Algorithms and Data Structures for Data Science
Binary Search Tree

CS 277
Brad Solomon

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UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Computer Science

Learning Objectives

Review understanding of Binary Trees

Practice tree traversals

Introduce the dictionary ADT

Extend ADT to Binary Search Trees

Practice recursion in the context of trees

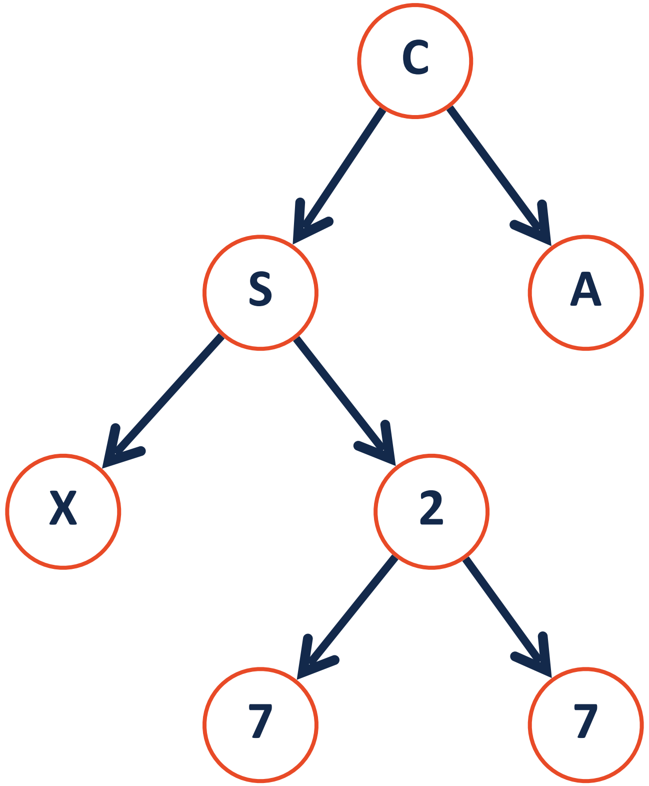
(Binary) Tree Recursion

A **binary tree** is a tree T such that:

$$T = None$$

or

$$T = treeNode(val, T_L, T_R)$$




```
class treeNode:
    def __init__(self, val, left=None, right=None):
        self.val = val
        self.left = left
        self.right = right
```

```
class binaryTree:
    def __init__(self):
        self.root = None
```

Tree ADT

Constructor: Build a new (empty) tree

Insert: Add an object into tree

Remove: Remove a specific object from tree

Traverse: Visit every node in tree (all objects)

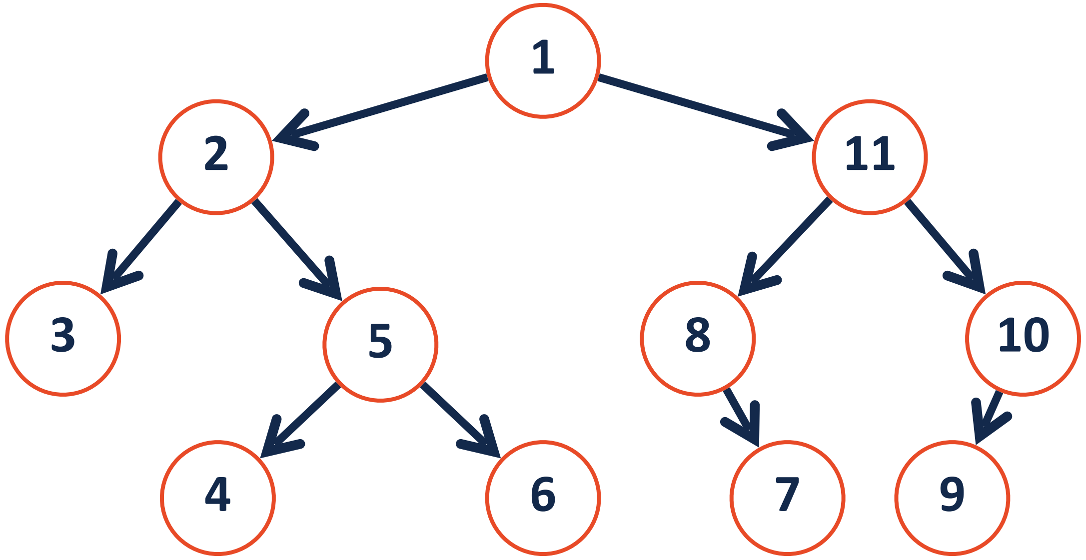
Search: Find a specific object in the tree

Binary Tree Insert and Remove

Last class we implemented insert and remove where we are given the parent node and direction (allowing us to reach the node of interest)

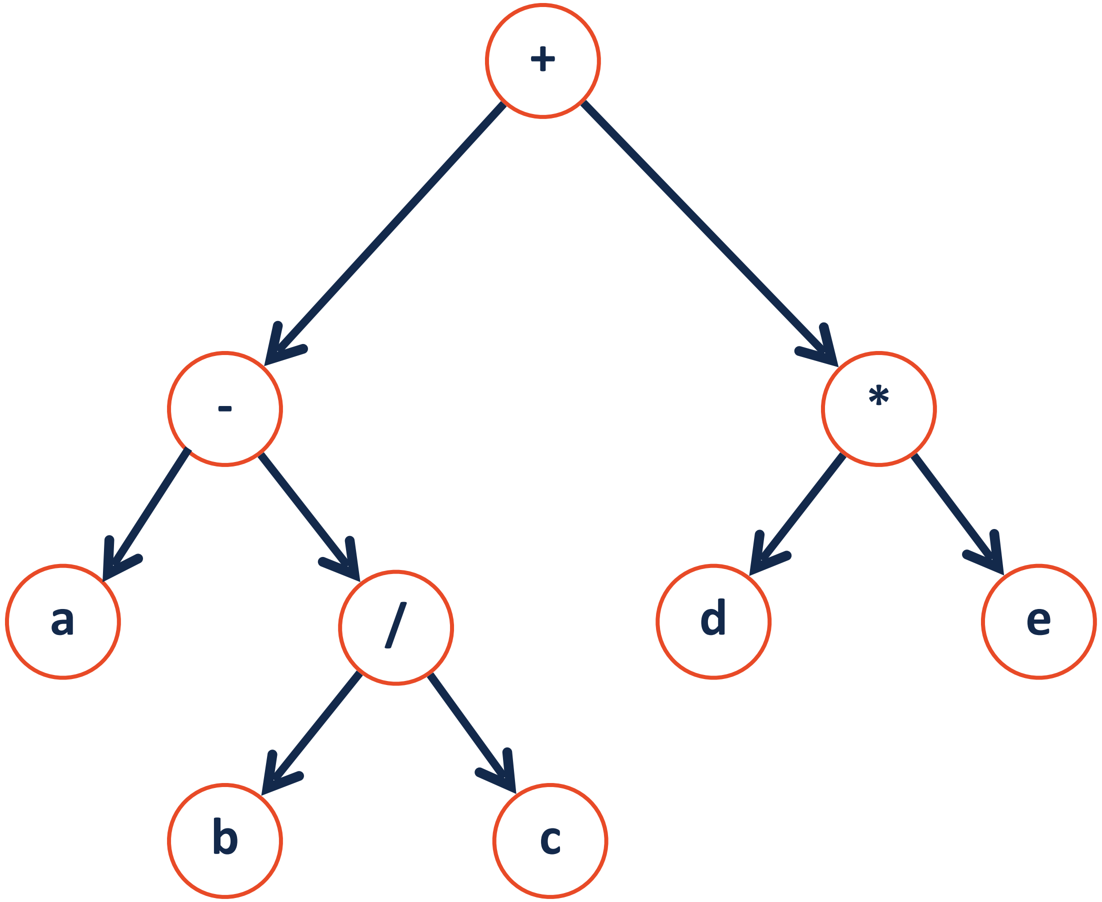
Insert was a lot like what previous data structure:

Remove has one bad case, which was:

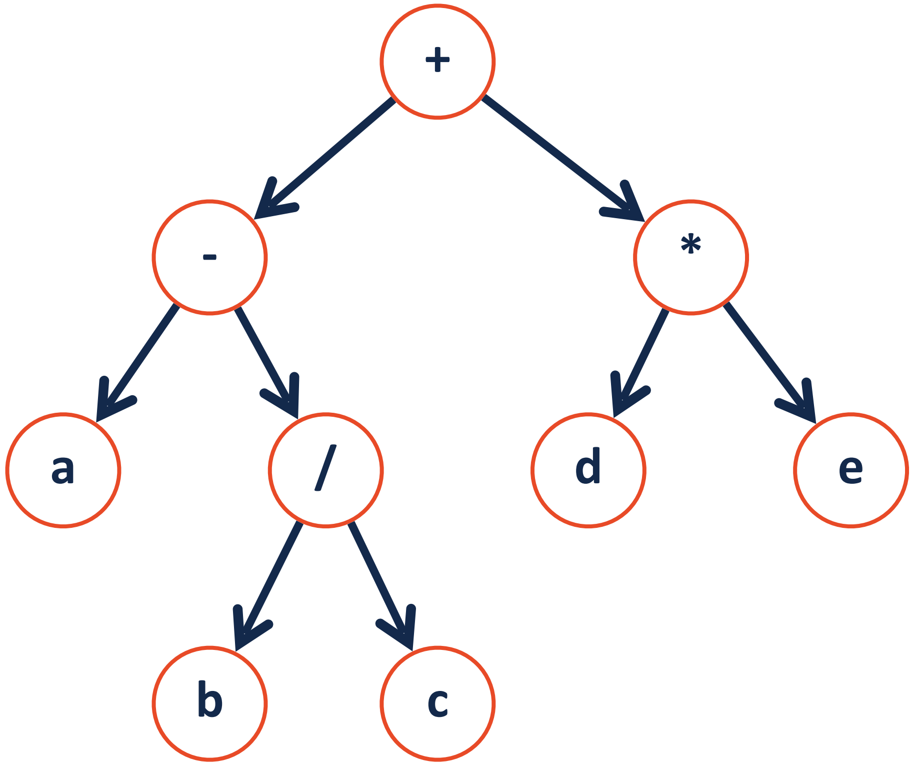


Tree Traversal

A **traversal** of a tree T is an ordered way of visiting every node once.



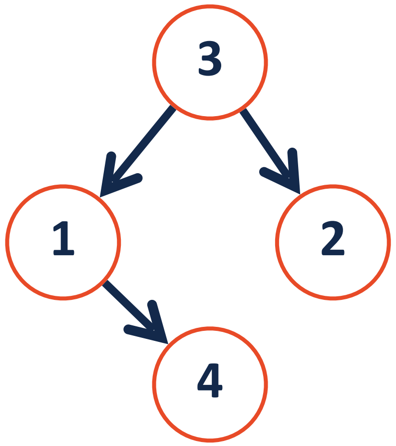
Pre-order Traversal




```
def preorderTraversal(node):
    if node:
        print(node.val)
        preorderTraversal(node.left)
        preorderTraversal(node.right)
```

Pre-order:

Pre-order Traversal Visualized



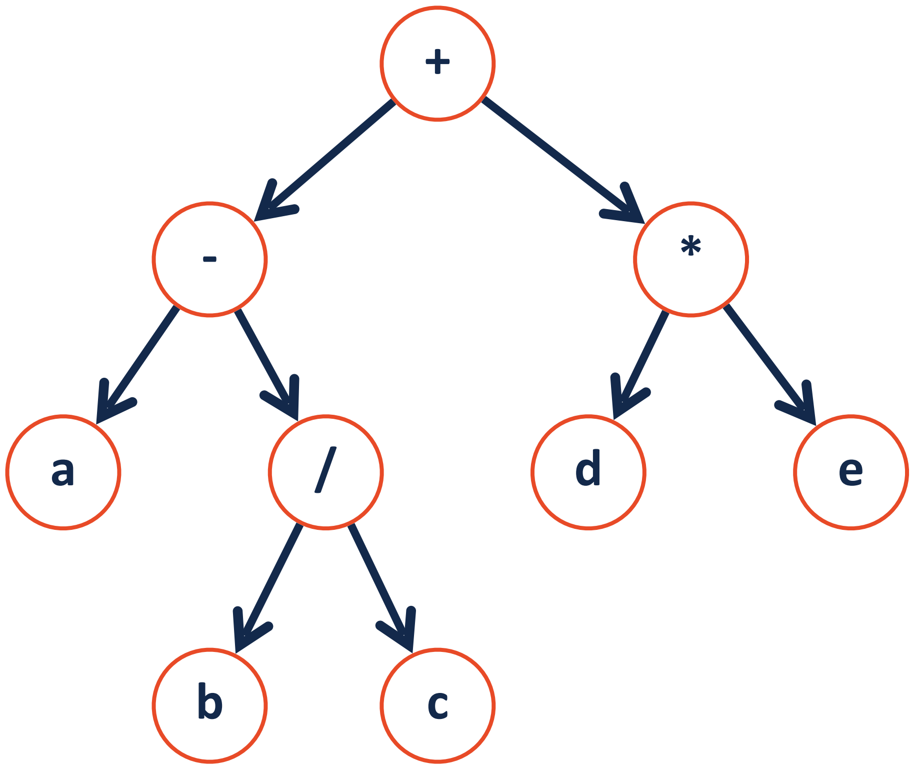

```
def preorderTraversal(<3>):
    if node:
        print(<3>.val)

        preorderTraversal(<3>.left)

        preorderTraversal(<3>.right)
```

```
def preorderTraversal(<1>):
    if node:
        print(<1>.val)

        preorderTraversal(<1>.left)

        preorderTraversal(<1>.right)
```

```
def preorderTraversal(<2>):
    if node:
        print(<2>.val)

        preorderTraversal(<2>.left)

        preorderTraversal(<2>.right)
```

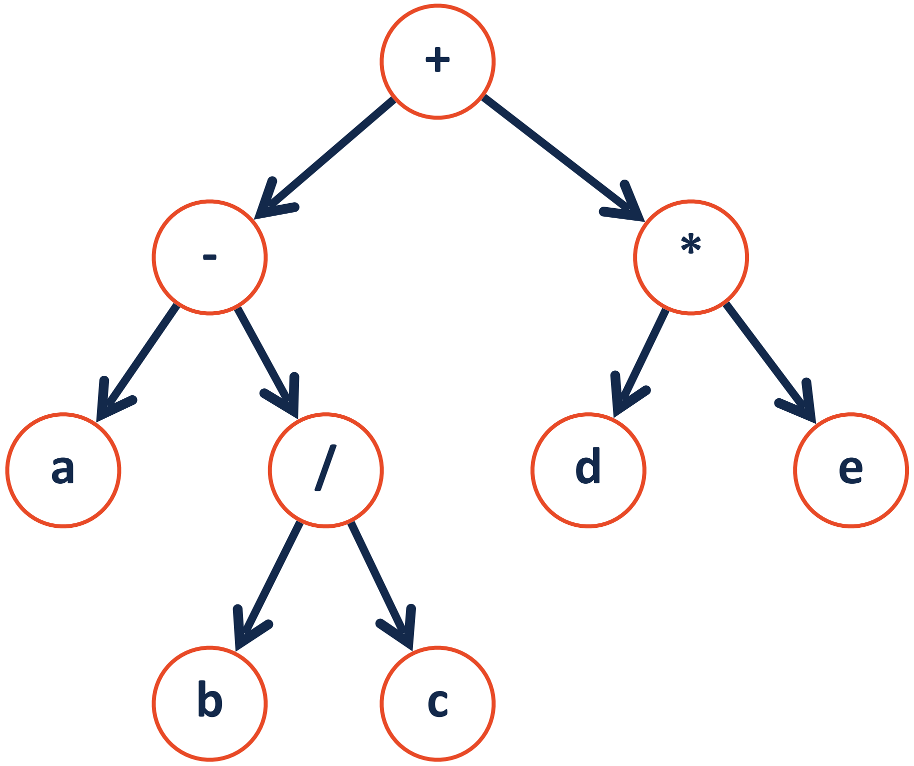
```
def preorderTraversal(<4>):
    if node:
        print(<4>.val)

        preorderTraversal(<4>.left)

        preorderTraversal(<4>.right)
```

In-order Traversal



In-order:

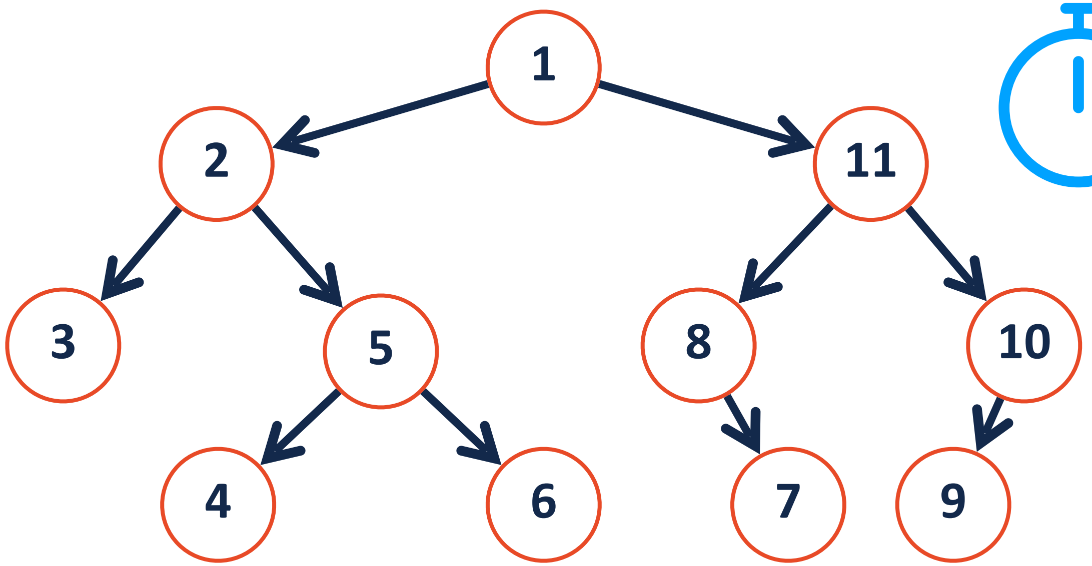
Post-order Traversal



Post-order:

Tree Traversals

Lets practice our traversals!

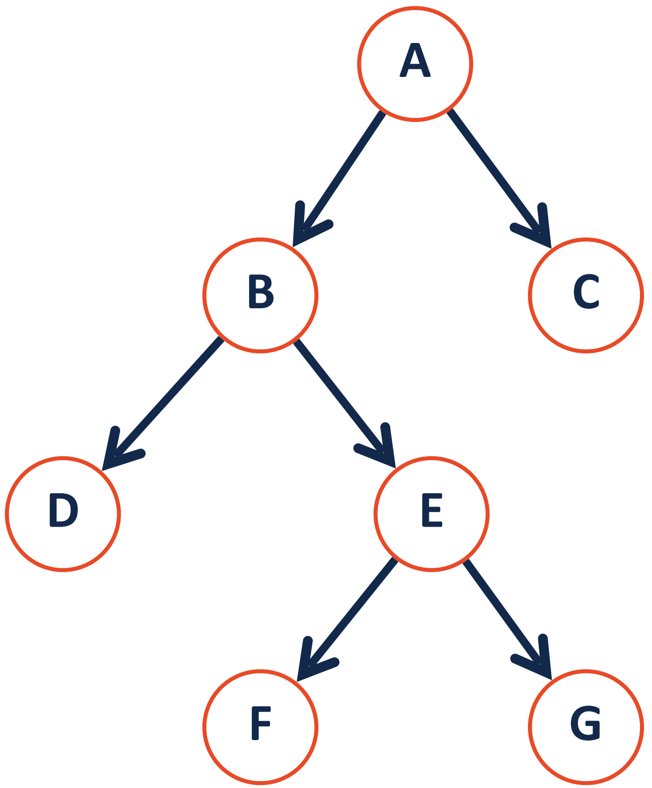


Pre-order:

In-order:

Post-order:

Traversal vs Search

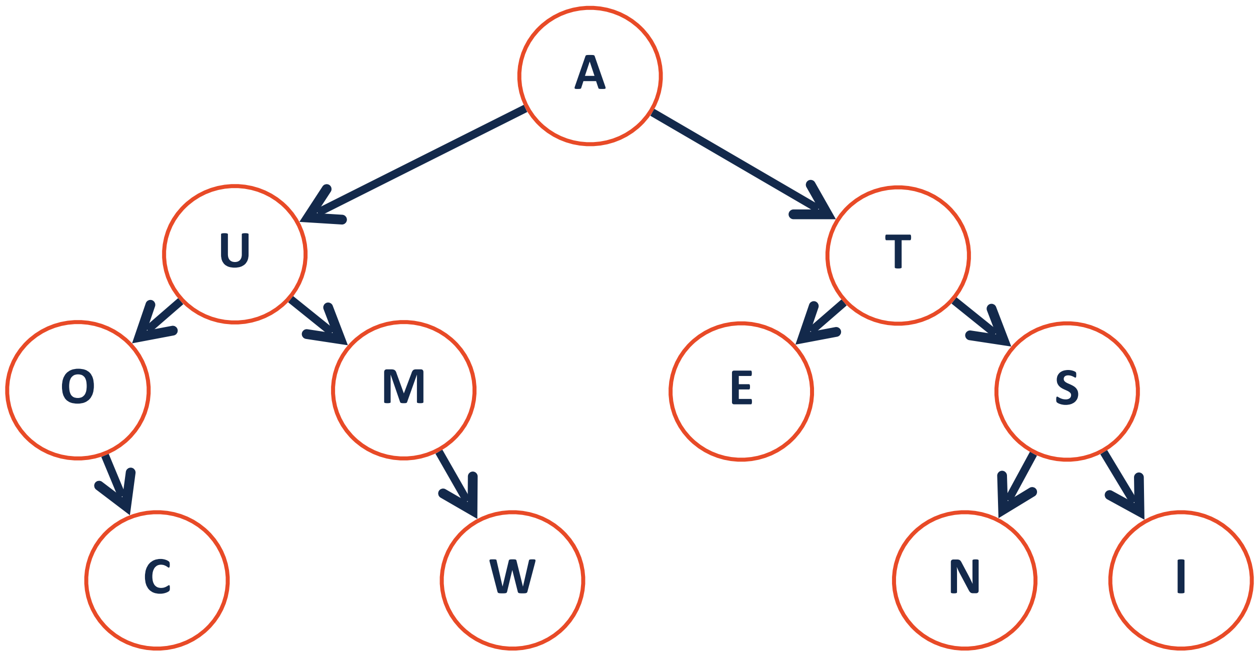


Traversal

Search

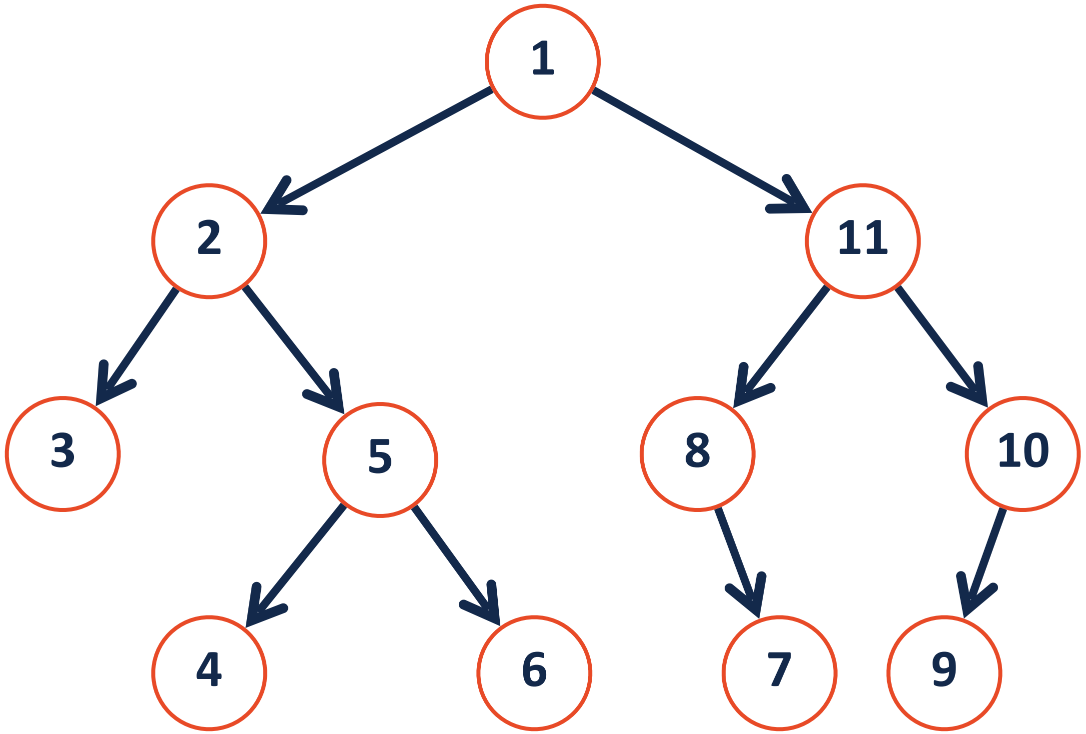
Searching a Binary Tree

There are two main approaches to searching a binary tree:



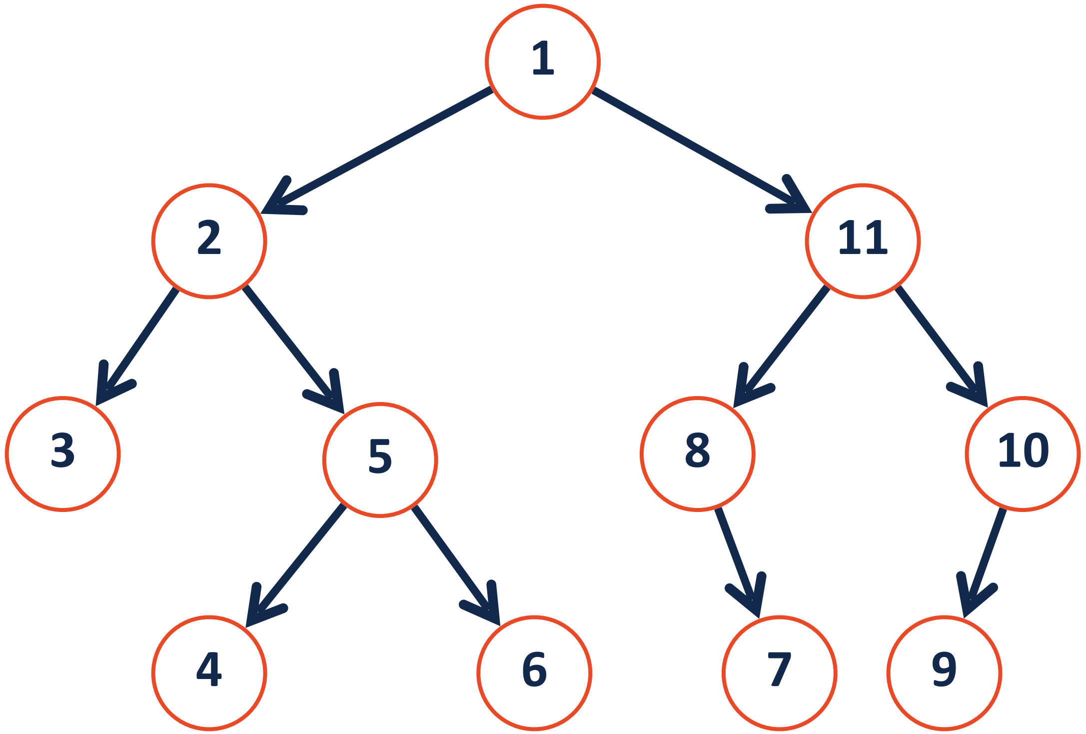
Depth First Search

Explore as far along one path as possible before backtracking



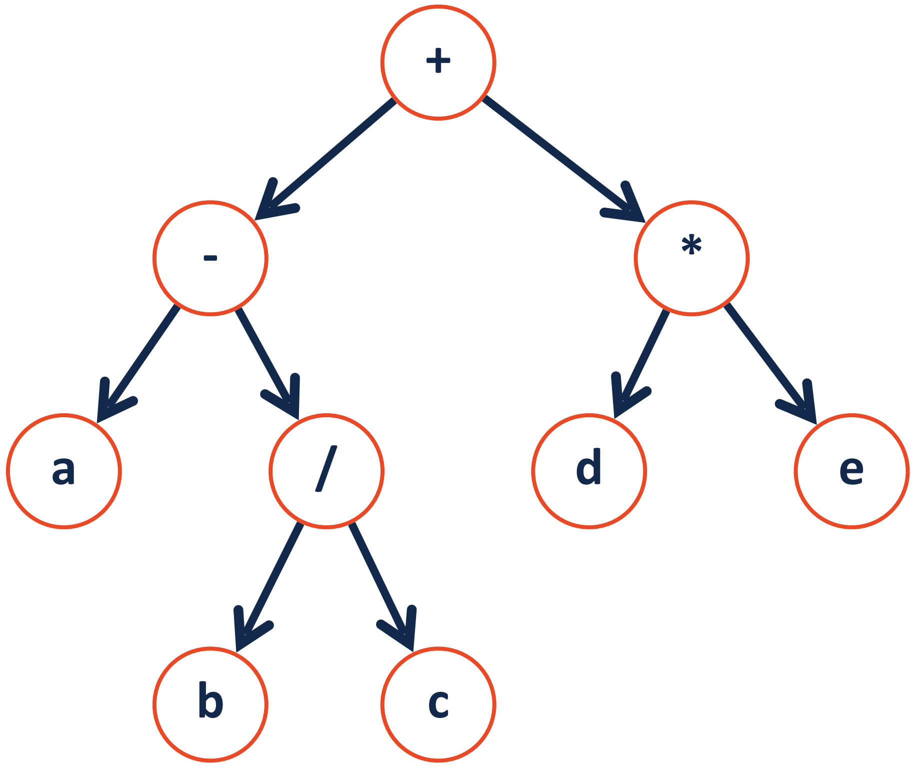
Breadth First Search

Fully explore depth i before exploring depth i+1



Traversal vs Search II

Pre-order, in-order, and post-order are three ways of doing which search?



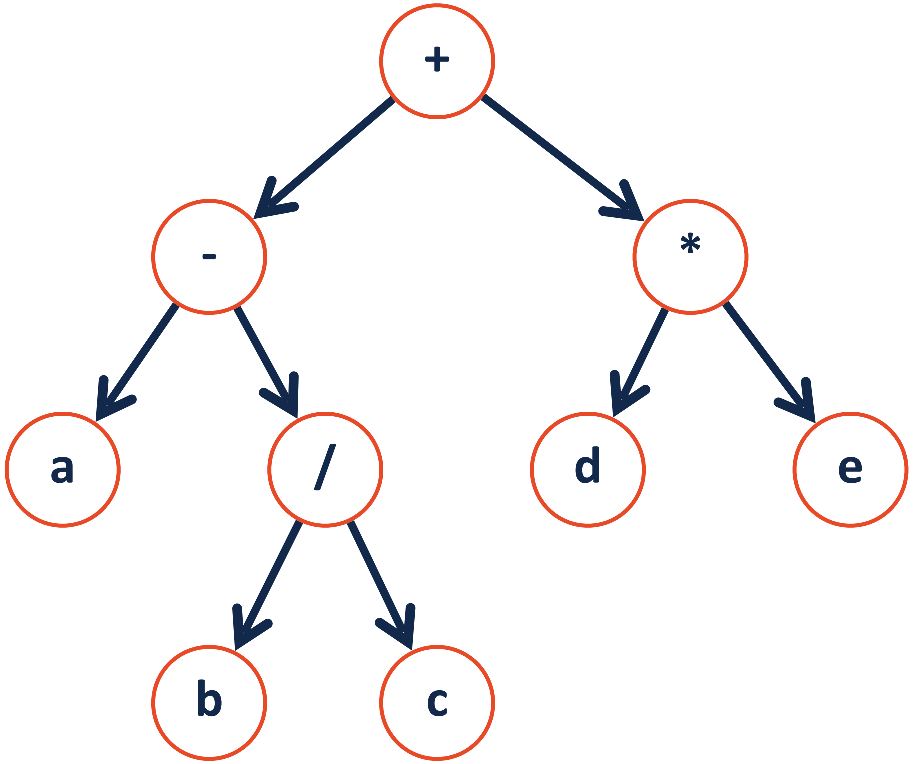
Pre-order: + - a / b c * d e

In-order: a - b / c + d * e

Post-order: a b c / - d e * +

Level-Order Traversal

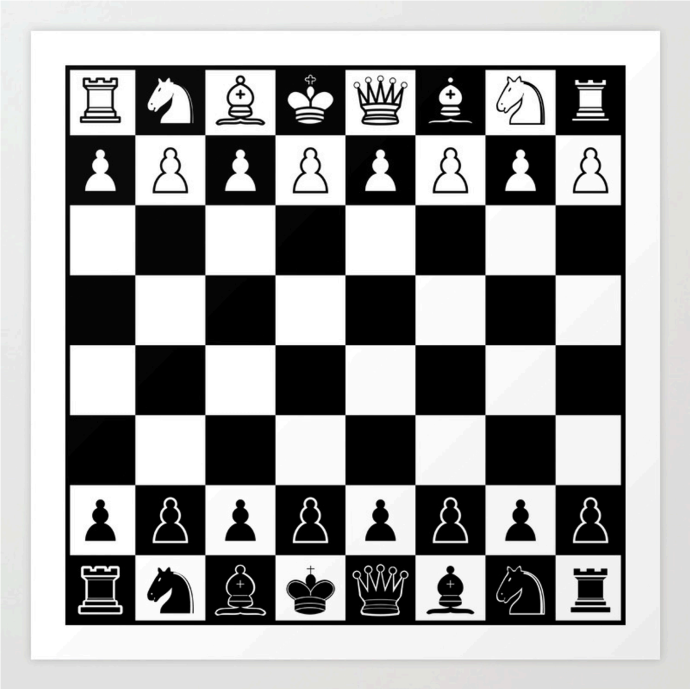
A tricky recursive implementation but an easier queue implementation!



Level-order:

What search algorithm is best?

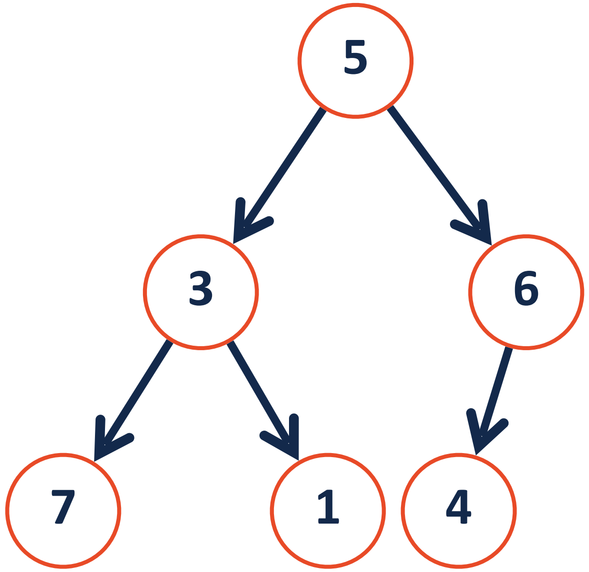
The average 'branch factor' for a game of chess is ~31. If you were searching a decision tree for chess, which search algorithm would you use?

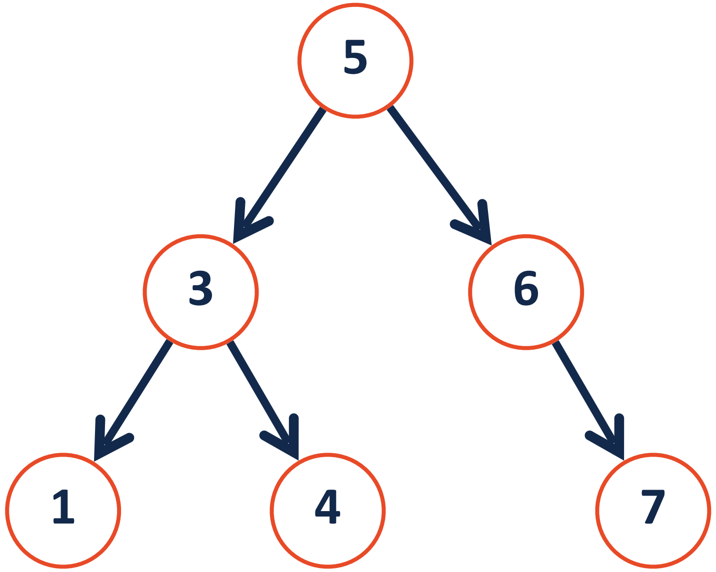


Improved search on a binary tree

5	3	6	7	1	4

1	3	4	5	6	7



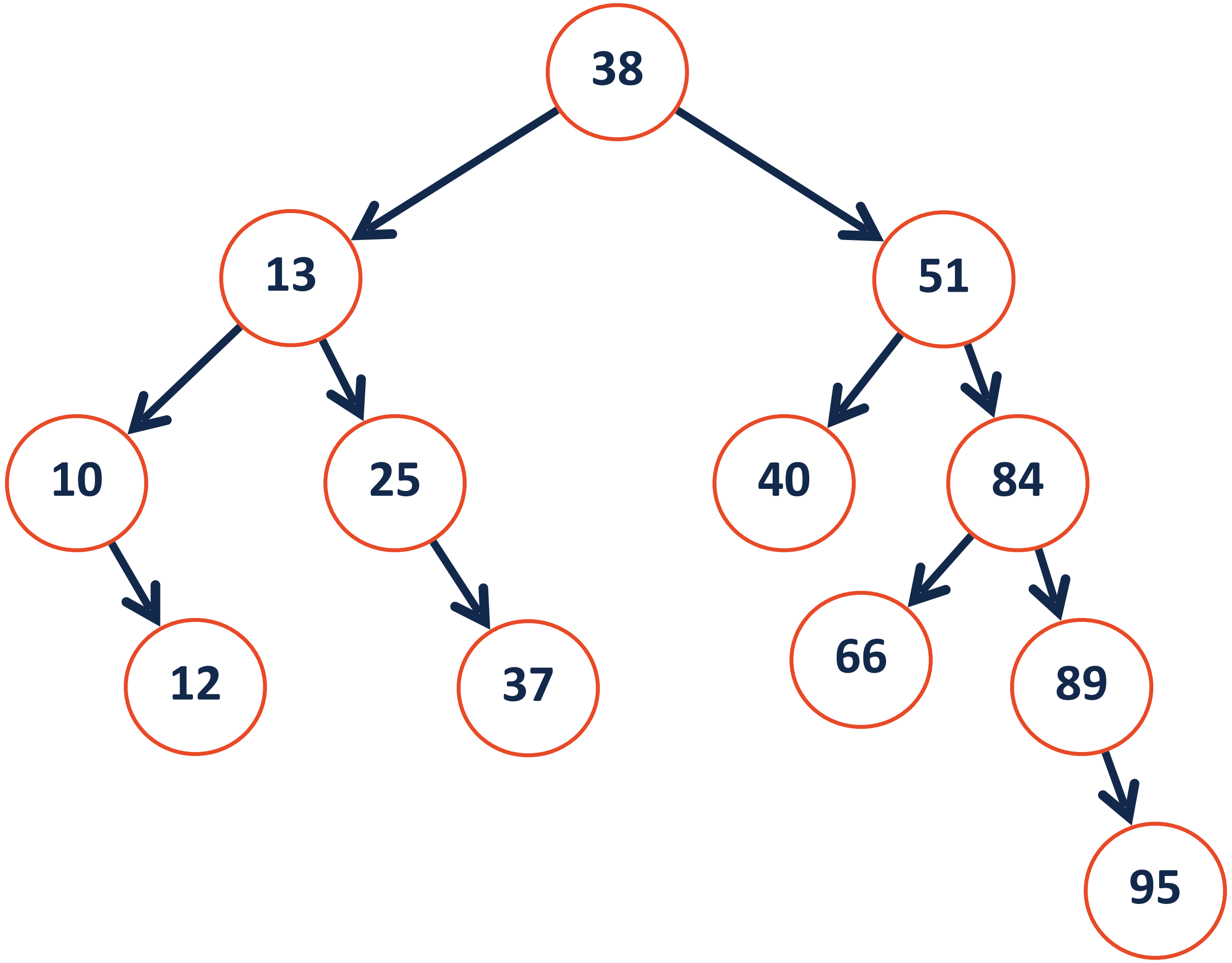


Binary Search Tree (BST)

A **BST** is a binary tree $T = treeNode(val, T_L, T_r)$ such that:

$$\forall n \in T_L, \ n\,.\,val < T\,.\,val$$

$$\forall n \in T_R, \ n\,.\,val > T\,.\,val$$



Improved search on a binary tree

5	3	6	7	1	4

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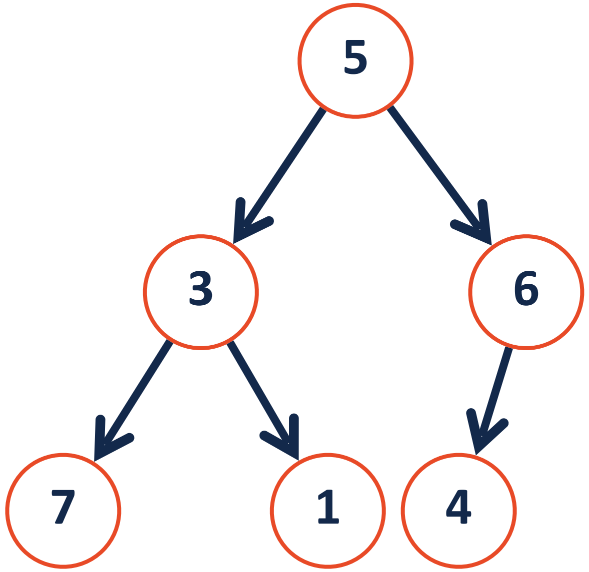

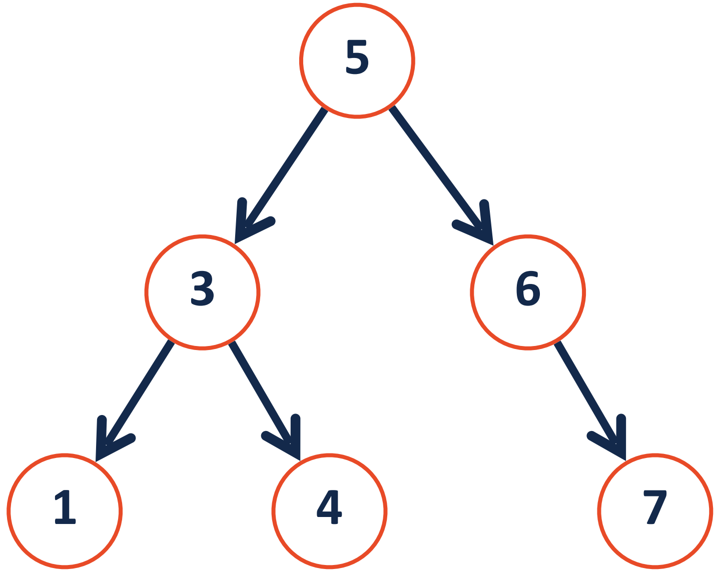

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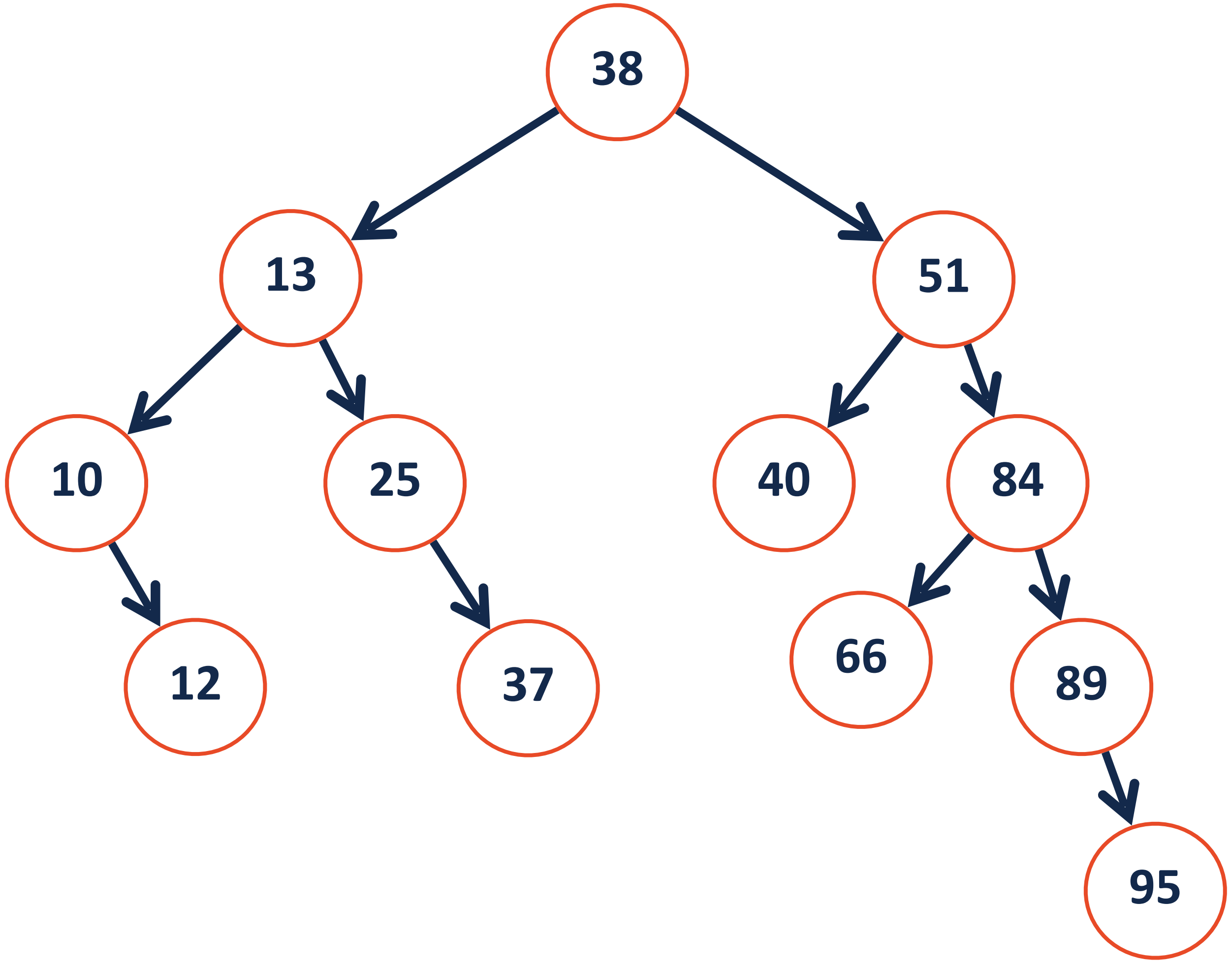


Dictionary ADT

Data is often organized into key/value pairs:

Word ➔ Definition

Course Number ➔ Lecture/Lab Schedule

Node ➔ Edges

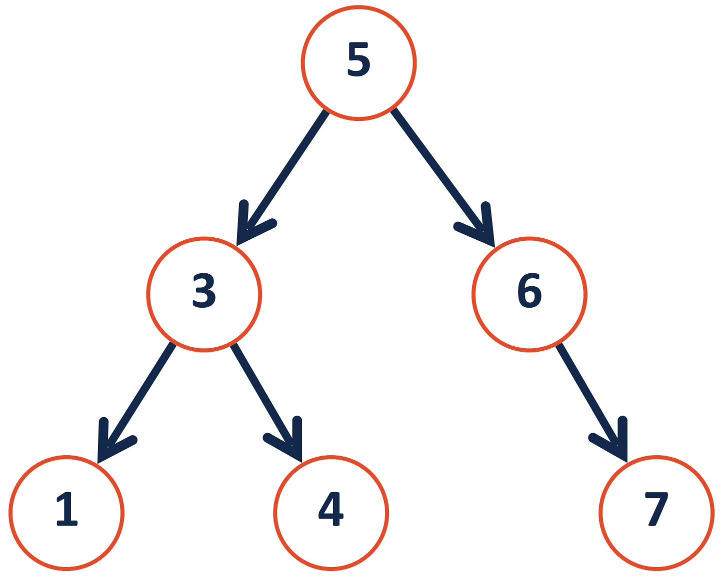
Flight Number ➔ Arrival Information

URL ➔ HTML Page

Average Image Color ➔ File Location of Image

Dictionary as a Binary Search Tree

```
class bstNode:
    def __init__(self, key, val, left=None, right=None):
        self.key = key
        self.val = val
        self.left = left
        self.right = right
```



Key	5	3	6	7	1	4
Value	A	B	C	D	E	F

Binary Search Tree ADT

Constructor: Build a new (empty) tree

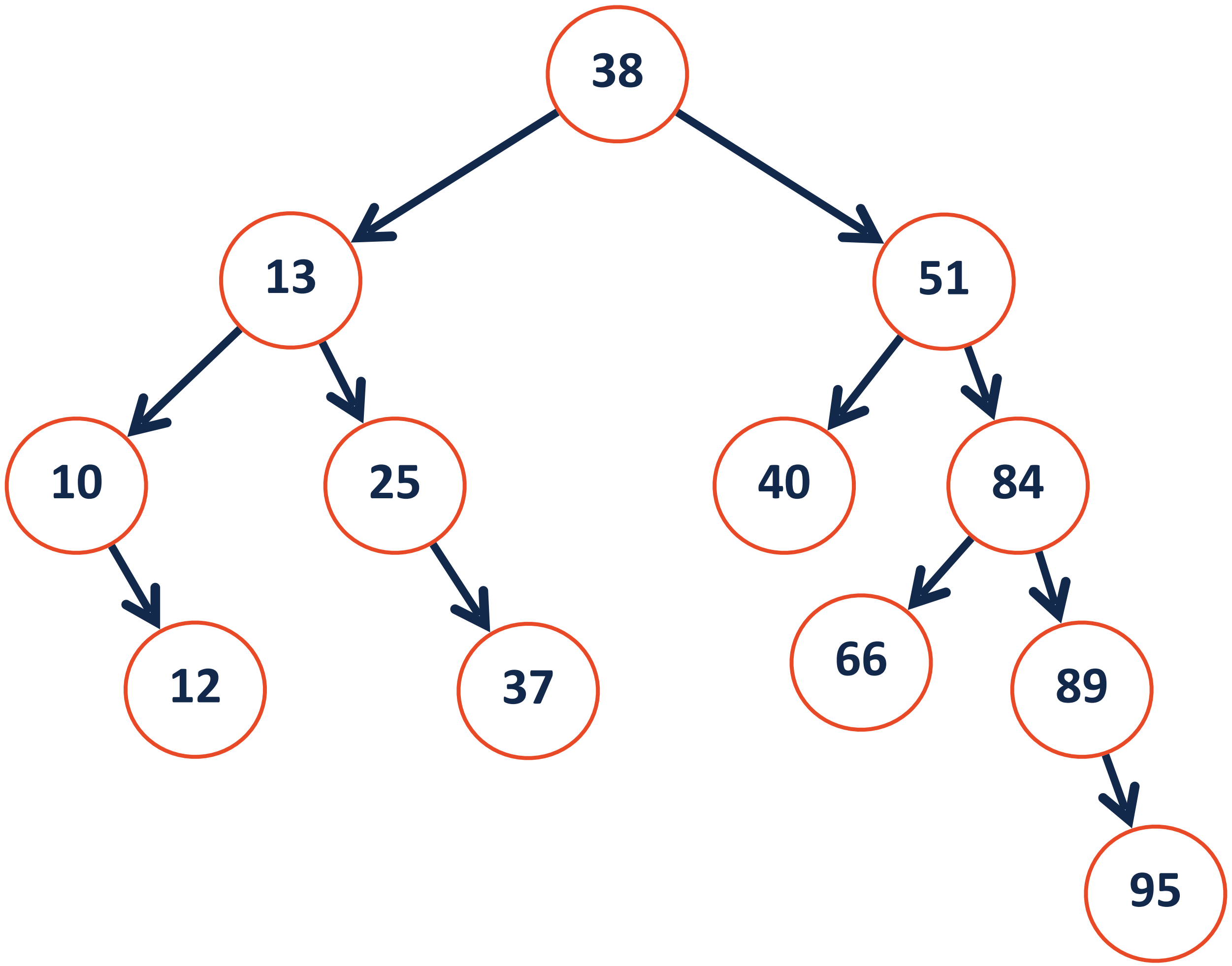
Insert: Add an object into tree

Remove: Remove a specific object from tree

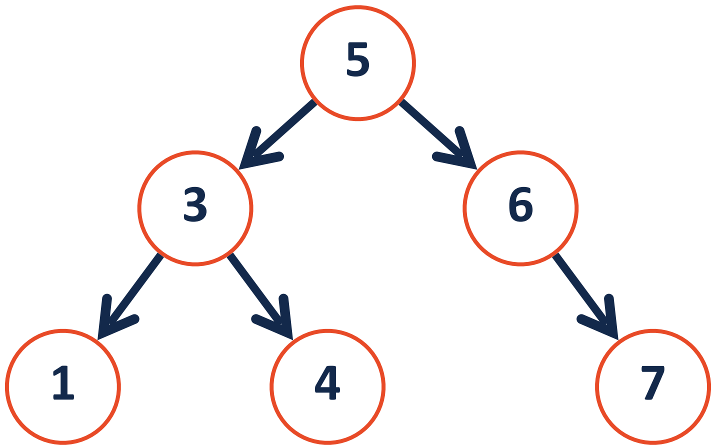
Traverse: Visit every node in tree (all objects)

Search: Find a specific object in the tree

BST In-Order Traversal



BST Insert



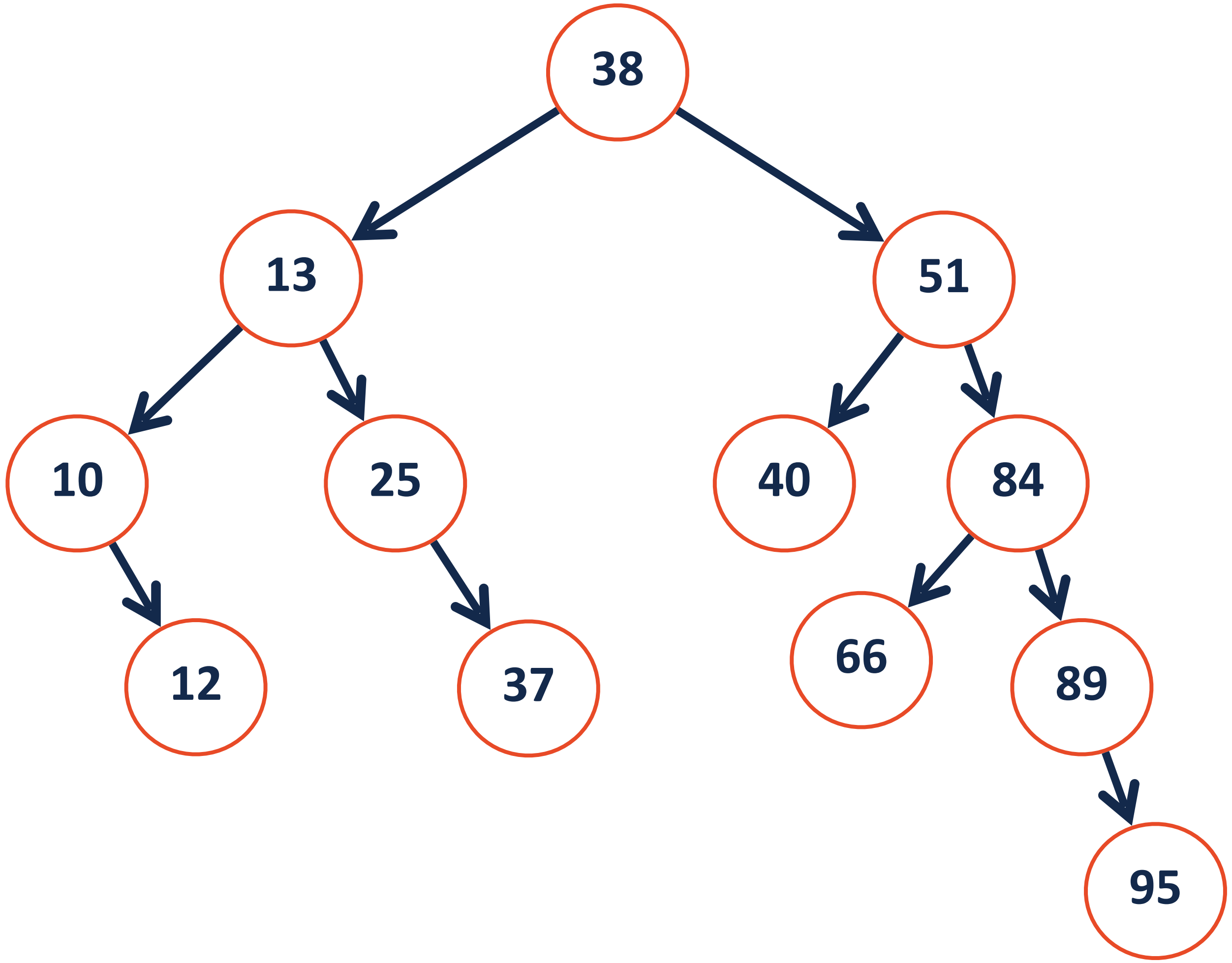
Base Case:

Recursive Step:

Combining:

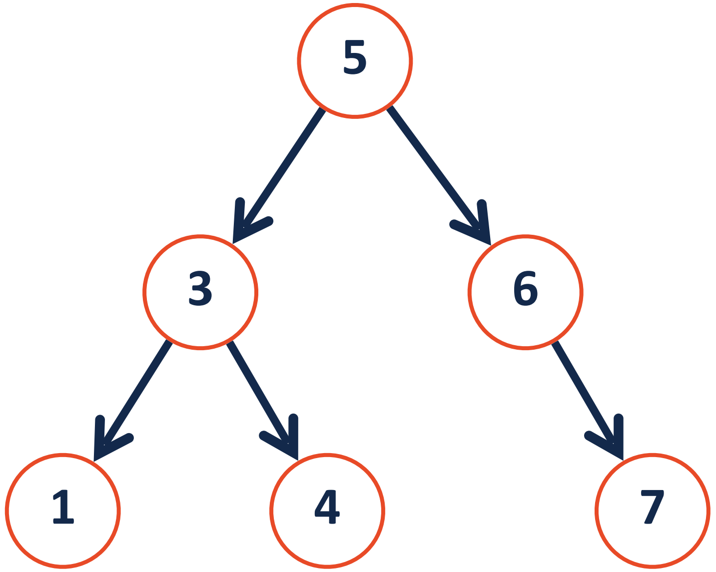
BST Insert

`insert(33)`



BST Insert

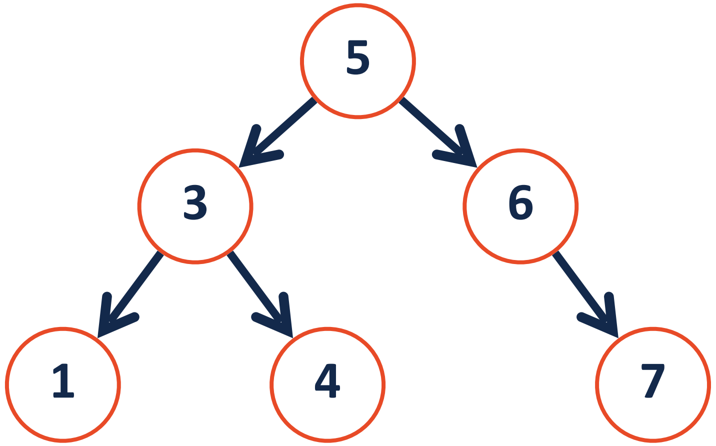
```
# inside class bst
def insert(self, key, val):
    self.root = self.insert_helper(self.root, key, val)

def insert_helper(self, node, key, val):
```



BST Insert

What binary would be formed by inserting the following sequence of integers: `[3, 7, 2, 1, 4, 8, 0]`

BST Find



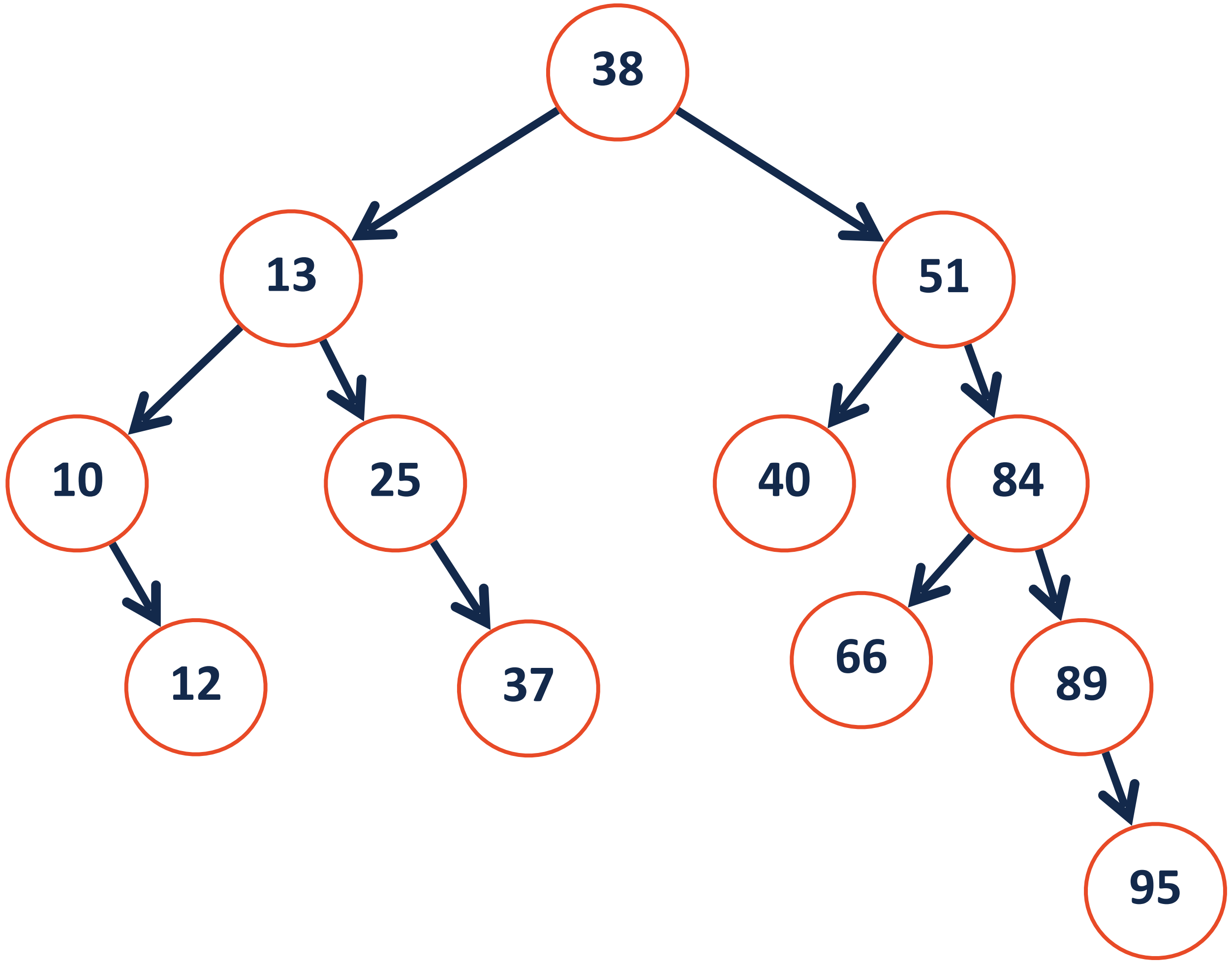
Base Case:

Recursive Step:

Combining:

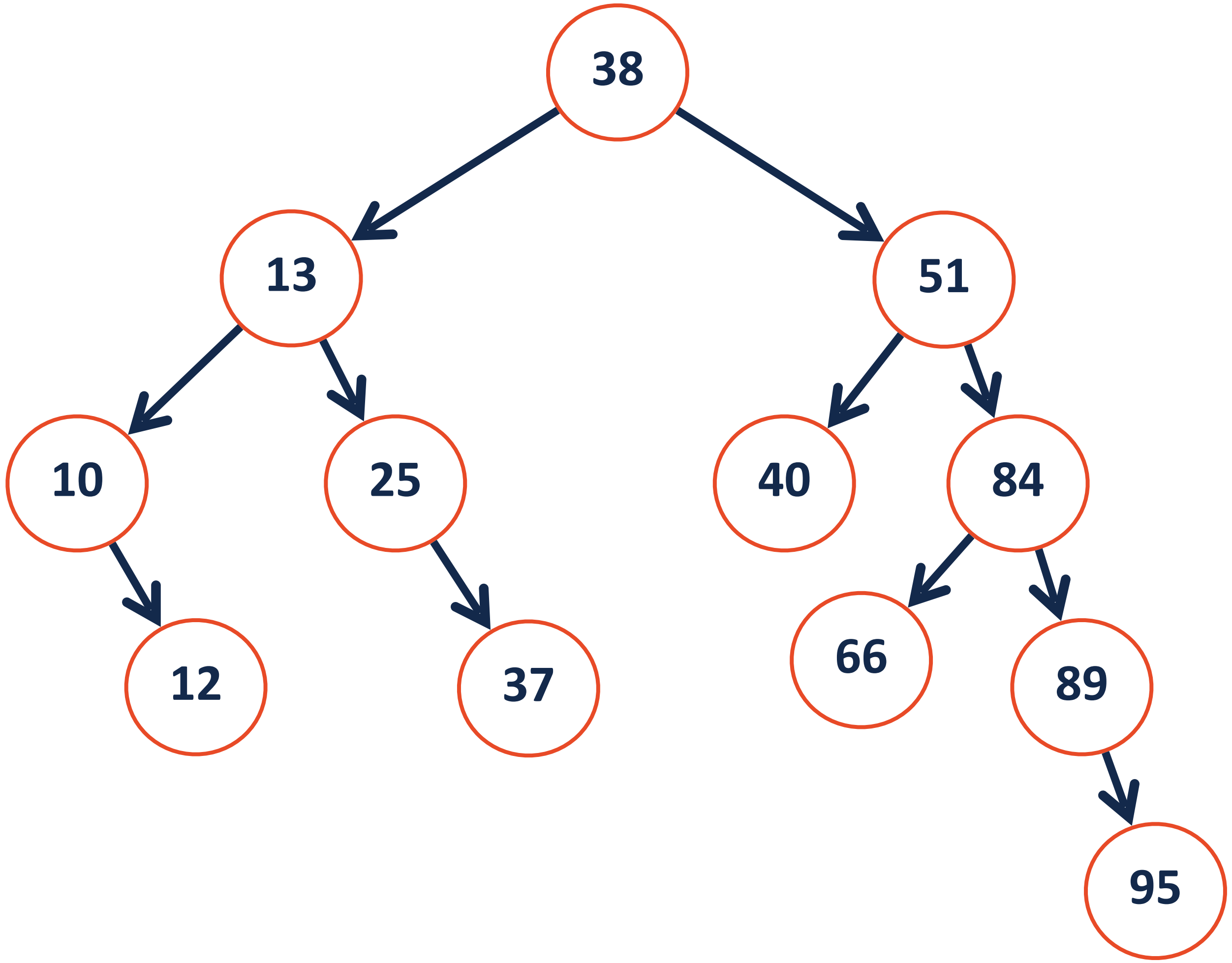
BST Find

`find(66)`



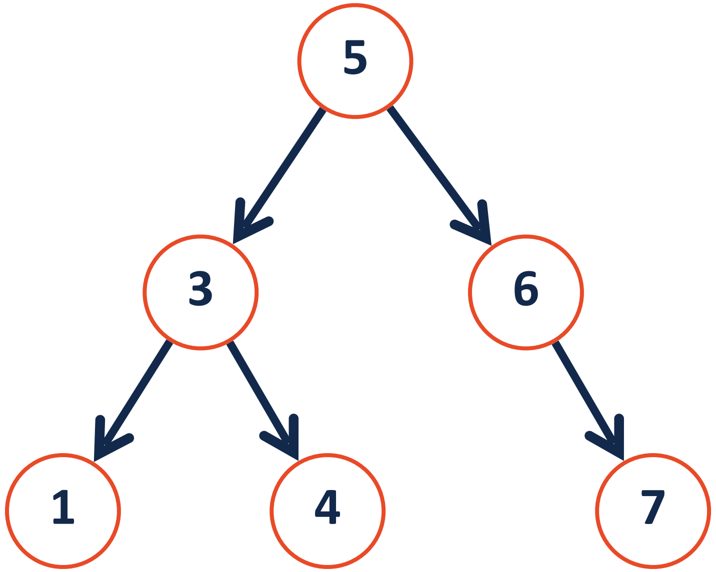
BST Find

`find(9)`



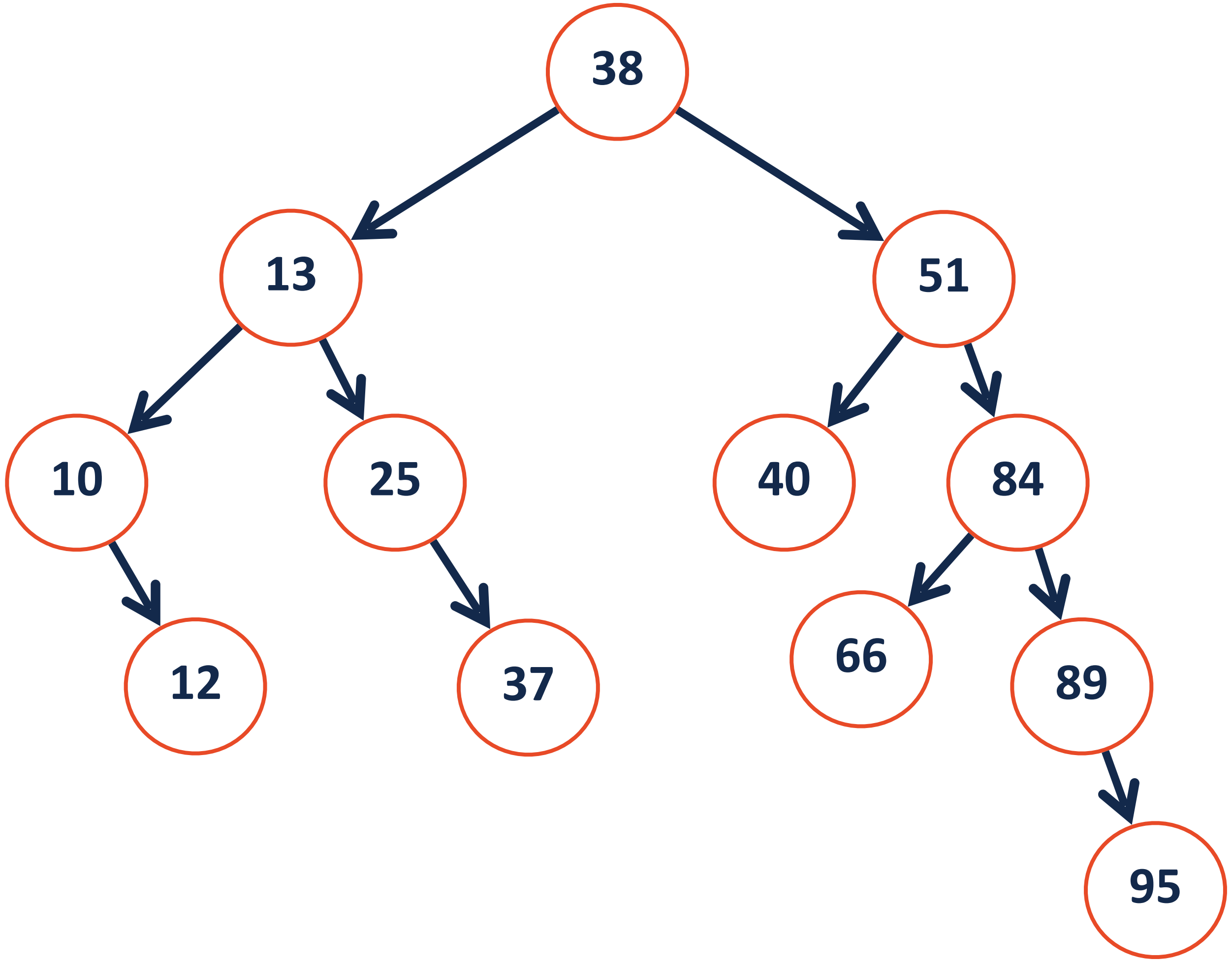
BST Find

```
#inside class bst
def find(self, key):






def find_helper(self, node, key):
```



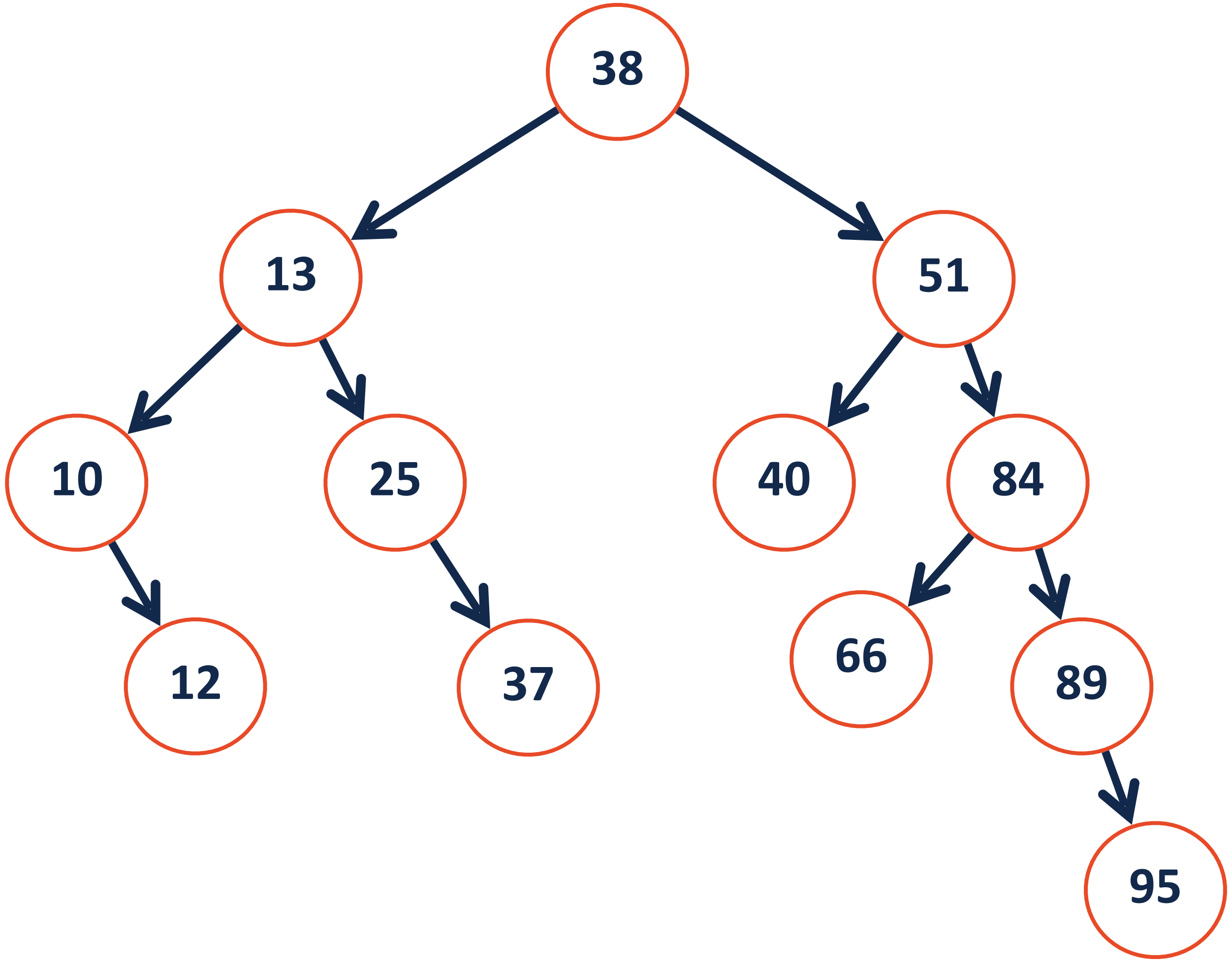
BST Remove

`remove(40)`



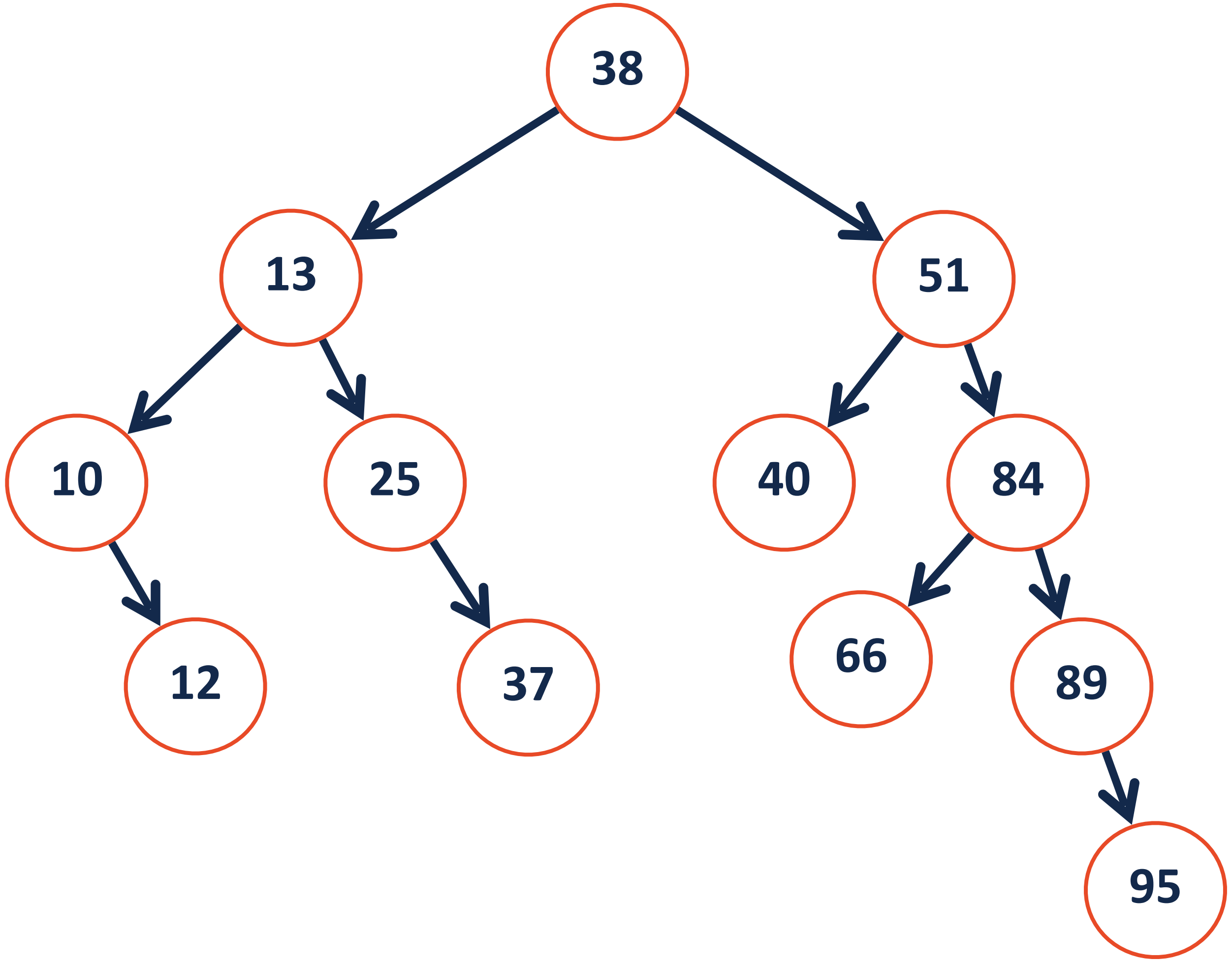
BST Remove

`remove(25)`



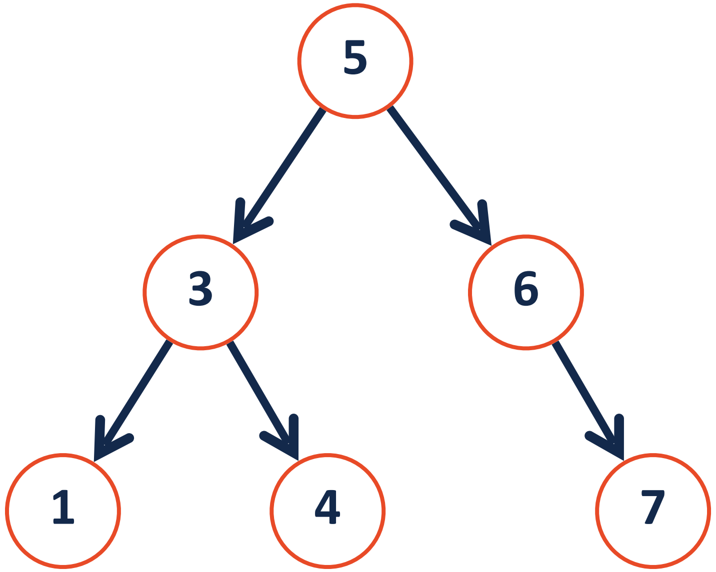
BST Remove

`remove(13)`



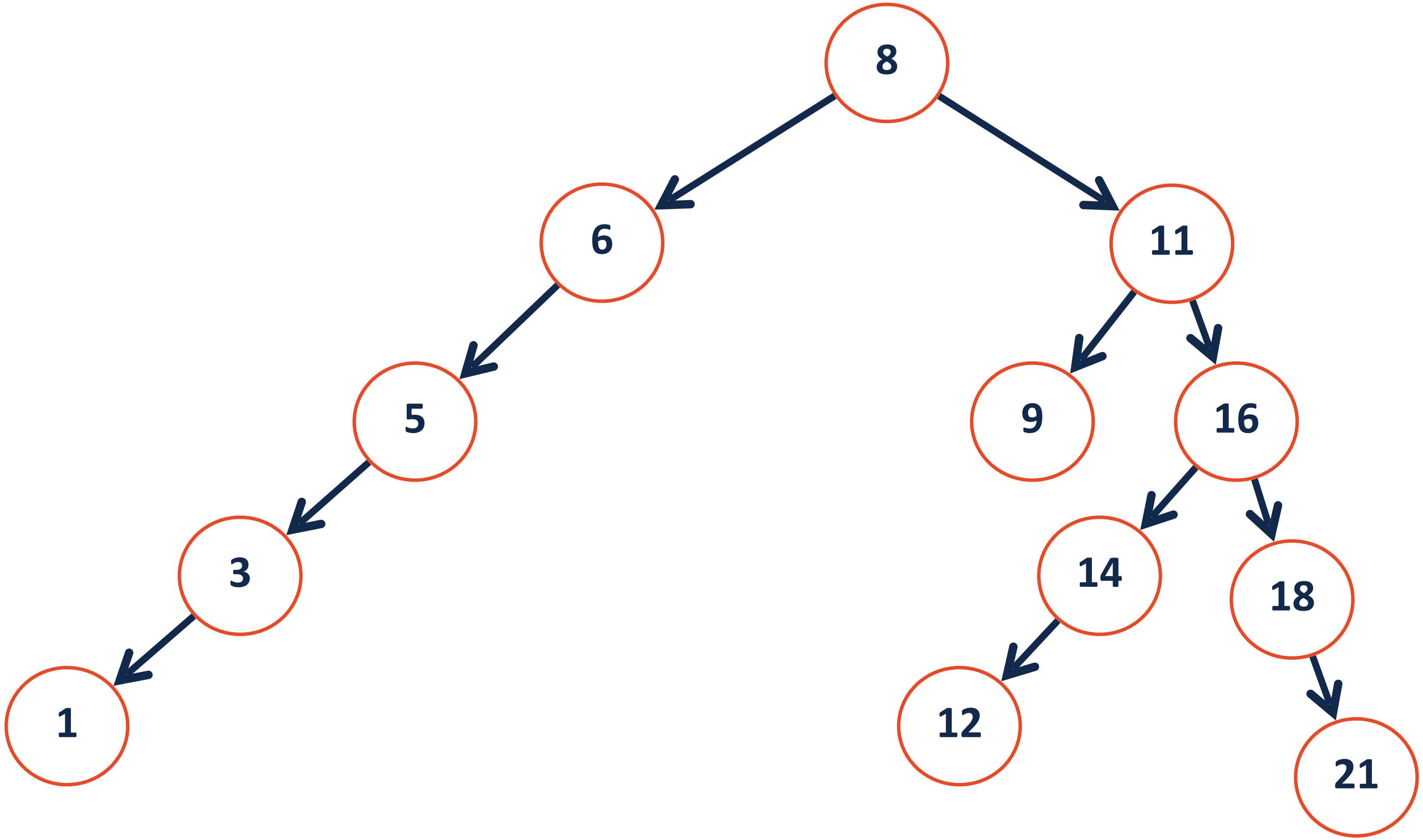
BST Remove

```
def remove(self, key):
    self.root = self.remove_helper(self.root, key)

def remove_helper(self, node, key):
```



BST Remove

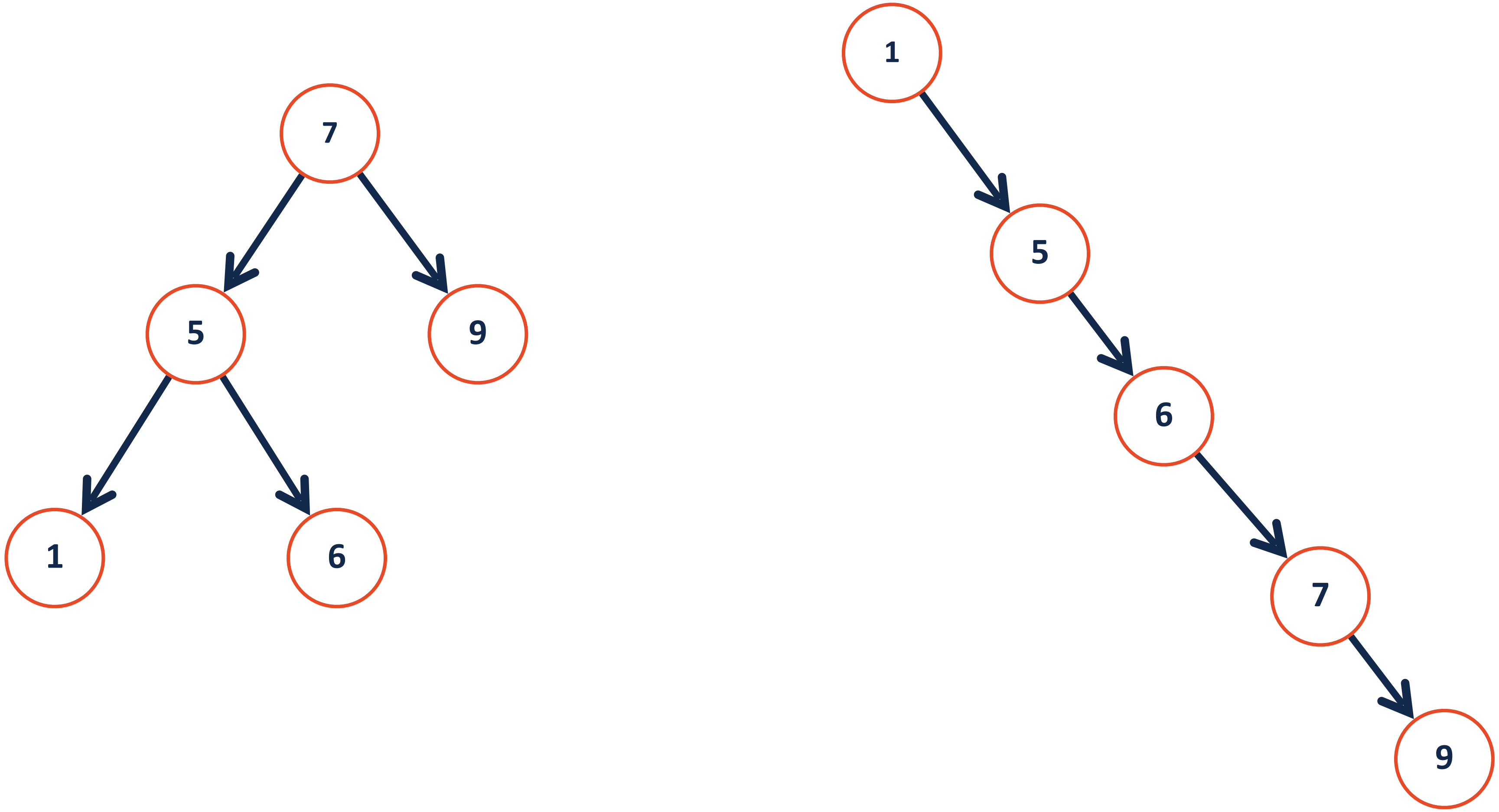
What will the tree structure look like if we remove node 16 using IOS?



BST Analysis – Running Time

Operation	BST Worst Case
find	
insert	
delete	
traverse	

Limiting the height of a tree



Option A: Correcting bad insert order

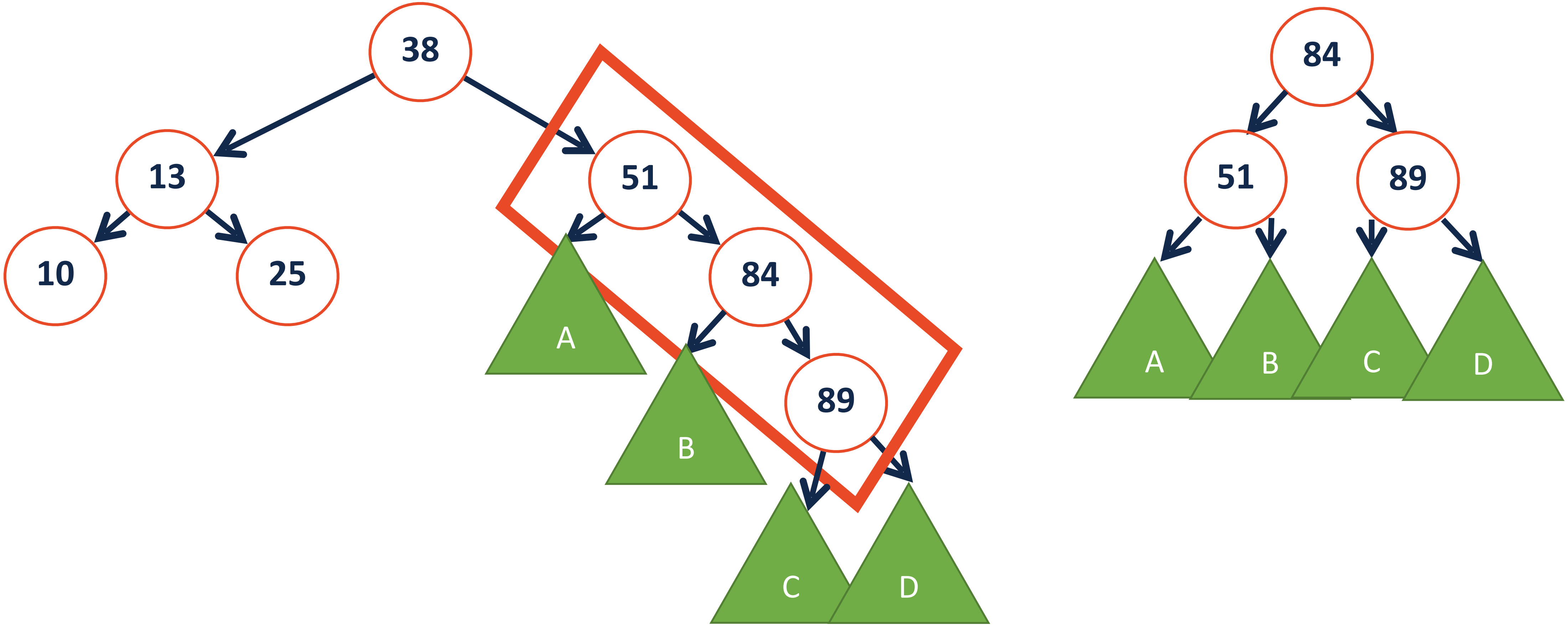
The height of a BST depends on the order in which the data was inserted

Insert Order: [1, 3, 2, 4, 5, 6, 7]

Insert Order: [4, 2, 3, 6, 7, 1, 5]

AVL-Tree: A self-balancing binary search tree

Rather than fixing an insertion order, just correct the tree as needed!



When would we use a tree?

Pretend for a moment that we always have an optimal BST.

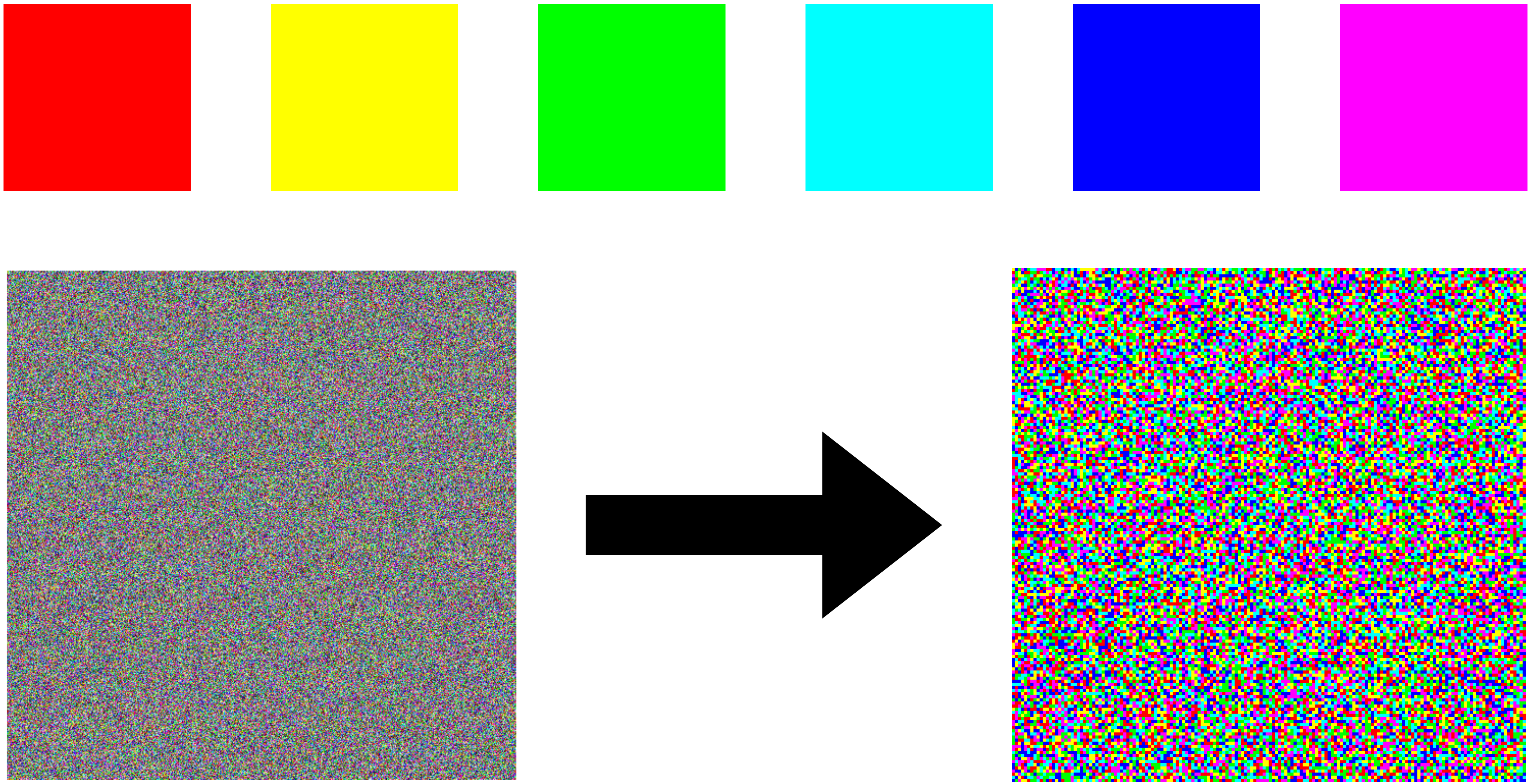
What is the running time of **find**?

What is the running time of **insert**?

What is the running time of **remove**?

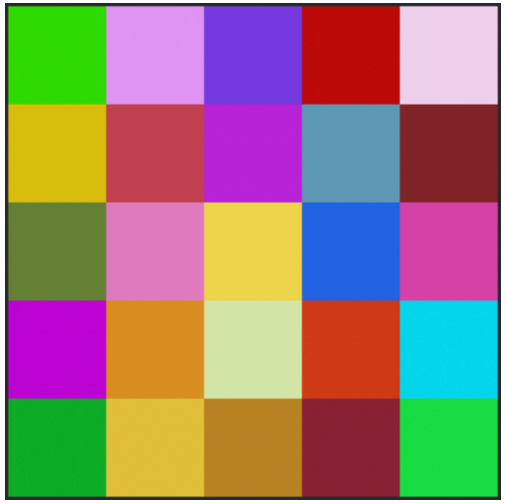
Is there a data structure with a *better* running time for all of these?

Real World Use Case: Nearest neighbor search



Real World Use Case: Nearest neighbor search

(255, 0, 0) (255, 255, 0) (0, 255, 0) (0, 255, 255) (0,0,255) (255,0,255)



```
[[ 45 218   0],[223 147 243],[116  57 223],[187   9   9],[238 208 236]]
[[216 190  15],[193  64  80],[184  35 215],[ 95 152 180],[128  36  41]]
[[101 128  53],[224 122 191],[237 212  74],[ 35  98 227],[214  66 167]]
[[188   3 211],[217 142  33],[210 229 167],[208  57  22],[  3 213 235]]
[[ 11 172  37],[225 191  57],[184 130  34],[136  33  51],[ 26 220  67]]
```

Real World Use Case: Nearest neighbor search

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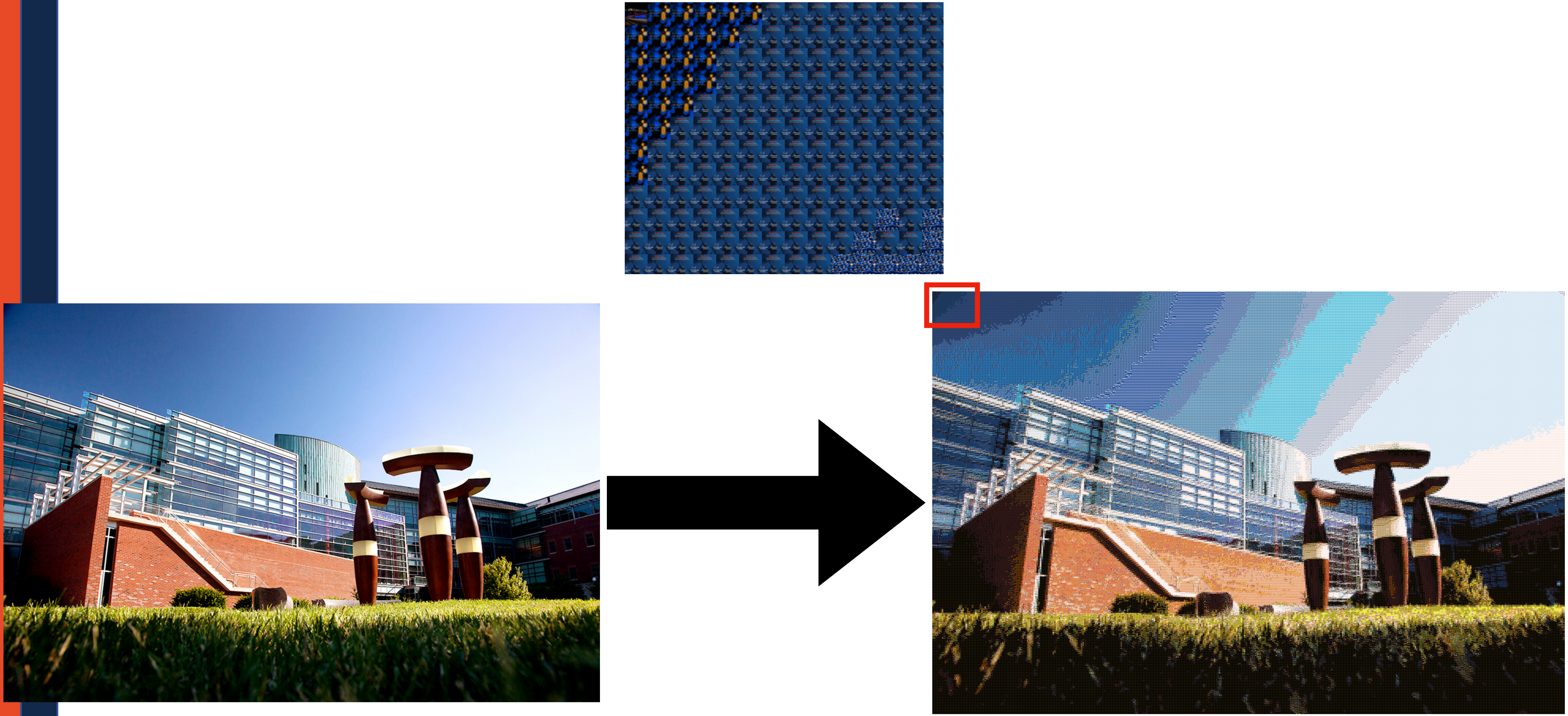

Real World Use Case: Nearest neighbor search

Given an input image, how can we find the closest match from a collection collection of other images?