Algorithms and Data Structures for Data Science

Trees

CS 277
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February 26, 2024

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Computer Science

Exam 1 next week

Multiple Choice / Fill in the blank exam

Covers content through Monday February 19th

See website for details

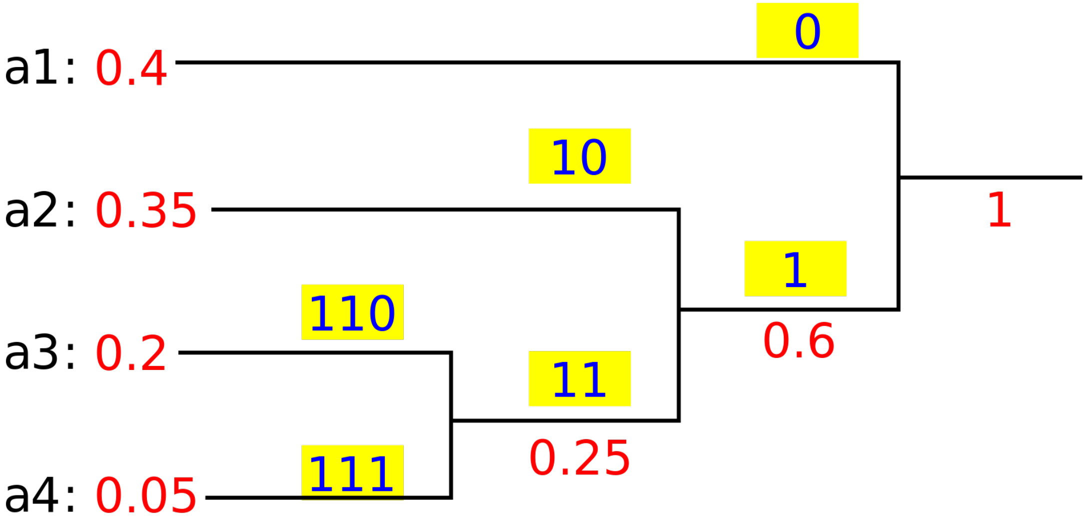
Learning Objectives

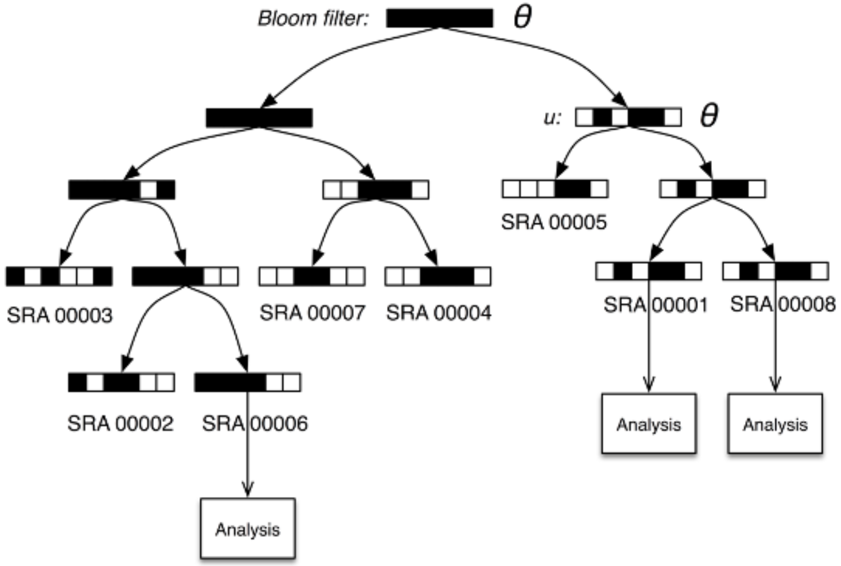
Build an understanding of the tree ADT

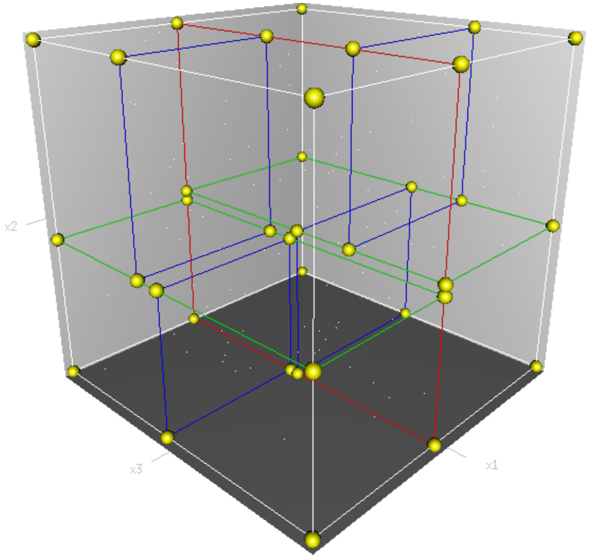
See the implementation details of a binary tree

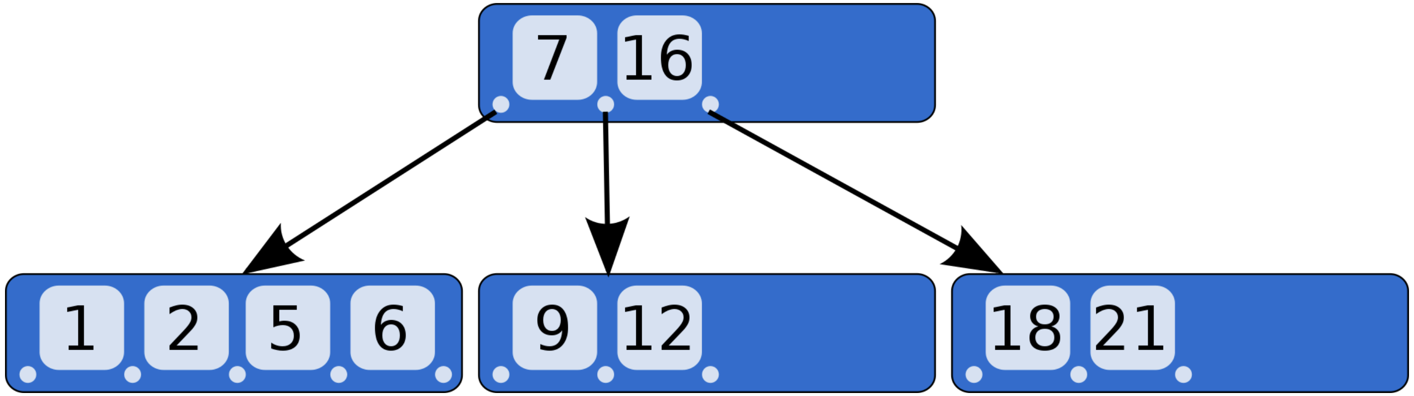
Practice recursion in the context of trees

There are many *types* of trees









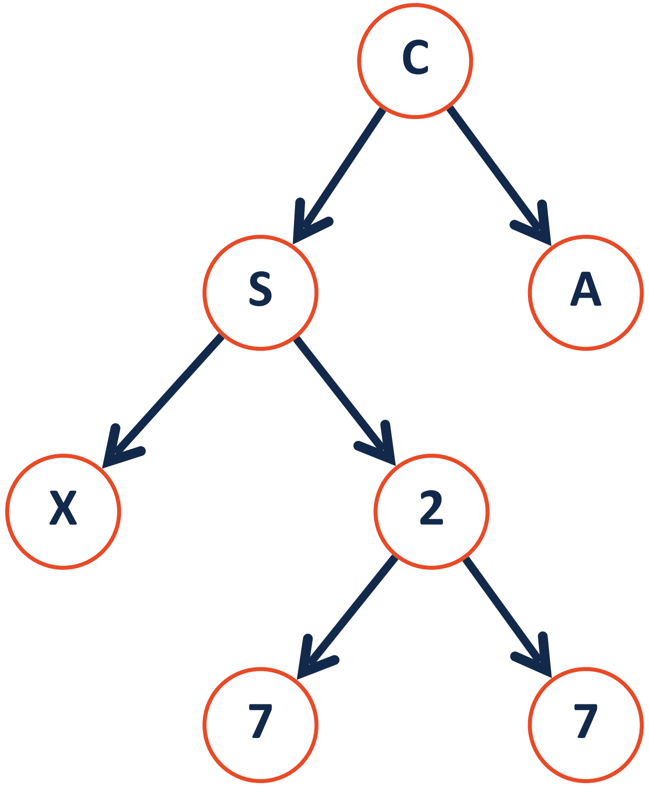
(Binary) Tree Recursion

A **binary tree** is a tree T such that:

$$T = None$$

or

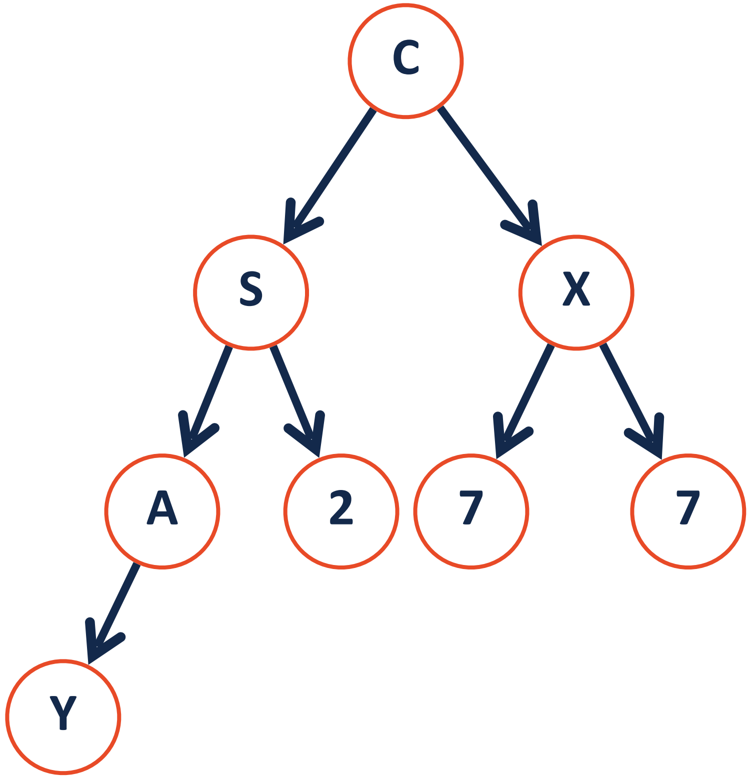
$$T = treeNode(val, T_L, T_R)$$

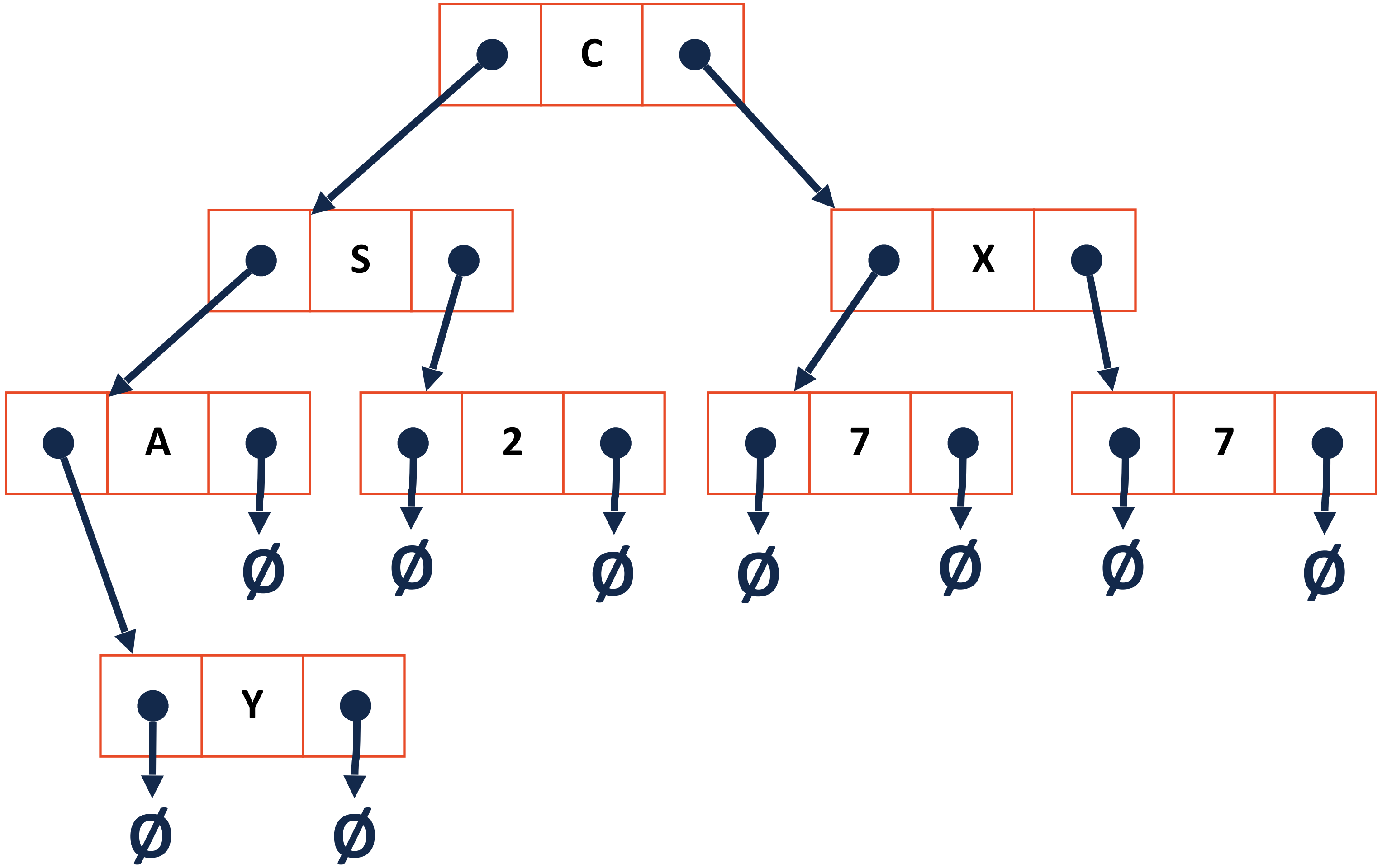



```
class treeNode:
    def __init__(self, val, left=None, right=None):
        self.val = val
        self.left = left
        self.right = right
```

```
class binaryTree:
    def __init__(self):
        self.root = None
```

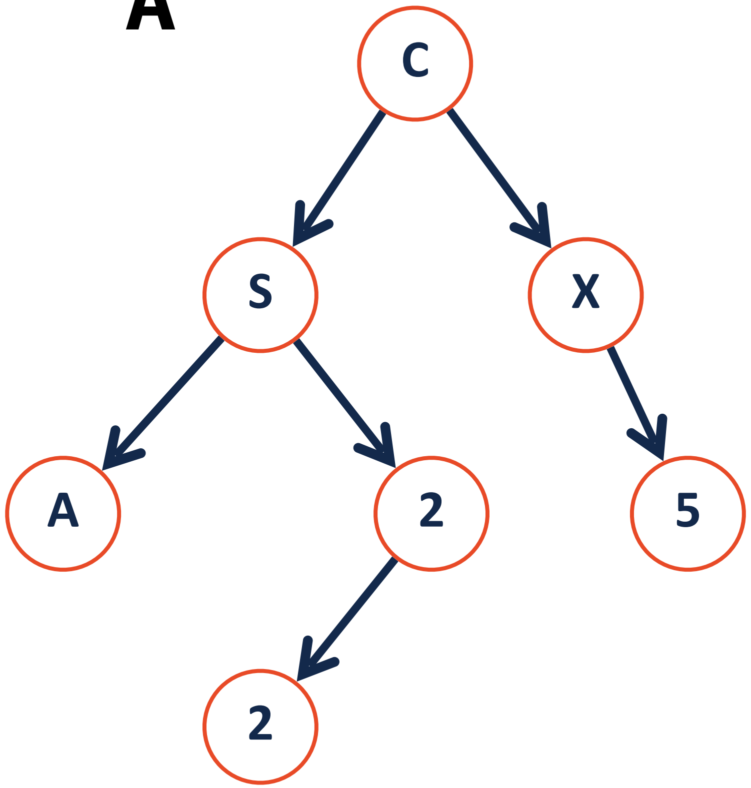
Visualizing trees



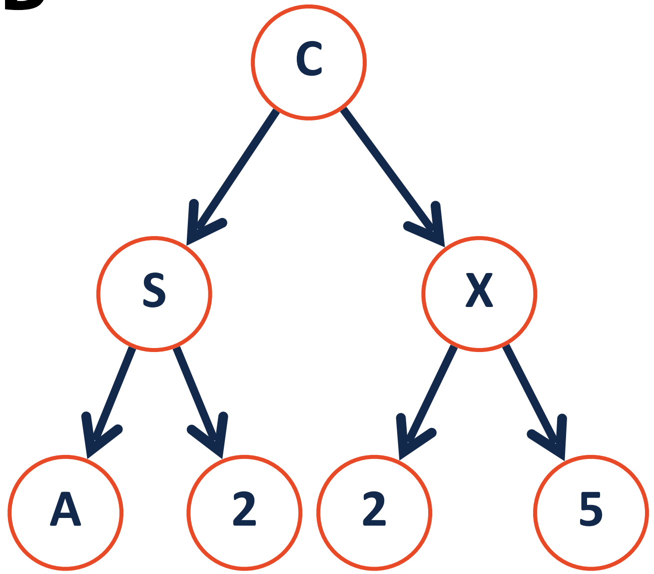


Which of the following are binary trees?

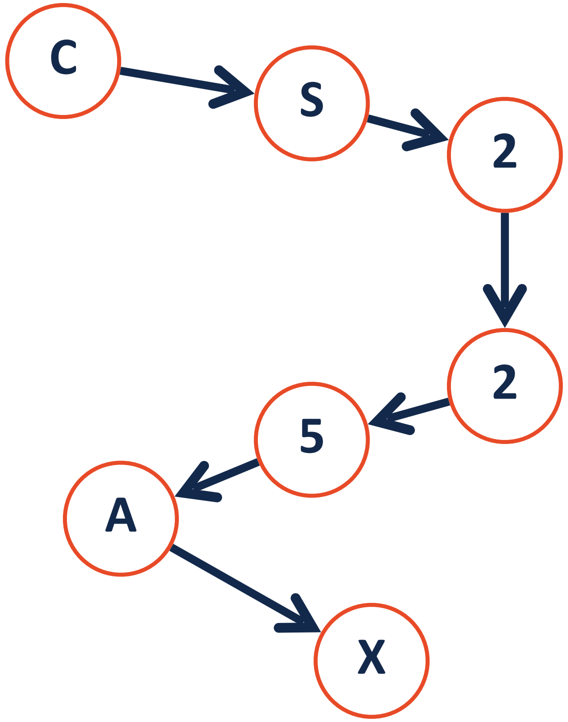
A



B



C



Tree ADT

Tree ADT

Constructor: Build a new (empty) tree

Insert: Add an object into tree

Remove: Remove a specific object from tree

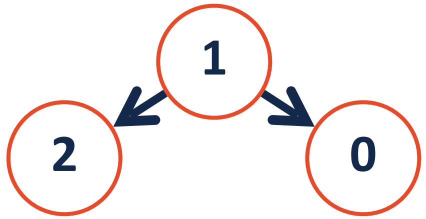
Traverse: Visit every node in tree (all objects)

Search: Find a specific object in the tree

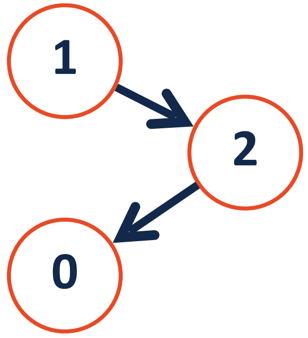
Recursion Practice: build_random_tree()

```
def build_random_tree(size, seed=None):
    random.seed(seed)
    keys = list(range(size))
    random.shuffle(keys)

    root = random_tree_helper(keys)
    return root
```

Ex: build_random_tree(3, 1)



Ex: build_random_tree(3, 1001)

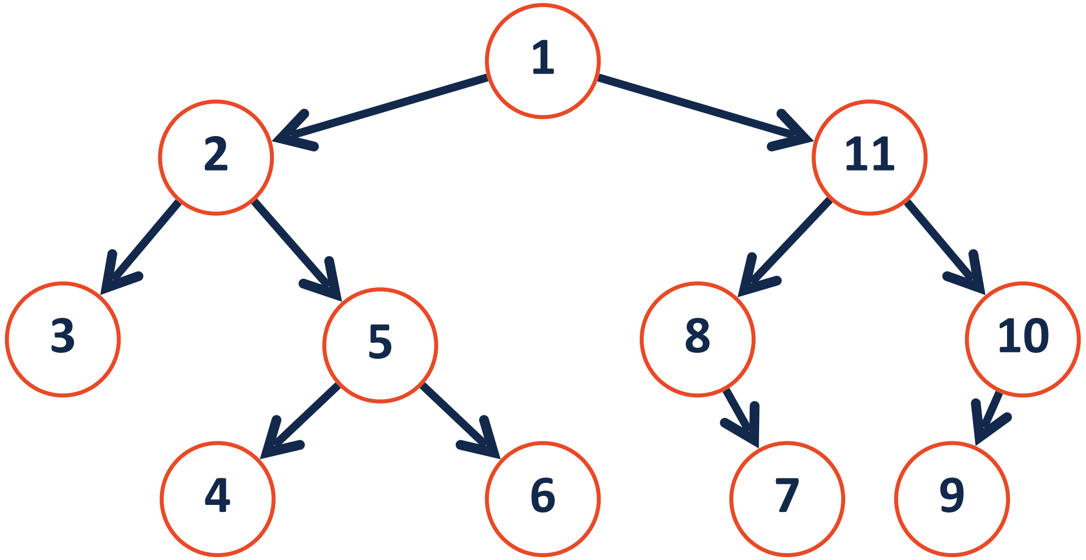



```
def random_tree_helper(keyList):
    # Base Case
    if len(keyList) == 0:
        return None
    if len(keyList) == 1:
        return treeNode(keyList[0])

    # Reduction Step
    node = treeNode(keyList.pop(0))

    # Combining Step
    partition = random.randint(0, len(keyList))
    leftList = keyList[:partition]
    rightList = keyList[partition:]

    node.left = random_tree_helper(leftList)
    node.right = random_tree_helper(rightList)

    return node
```

Binary Tree Insert

If I want to insert a value into my tree, what information do I need?

Ex: I want to insert the value ’13’.



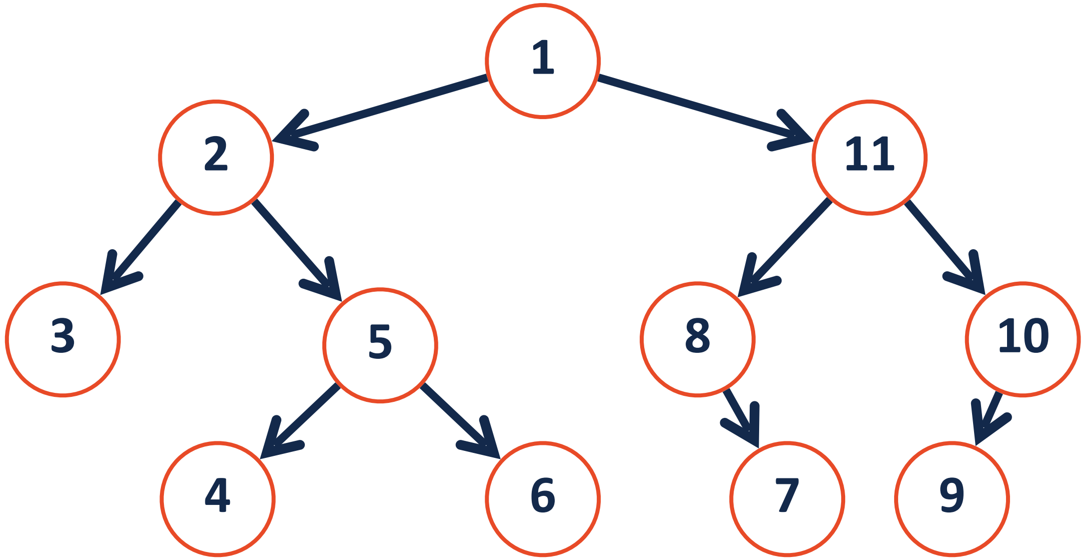
Binary Tree Insert

Different implementations will have very different insert strategies!

In our case, we need to know the following:

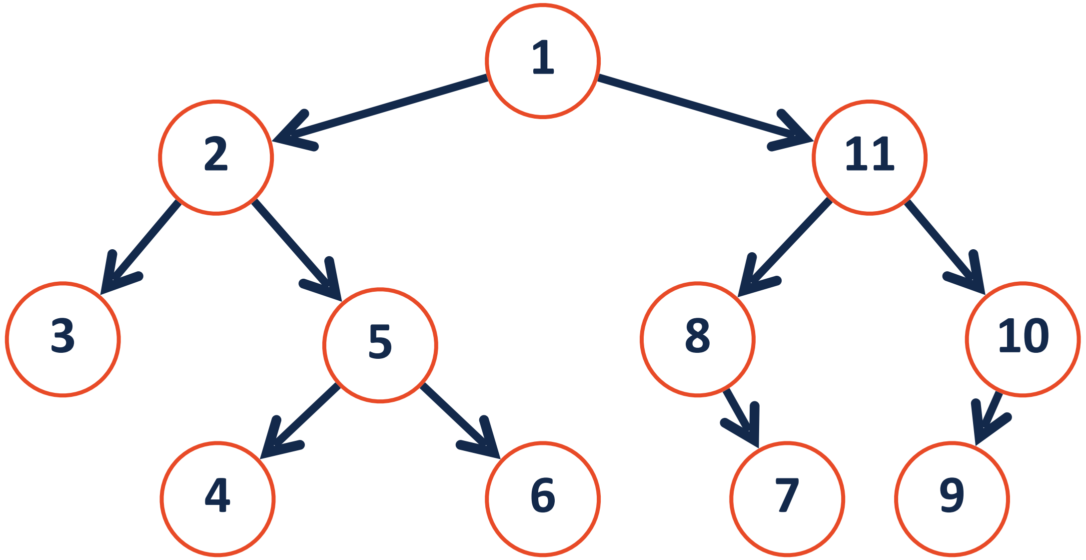
1. The exact insert location

2. The value we want to insert



Binary Tree Insert

Choice: What happens if a node already exists at our target location?



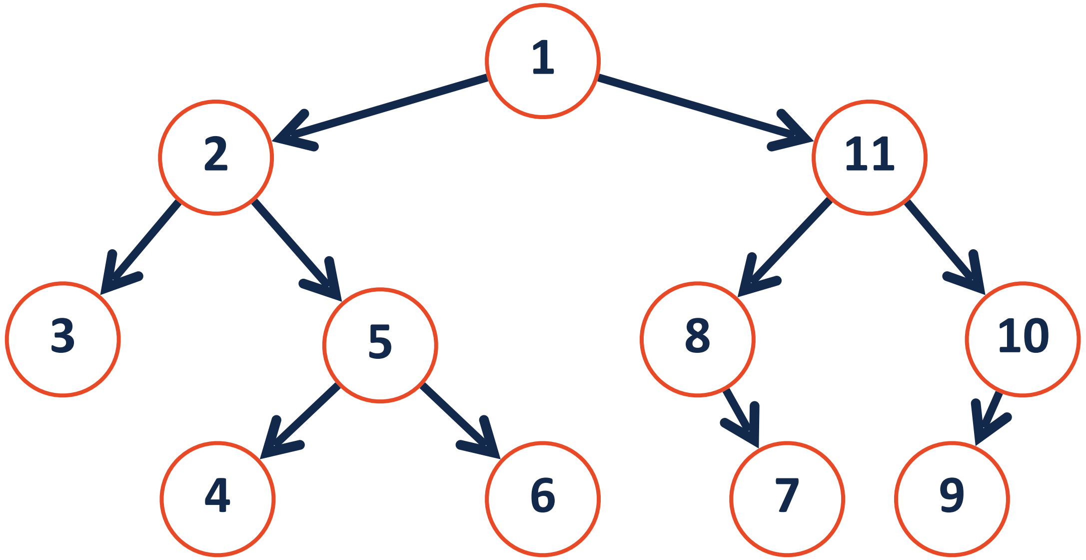
Lets code up our choice! What is the Big O?

Binary Tree Insert Big O

Binary Tree insert is similar to linked list insert.

If we are given the *previous* node (here, the parent node), its O(1).

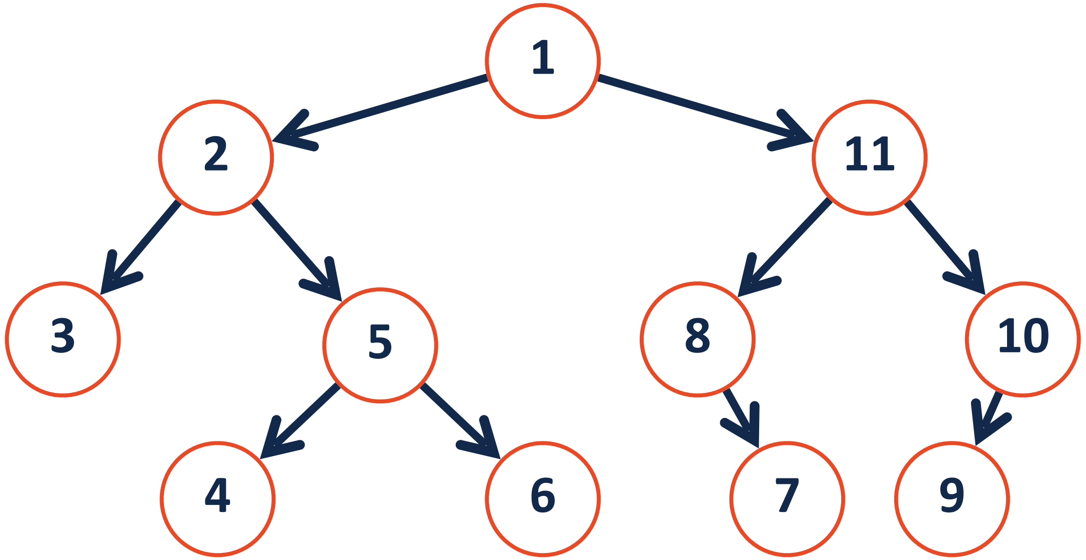
But the act of *finding* a node by value is more complicated (traversal)



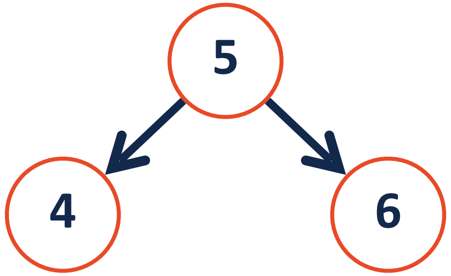
Binary Tree Remove

Removing a tree from a binary tree looks deceptively simple…

Ex: I want to remove the value ’4’.



Binary Tree Remove



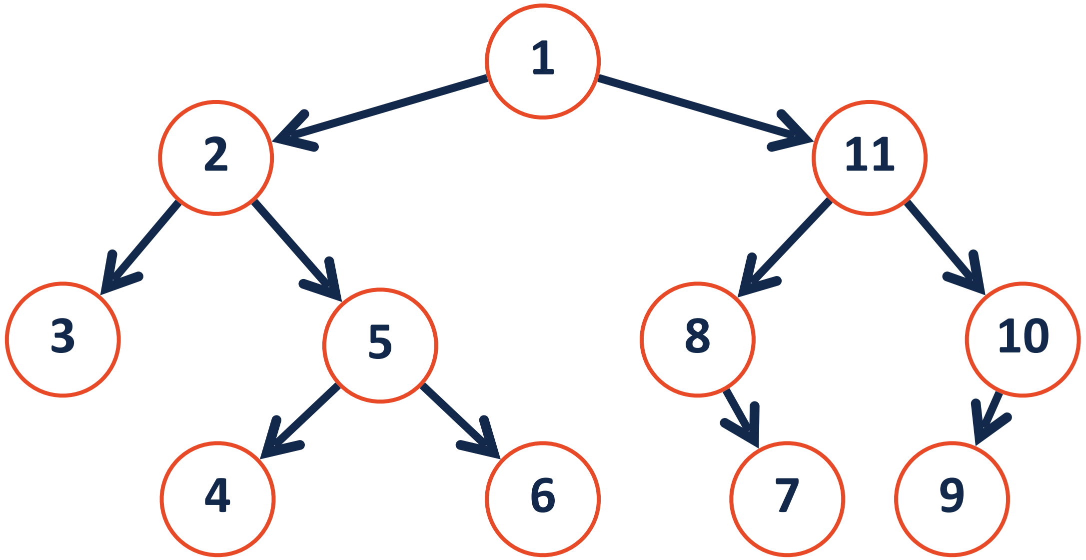
Choice: How do we adjust our tree given a removed node?

If the node being removed has 0 children:

Binary Tree Remove

When we remove, we have to be careful not to delete a tree branch!

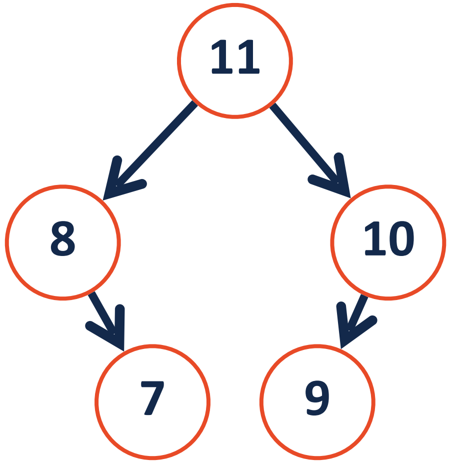
Ex: I want to remove the value '8'.



Binary Tree Remove

Choice: How do we adjust our tree given a removed node?

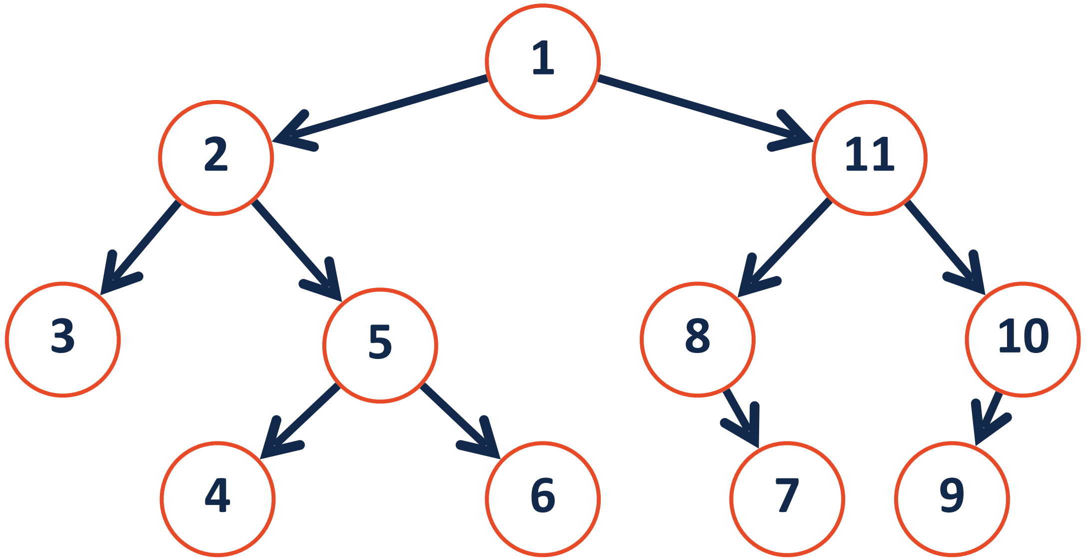
If the node being removed has 1 child:



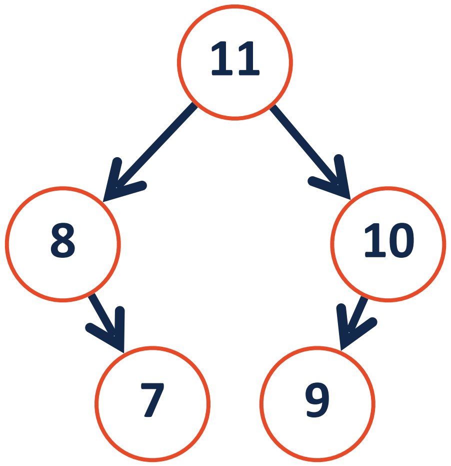
Binary Tree Remove

When we remove, we have to be careful not to delete a tree branch!

Ex: I want to remove the value ’11’.



Binary Tree Remove



Choice: How do we adjust our tree given a removed node?

If the node being removed has 2 children:

Binary Tree Remove Big O

What is the Big O of our removal algorithm on a binary tree?

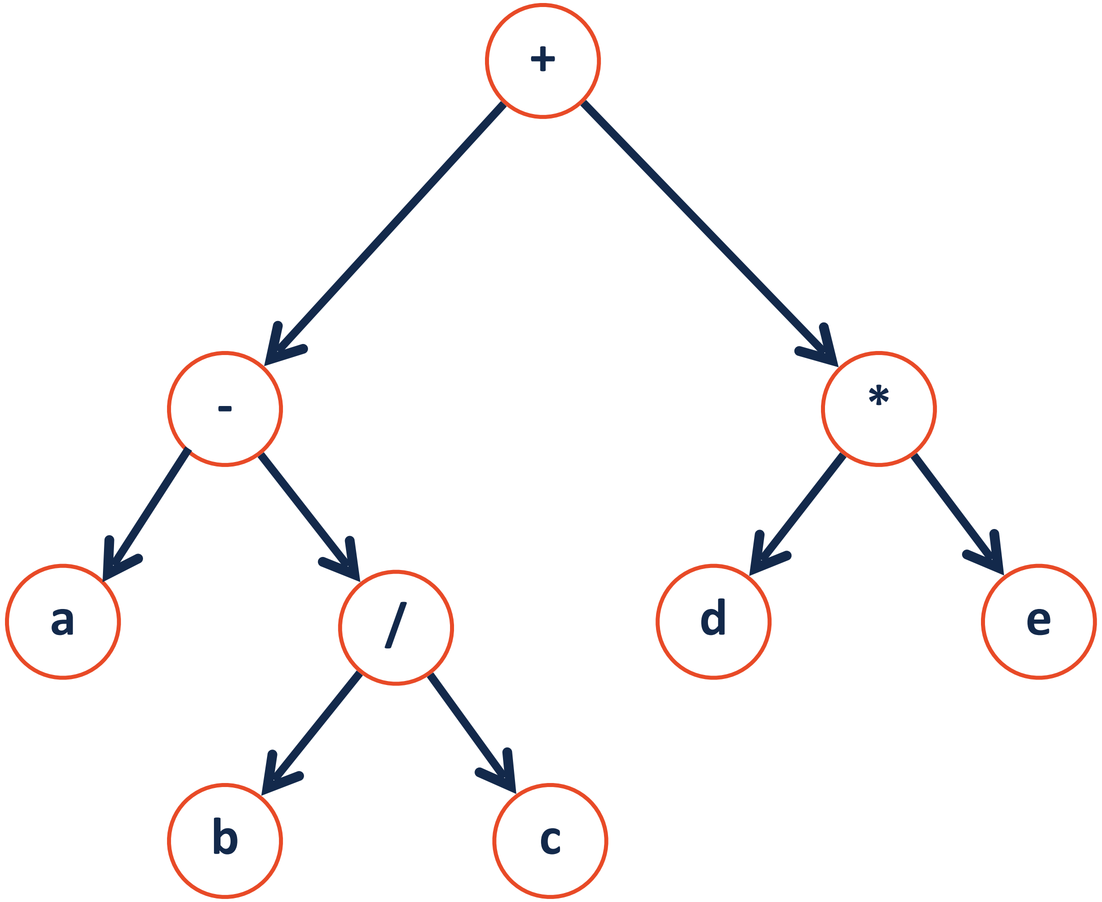
0 child:

1 child:

2 child:

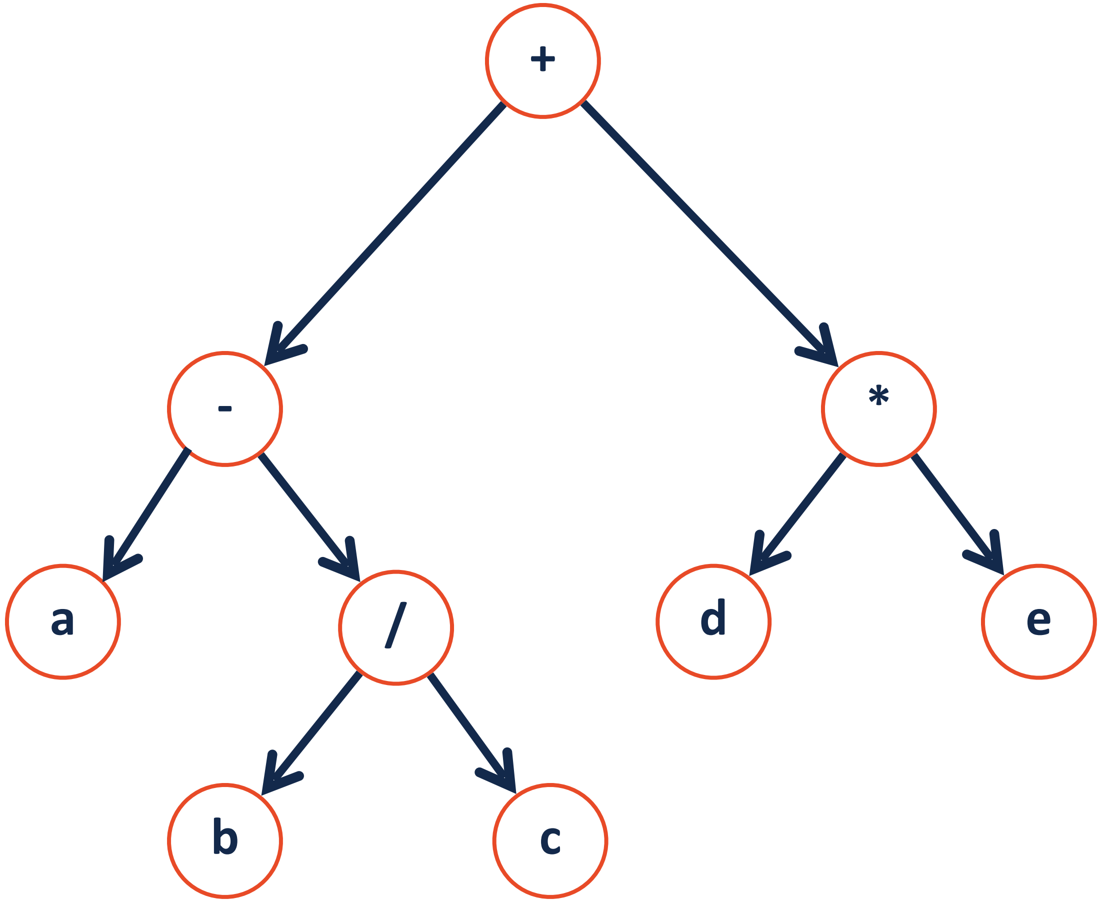
Tree Traversal

A **traversal** of a tree T is an ordered way of visiting every node once.

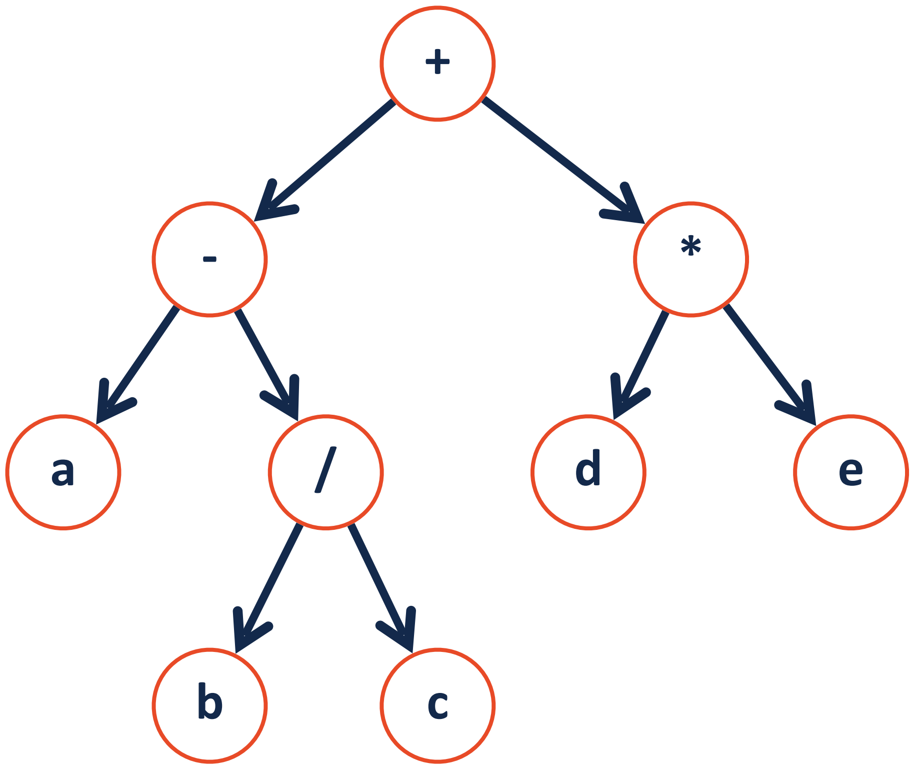


Tree Traversal

A **traversal** of a tree T is an ordered way of visiting every node once.



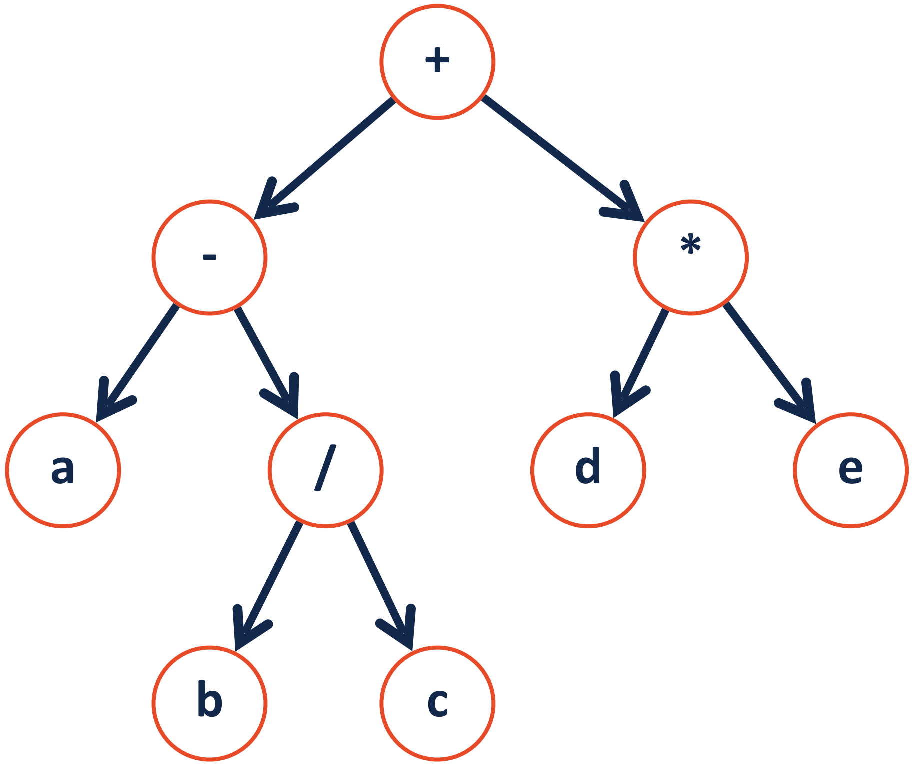
Pre-order Traversal




```
def preorderTraversal(node):
    if node:
        print(node.val)
        preorderTraversal(node.left)
        preorderTraversal(node.right)
```

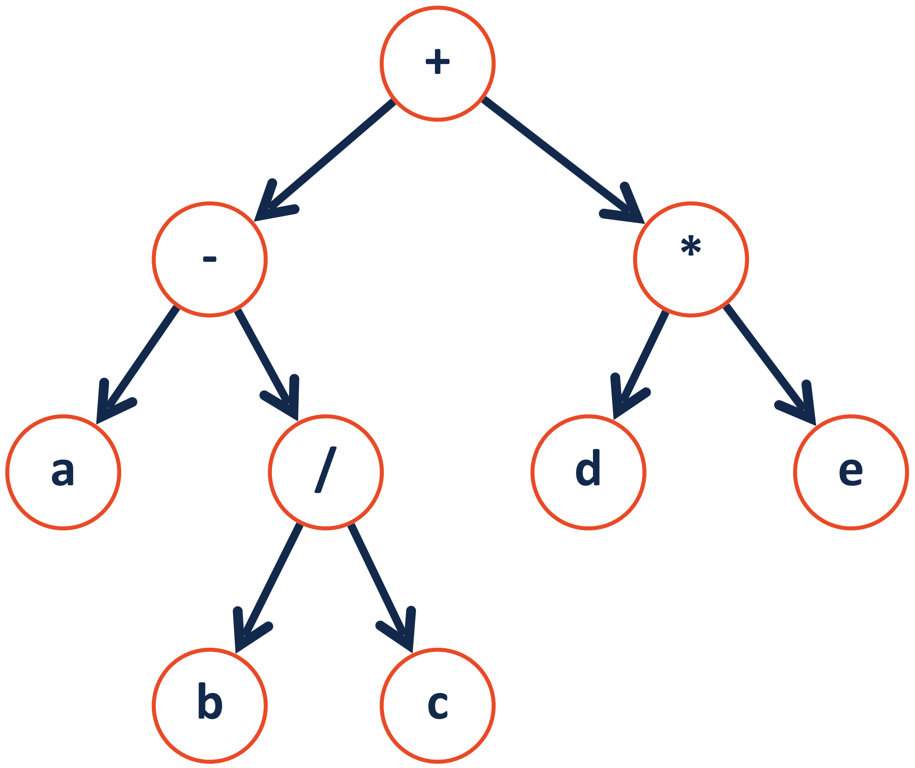
Pre-order:

In-order Traversal



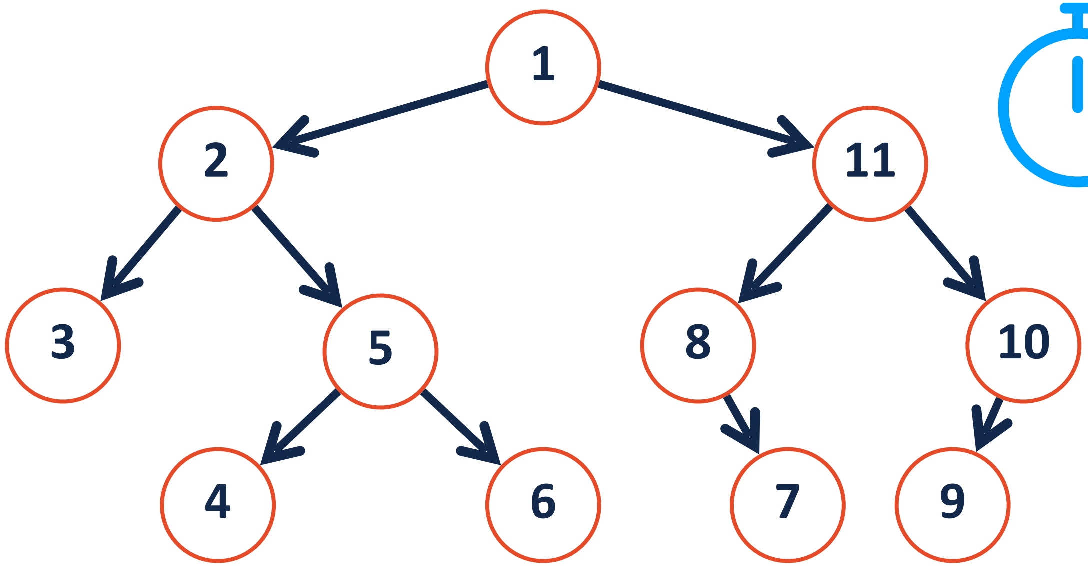
In-order:

Post-order Traversal



Post-order:

Tree Traversals



Lets practice our traversals!

Pre-order:

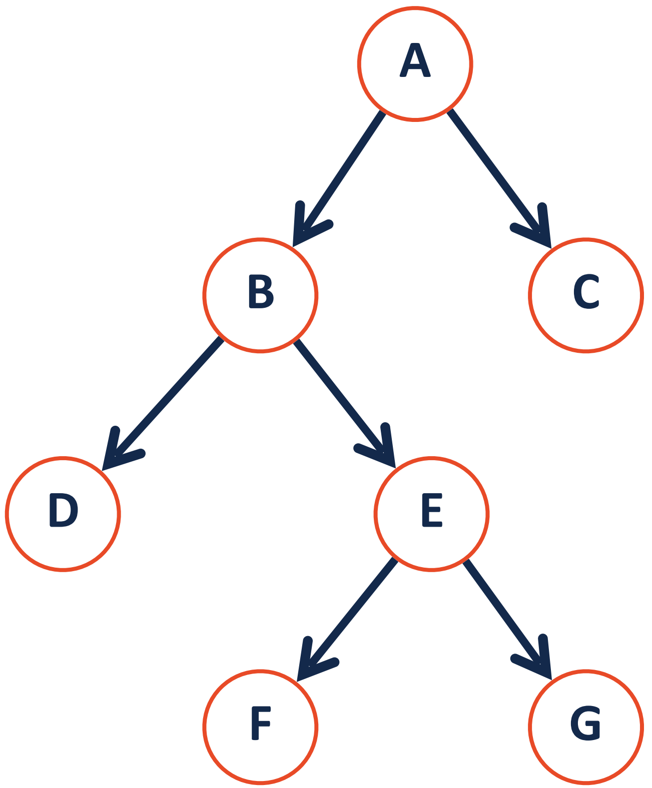
In-order:

Post-order:

Traversal vs Search

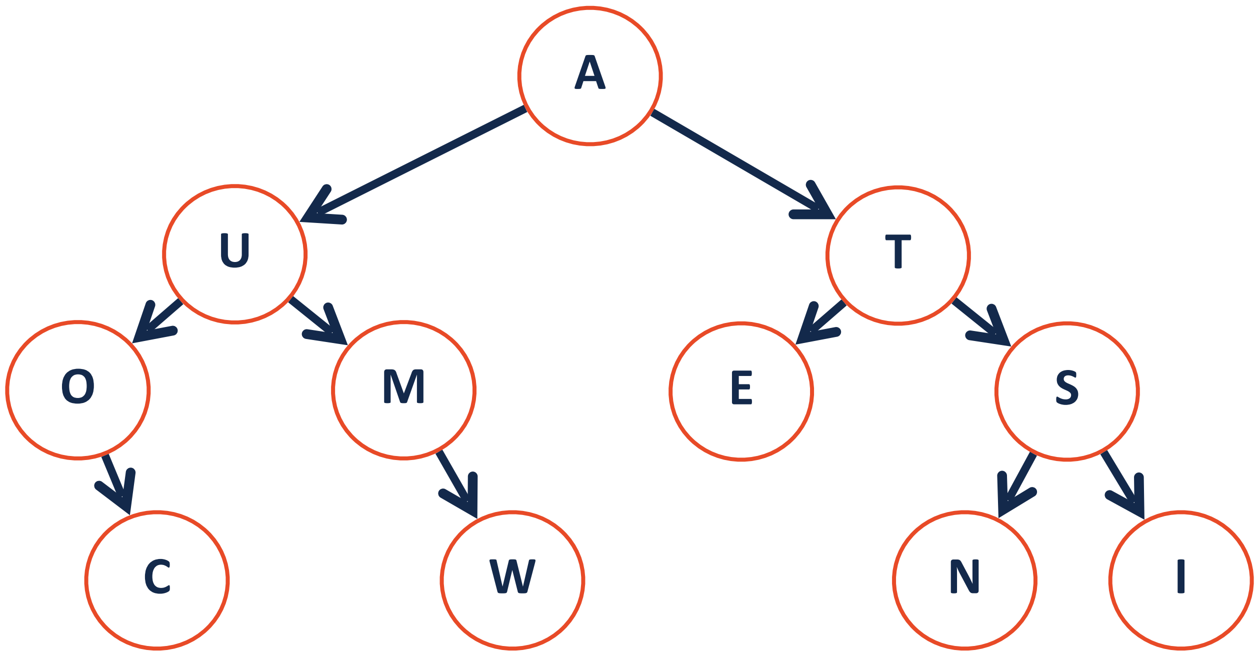
Traversal

Search



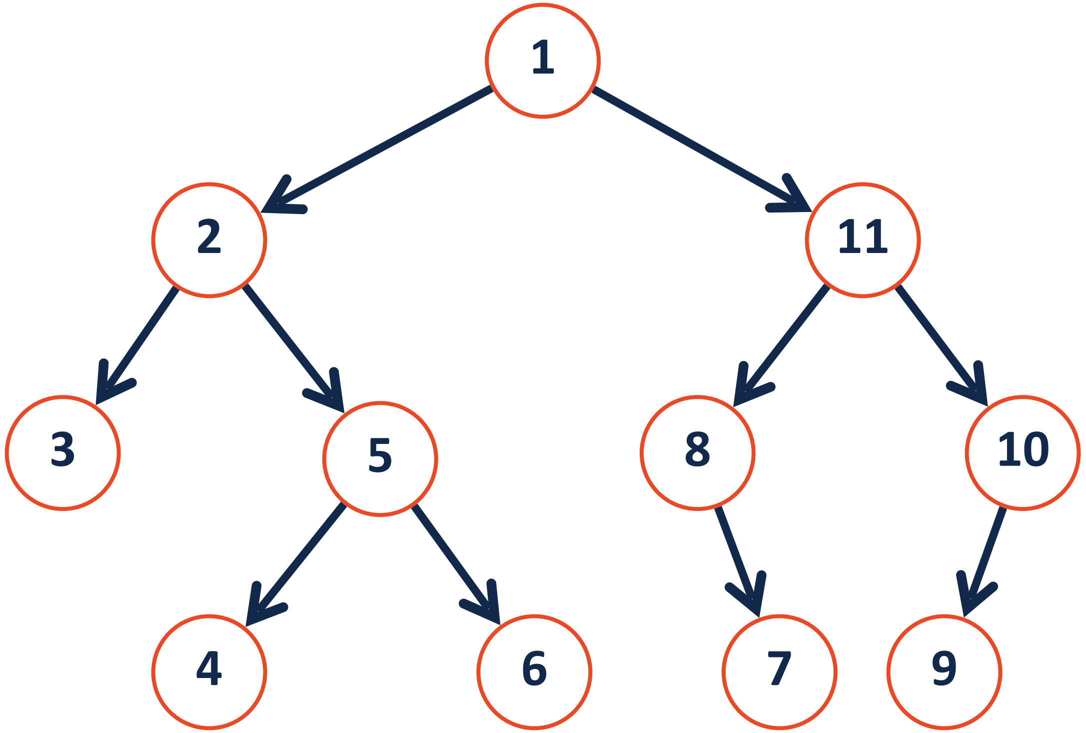
Searching a Binary Tree

There are two main approaches to searching a binary tree:



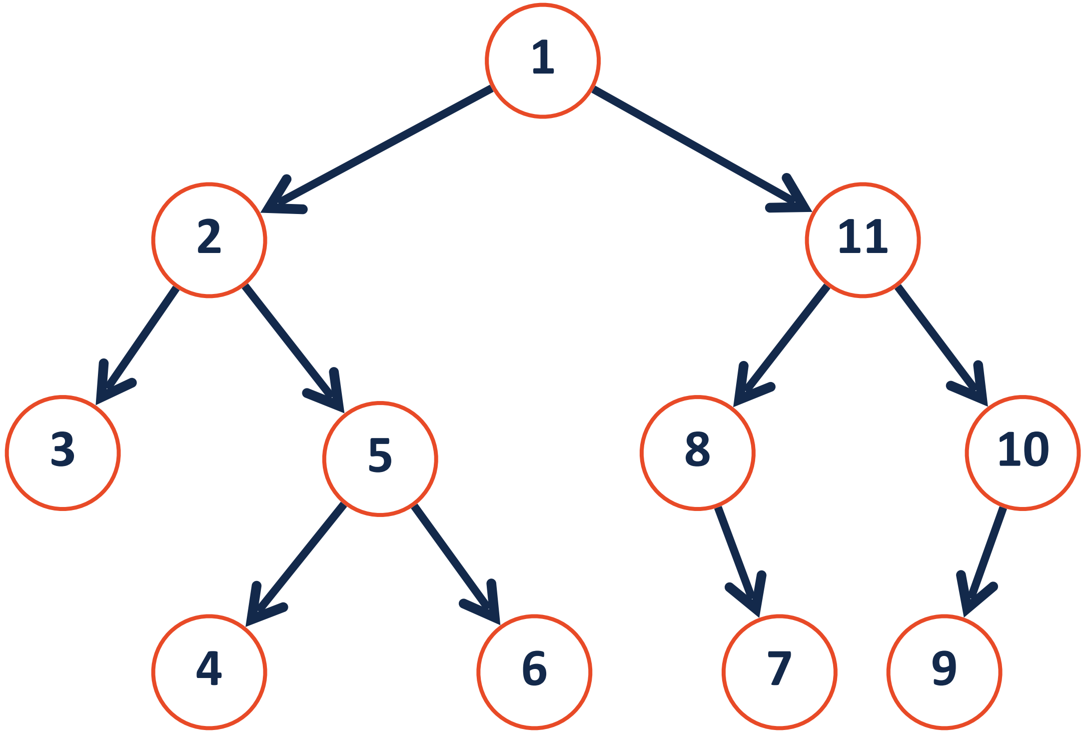
Depth First Search

Explore as far along one path as possible before backtracking



Breadth First Search

Fully explore depth i before exploring depth i+1



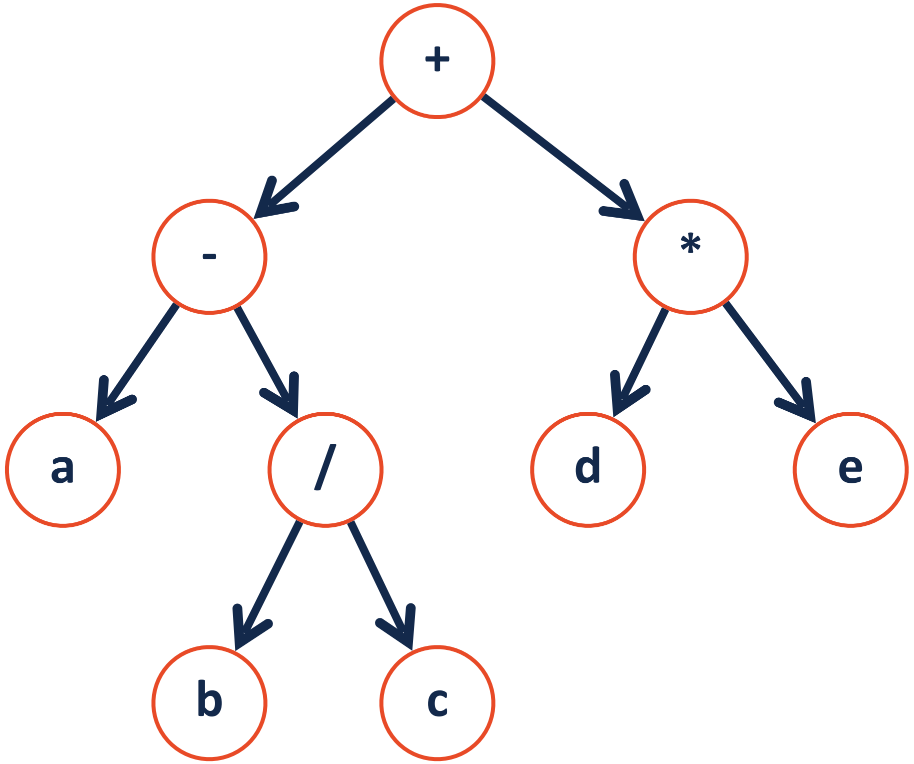
Traversal vs Search II

Pre-order, in-order, and post-order are three ways of doing which search?

Pre-order: + - a / b c * d e

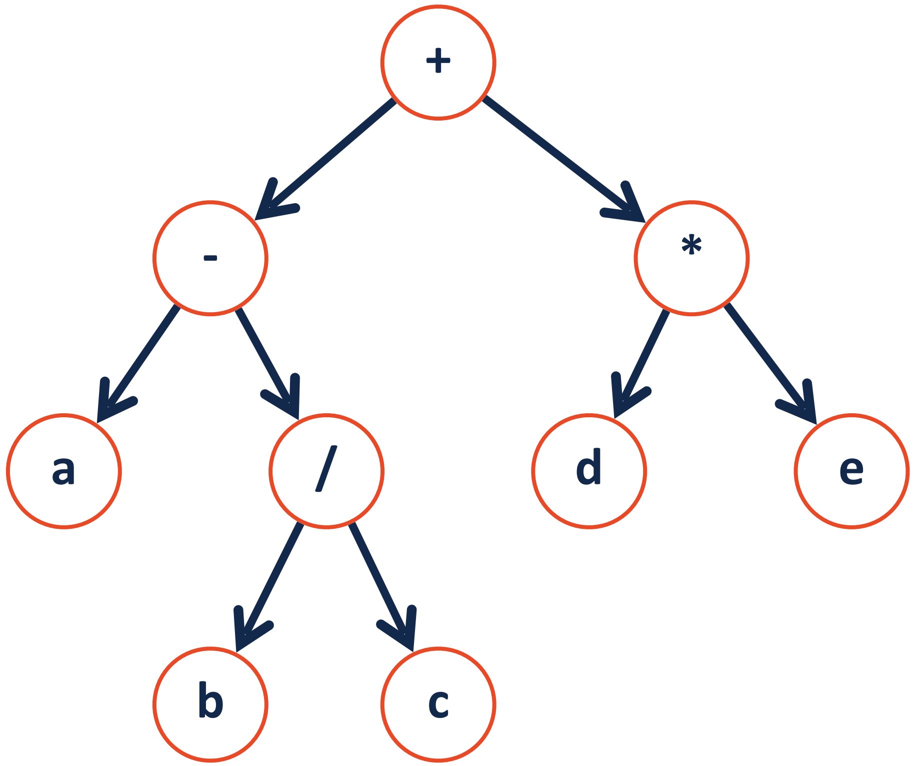
In-order: a - b / c + d * e

Post-order: a b c / - d e * +



Level-Order Traversal

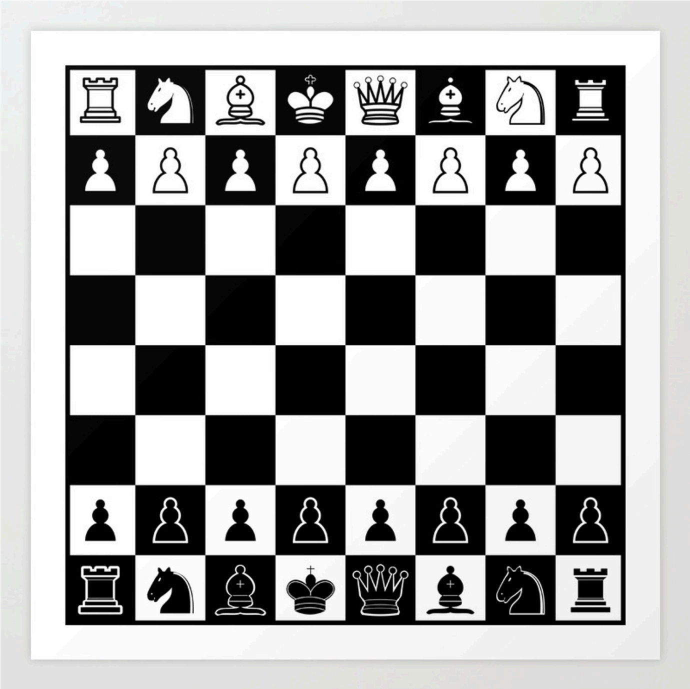
A tricky recursive implementation but an easier queue implementation!



Level-order:

What search algorithm is best?

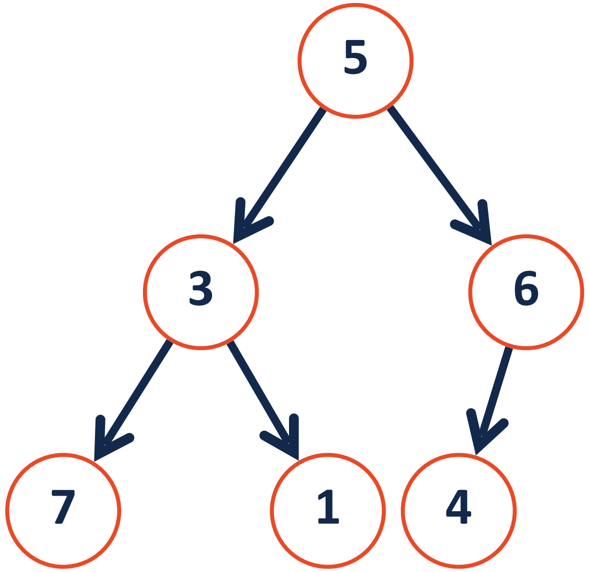
The average 'branch factor' for a game of chess is ~31. If you were searching a decision tree for chess, which search algorithm would you use?

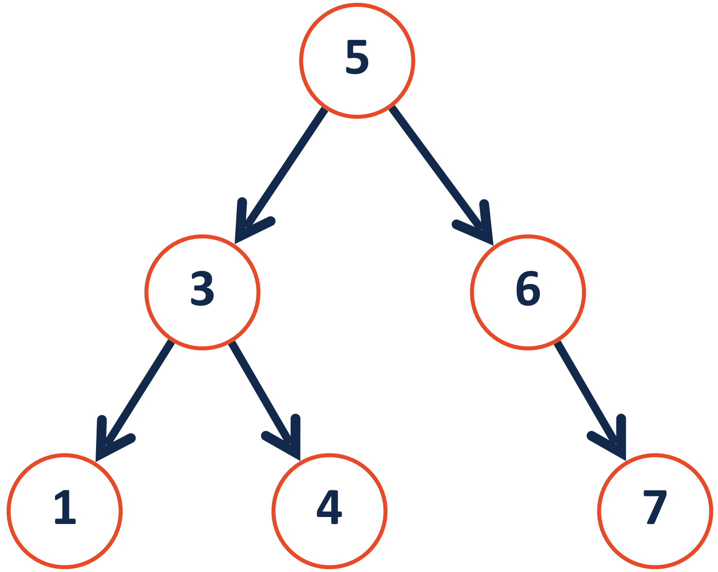


Improved search on a binary tree

5	3	6	7	1	4

1	3	4	5	6	7





Binary Search Tree (BST)

A **BST** is a binary tree $T = treeNode(val, T_L, T_r)$ such that:

$$\forall n \in T_L,\ n.val < T.val$$

$$\forall n \in T_R,\ n.val > T.val$$