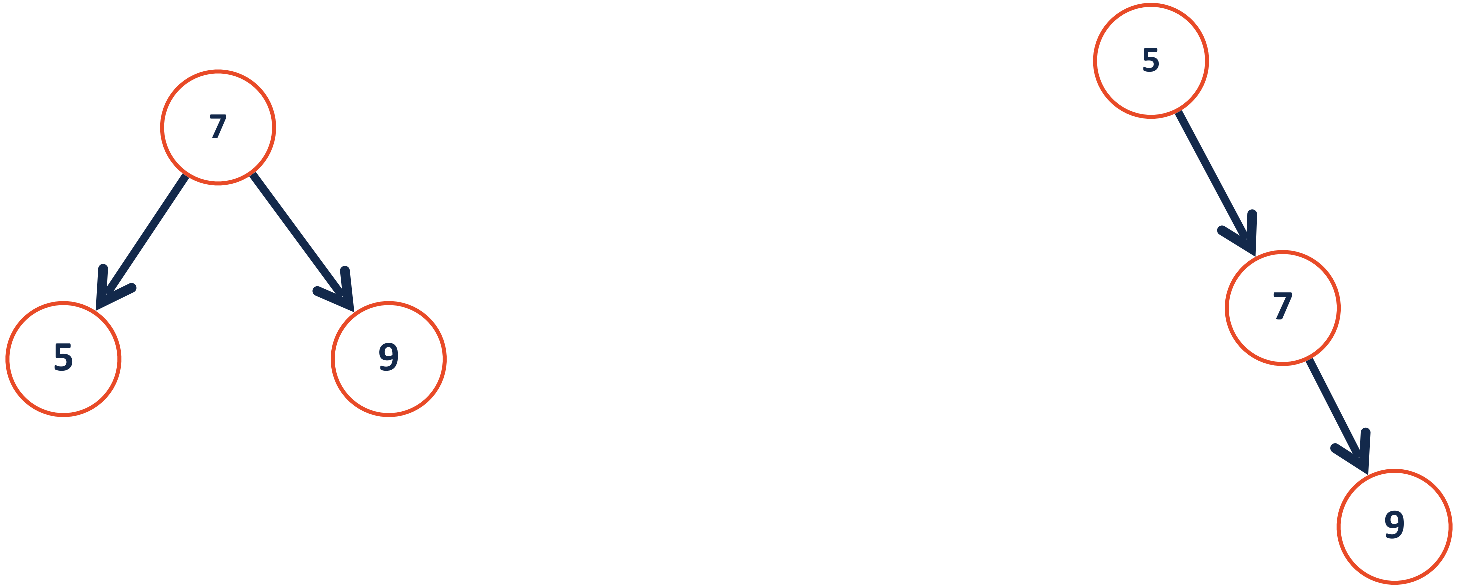




How would you describe this mathematically?

Height-Balanced Tree

What tree is better?



Height balance: $b = height(T_R) - height(T_L)$

A tree is "balanced" if:

Option A: Correcting bad insert order

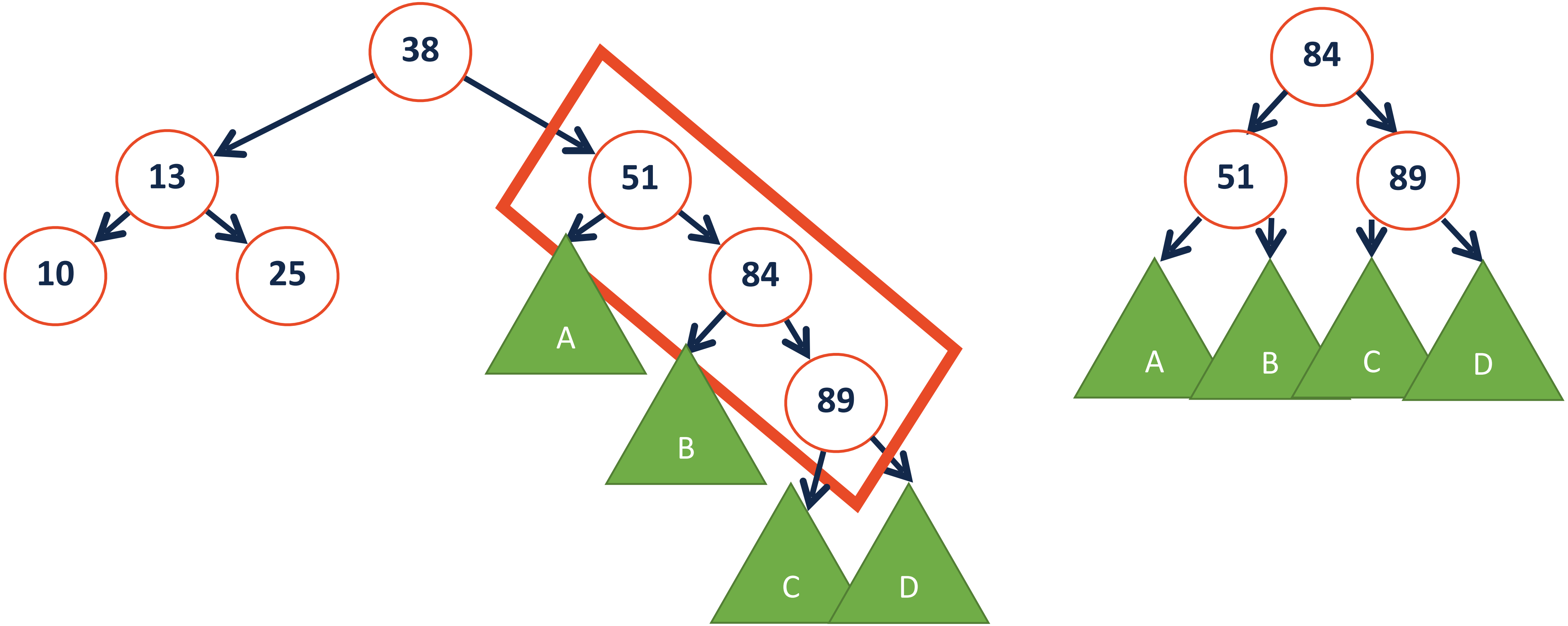
The height of a BST depends on the order in which the data was inserted

Insert Order: [1, 3, 2, 4, 5, 6, 7]

Insert Order: [4, 2, 3, 6, 7, 1, 5]

AVL-Tree: A self-balancing binary search tree

Rather than fixing an insertion order, just correct the tree as needed!



We will return to this topic… after spring break!

Optimal Storage Costs

Achieving an optimal storage cost for a dataset is often important

Let's use strings as an accessible example!

What is the minimum bits needed to encode the message:

Char	Binary
f	000
e	001
d	010
m	100
r	011
o	101
‘ ‘	110

`‘feed me more food’`

Optimal Storage Costs

Using three bits per character, we have 51 bits total. But can we do better?

`‘feed me more food’`

If we think about our input as a sorted list of frequencies, yes!

r : 1 | d : 2 | f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

Using binary trees for string encoding

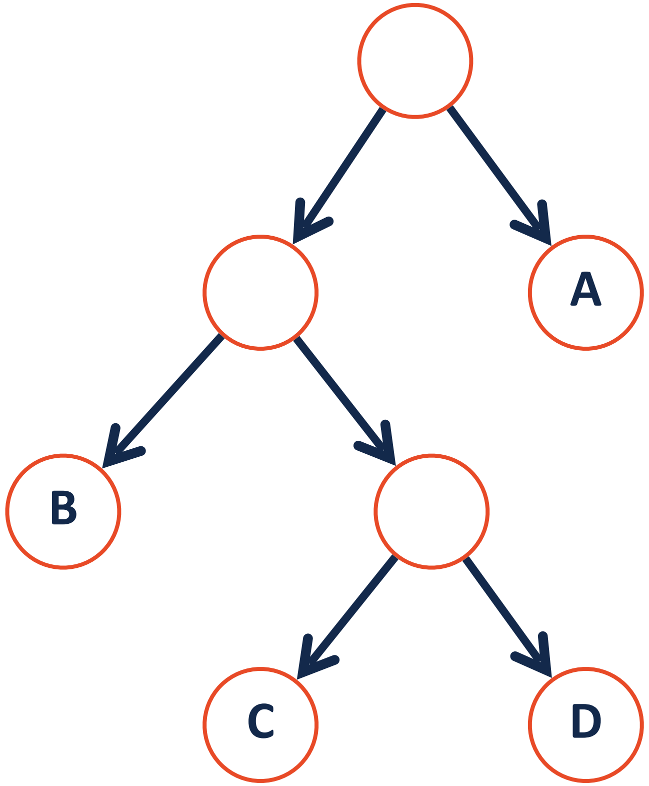
Lets define a tree with the following:

The keys are individual characters

The values are the frequencies of those characters

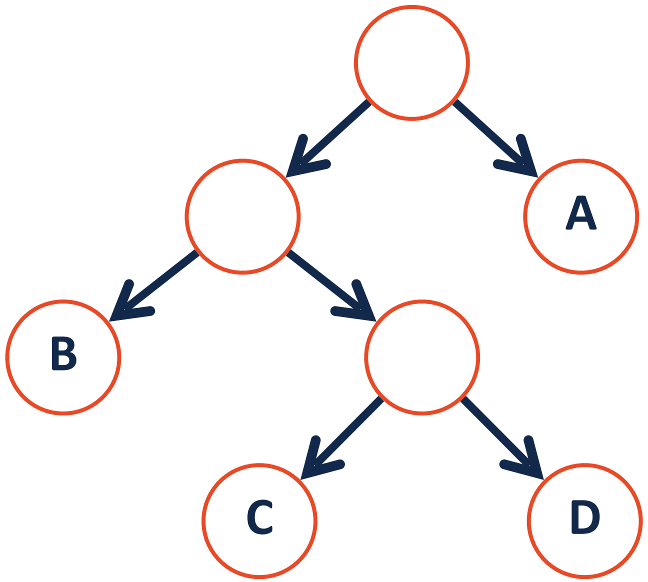
```
class treeNode:
    def __init__(self, key, val, left=None, right=None):
        self.key = key
        self.val = val
        self.left = left
        self.right = right
```

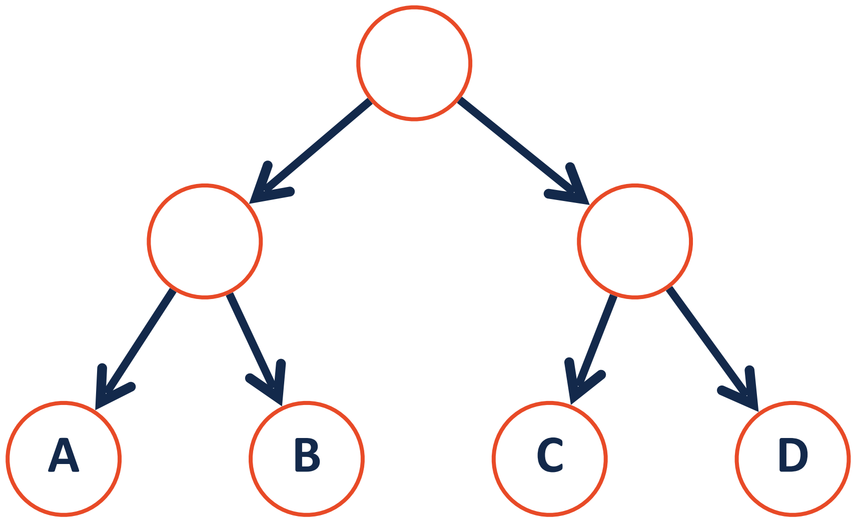
Key	A	B	C	D
Value	7	5	2	4



Binary Tree encoding

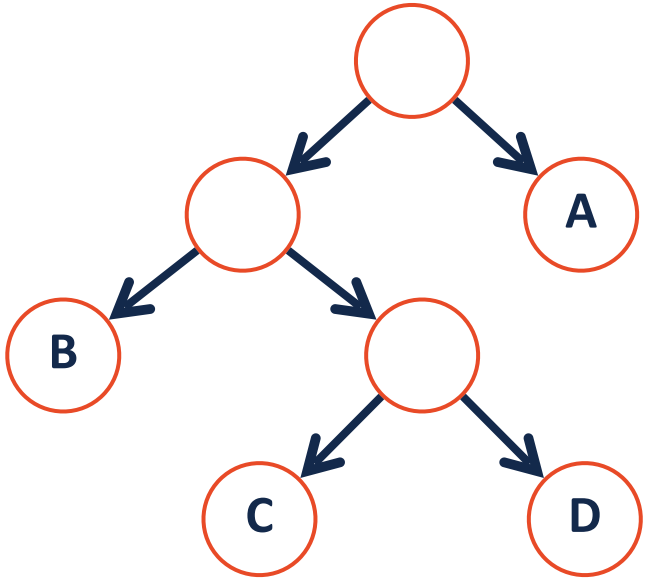
Given the following two trees, how might we define an encoding?



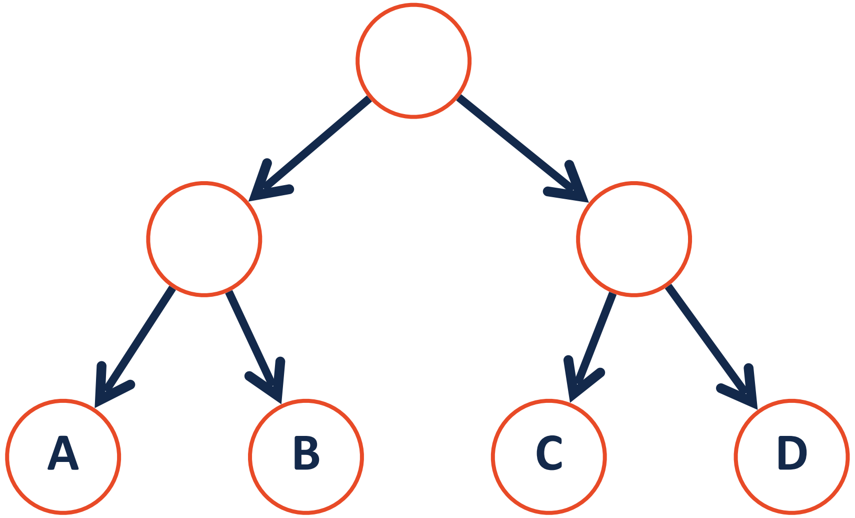


Binary Tree encoding

How did we produce this encoding?



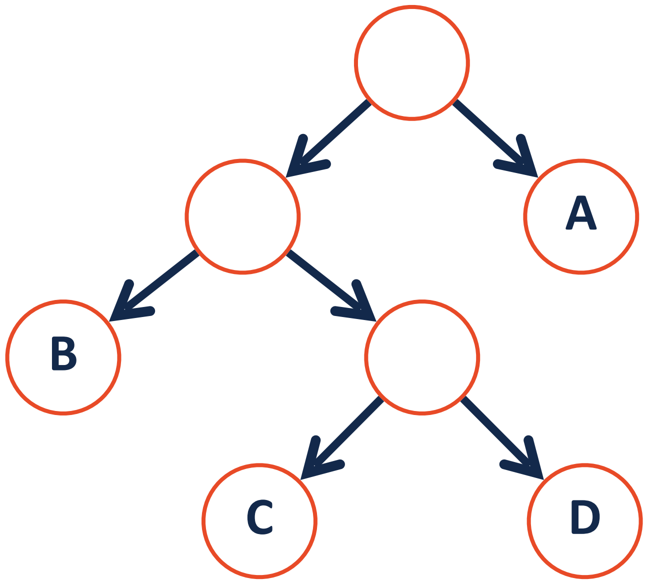
Char	Binary
A	1
B	00
C	010
D	011



Char	Binary
A	00
B	01
C	10
D	11

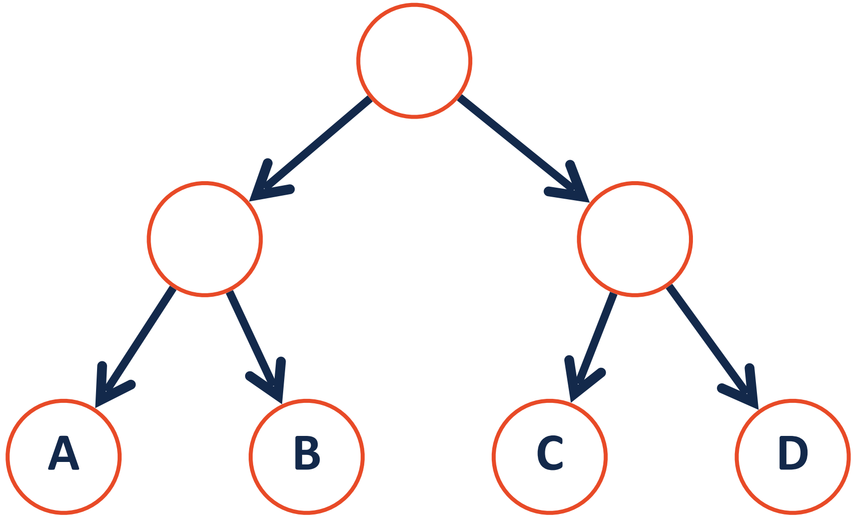
Binary Tree encoding

The **path** from root to leaf defines our encoding, but which tree is best?



Going left = 0

Going right = 1

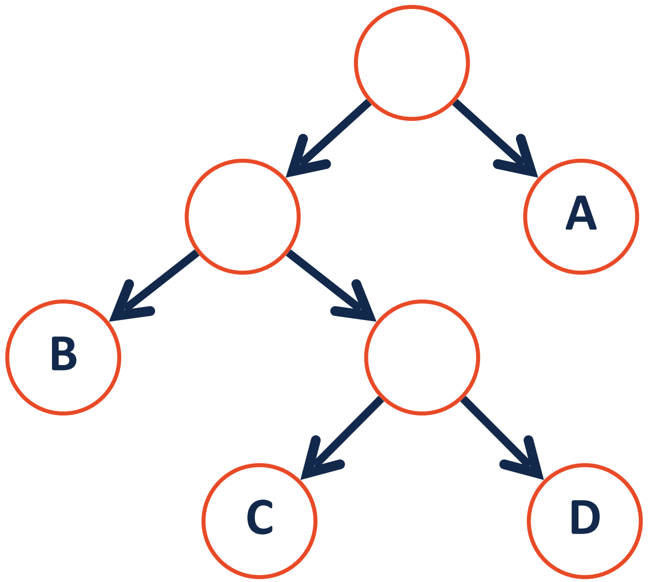


Char	Binary
A	1
B	00
C	010
D	011

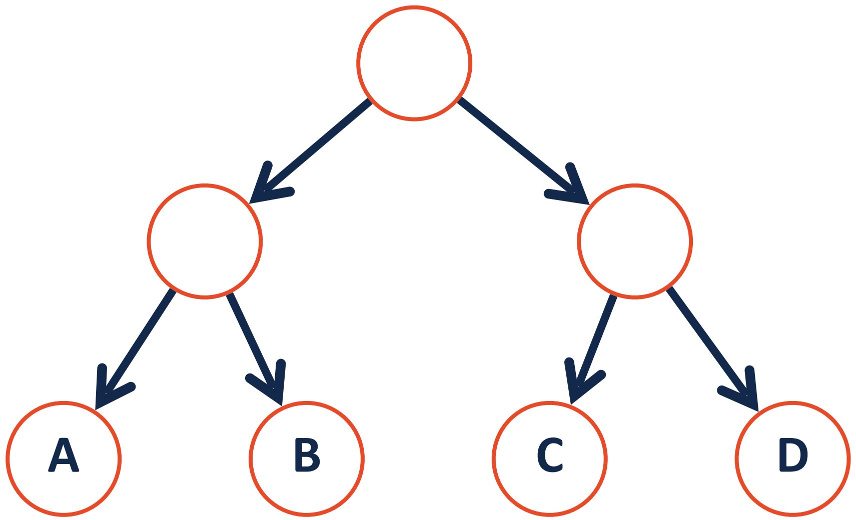
Char	Binary
A	00
B	01
C	10
D	11

Binary Tree encoding

If my frequencies are {A : 7 | B : 5 | C : 2 | D : 4 }, which tree was better?



Char	Binary
A	1
B	00
C	010
D	011



Char	Binary
A	00
B	01
C	10
D	11

Building the Huffman Tree

The **Huffman Tree** is the tree with the optimal total path length for a given set of characters and their frequencies.

Step 1: Calculate the frequency of every character in text and order by increasing frequency. Store in a queue (a sorted list).

`Input:` `’feed me more food’`

r : 1 | d : 2 | f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

Building the Huffman Tree

Step 2: Build a tree from the bottom up. Start by taking the two least frequent characters and merge them (create a parent node). Store the merged characters in a new queue.

Input:

r : 1 | d : 2 | f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

Building the Huffman Tree

Step 2: Build a tree from the bottom up. Start by taking the two least frequent characters and merge them (create a parent node). Store the merged characters in a new queue.

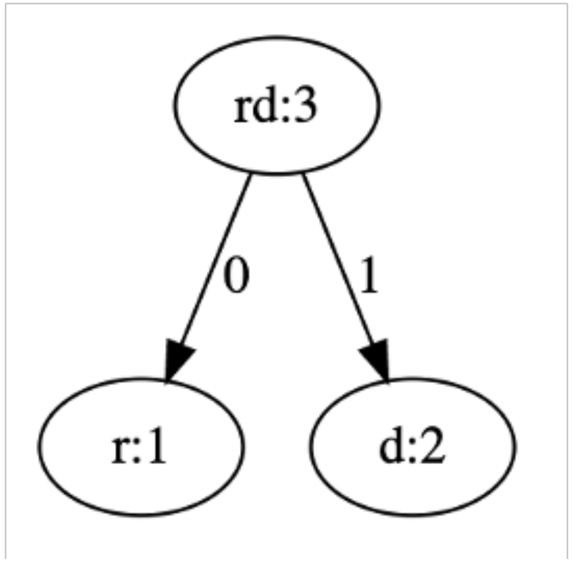
Input:

r : 1 | d : 2 | f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

Output:

Single: f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

Merged: rd : 3



Building the Huffman Tree

Step 3: Repeatedly merge the minimum two items from either list.
Be sure to **remove and return** the minimum item as seen below:

Input:

Single: f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

Merged: rd : 3

Building the Huffman Tree

Step 3: Repeatedly merge the minimum two items from either list.
Be sure to **remove and return** the minimum item as seen below:

Input:

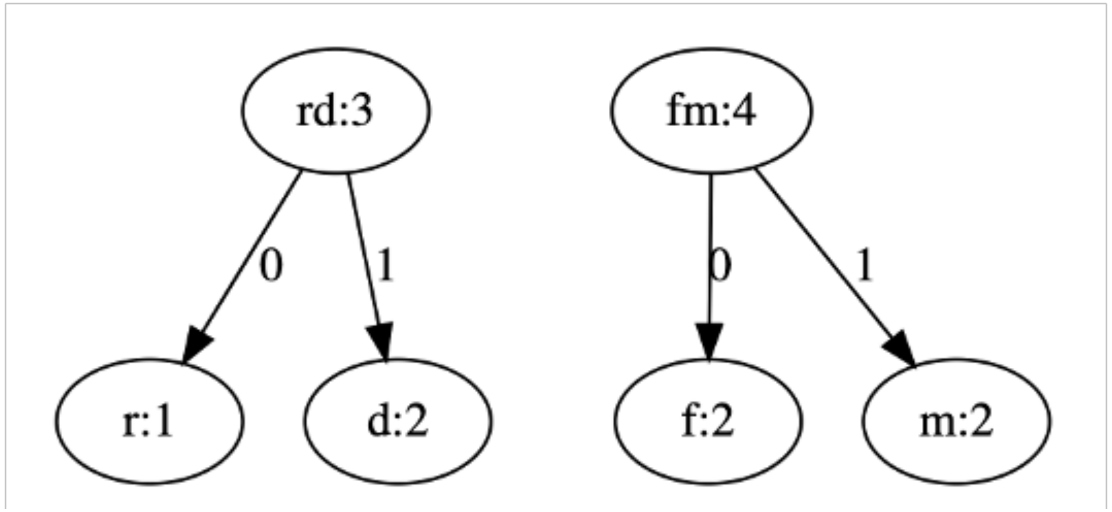
Single: f : 2 | m : 2 | o : 3 | 'SPACE' : 3 | e : 4

Merged: rd : 3

Output:

Single: o : 3 | 'SPACE' : 3 | e : 4

Merged: rd : 3 | fm : 4



Building the Huffman Tree

Step 3: Repeatedly merge the minimum two items. Note that **by inserting in the back** the merged items will always remain sorted!

Input:

Single: o : 3 | 'SPACE' : 3 | e : 4

Merged: rd : 3 | fm : 4

Building the Huffman Tree

Step 3: Repeatedly merge the minimum two items. Note that **by inserting in the back** the merged items will always remain sorted!

Input:

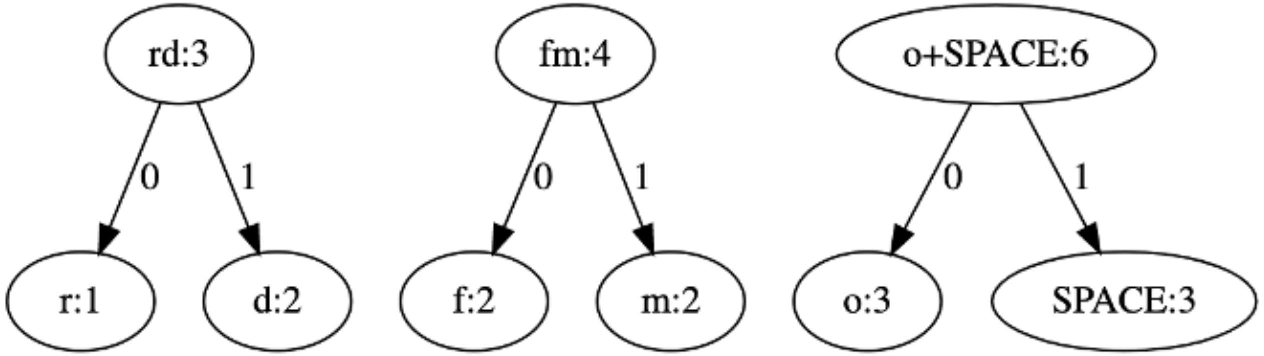
Single: o : 3 | 'SPACE' : 3 | e : 4

Merged: rd : 3 | fm : 4

Output:

Single: e : 4

Merged: rd : 3 | fm : 4 | o'SPACE' : 6



Building the Huffman Tree

Step 3: Once the 'single' character list has been exhausted, we can easily merge the rest of our list by taking the front two values in merged.

Input:

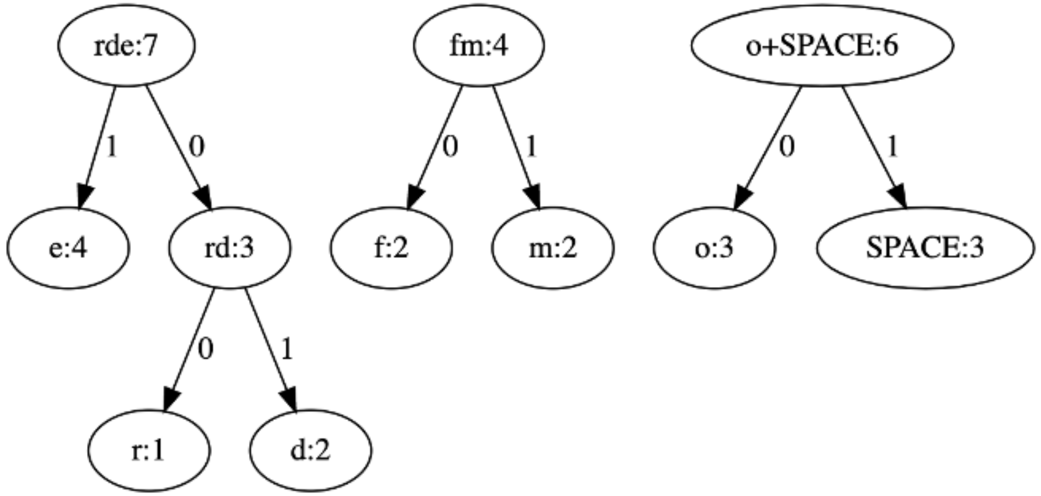
Single: e : 4

Merged: rd : 3 | fm : 4 | o'SPACE' : 6

Output:

Single:

Merged: fm : 4 | o'SPACE' : 6 | rde : 7



Building the Huffman Tree

Step 3: Once the 'single' character list has been exhausted, we can easily merge the rest of our list by taking the front two values in merged.

Input:

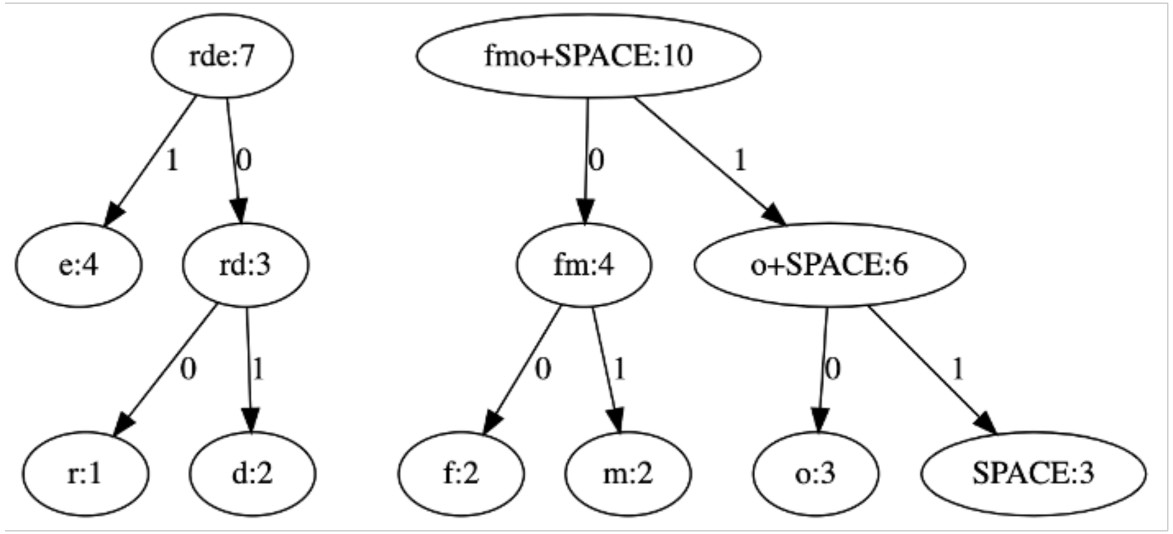
Single:

Merged: fm : 4 | o'SPACE' : 6 | rde : 7

Output:

Single:

Merged: rde : 7 | fmo'SPACE' : 10



Building the Huffman Tree

Step 4: Stop when there is only a single item in either queue. This is our complete binary tree!

Input:

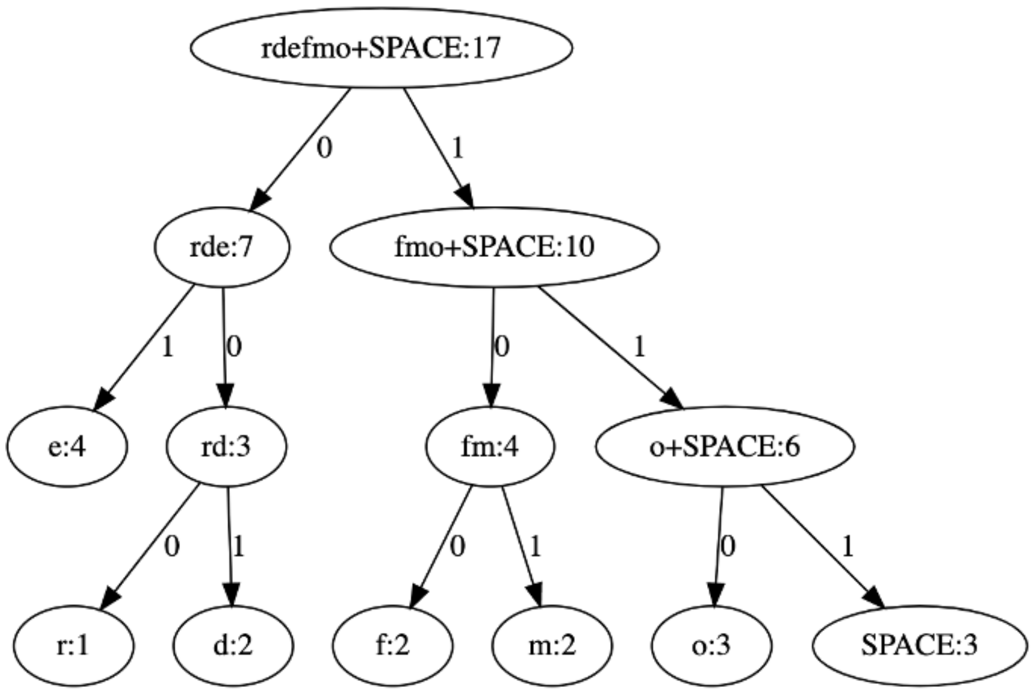
Single:

Merged: rde : 7 | fmo'SPACE' : 10

Output:

Single:

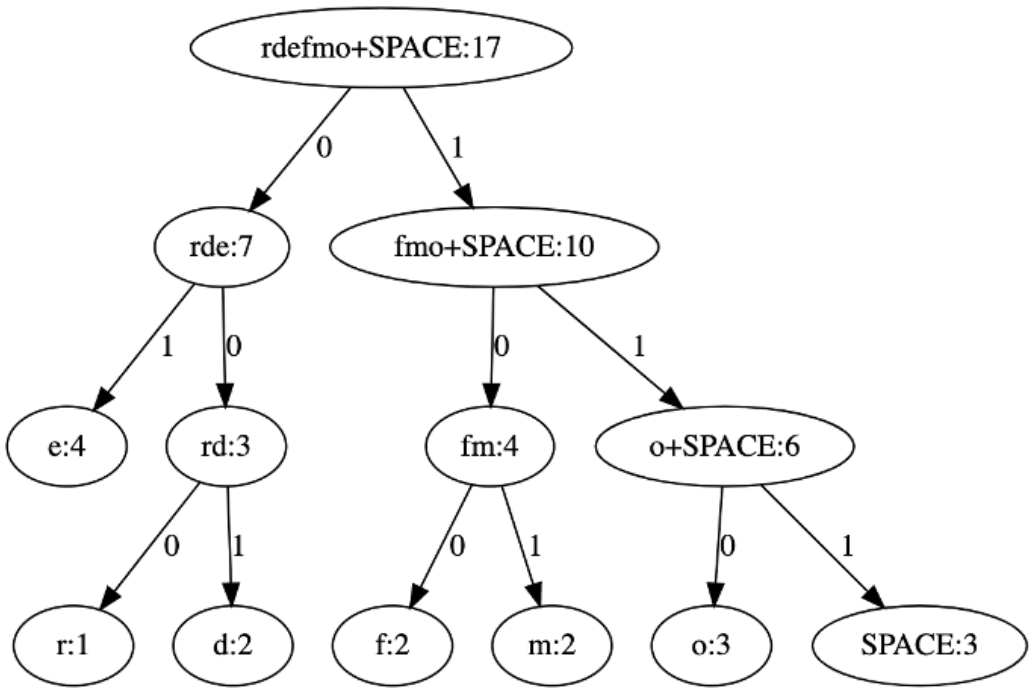
Merged: rdefmo'SPACE' : 17



Encoding using the Huffman Tree

The path through the tree defines each individual character's encoding!

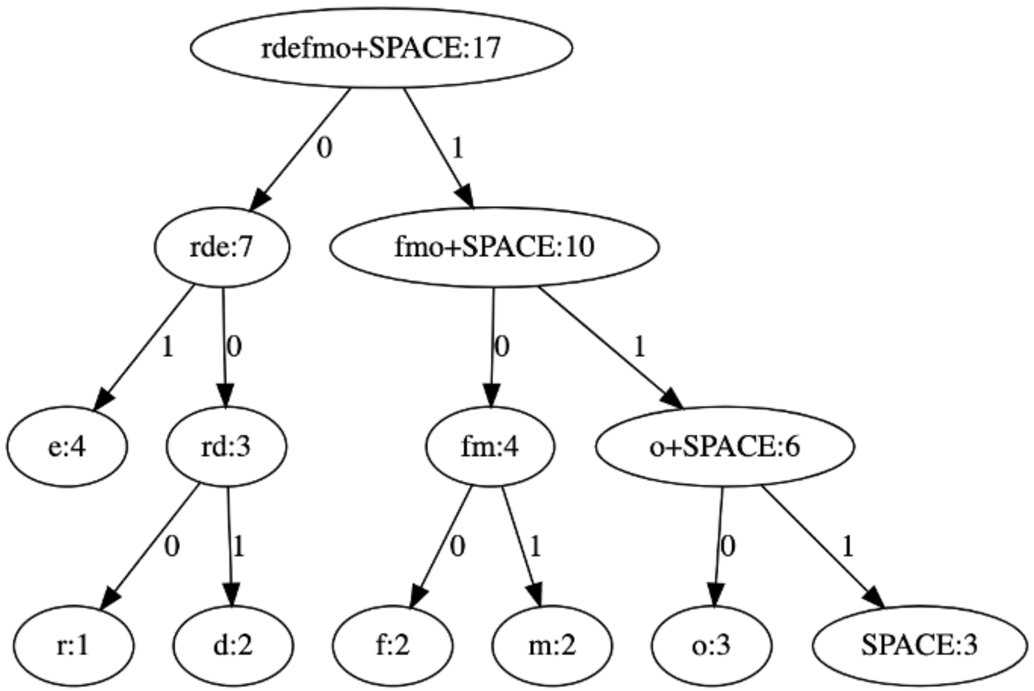
Char	Binary
f	
e	
d	
m	
r	
o	
‘ ‘	



Encoding using the Huffman Tree

The path through the tree defines each individual character's encoding!

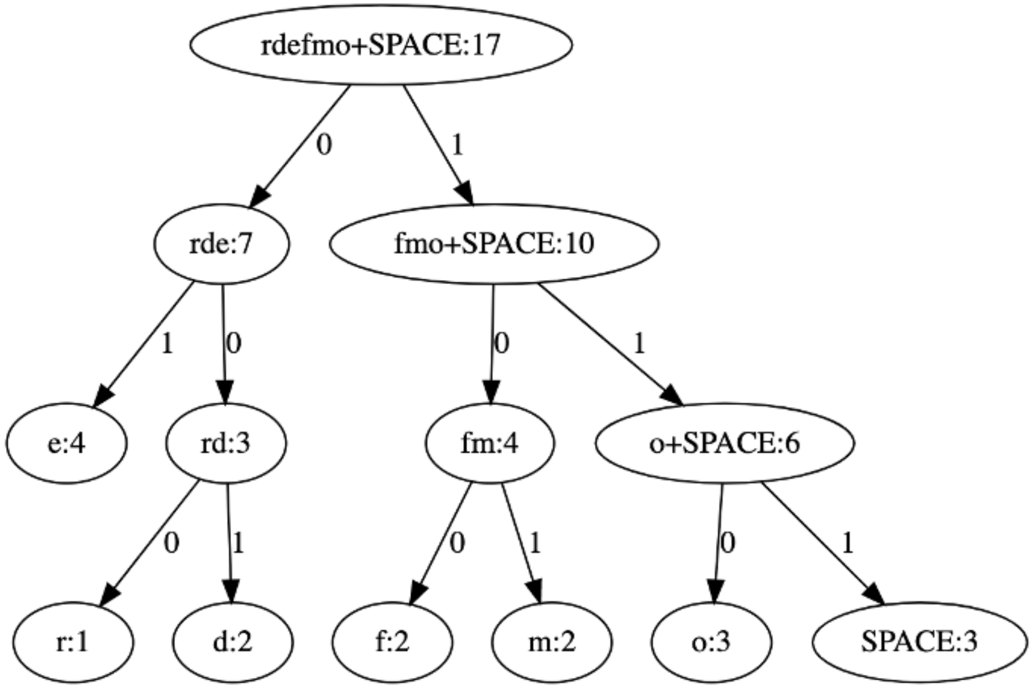
Char	Binary
f	100
e	01
d	001
m	101
r	000
o	110
‘ ‘	111



Decoding using the Huffman Tree

We can decode by walking through the tree using 0s and 1s as instructions!

Input: `100010100111110101`



Output: