Algorithms and Data Structures for Data Science
Recursion

CS 277
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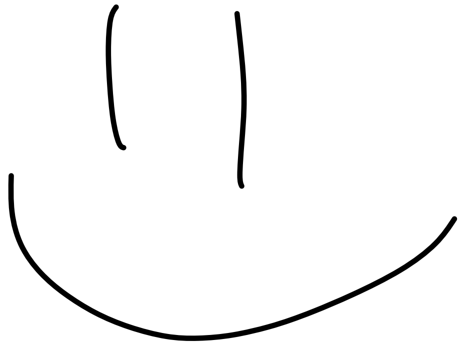
February 21, 2024



UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN
Department of Computer Science

Exam 1 next week

Multiple Choice / Fill in the blank exam

Covers content through Monday February 19th

See website for details

- attend assessments

↳ python fundamentals

↳ Big O

↳ Lists

↳ Stack & queue

Learning Objectives

Introduce recursion in the context of trees

Explore recursion in the context of loops

Practice recursion in the context of lists

Back to trees next week

↳ ADT → Implement (recursive) → Big O

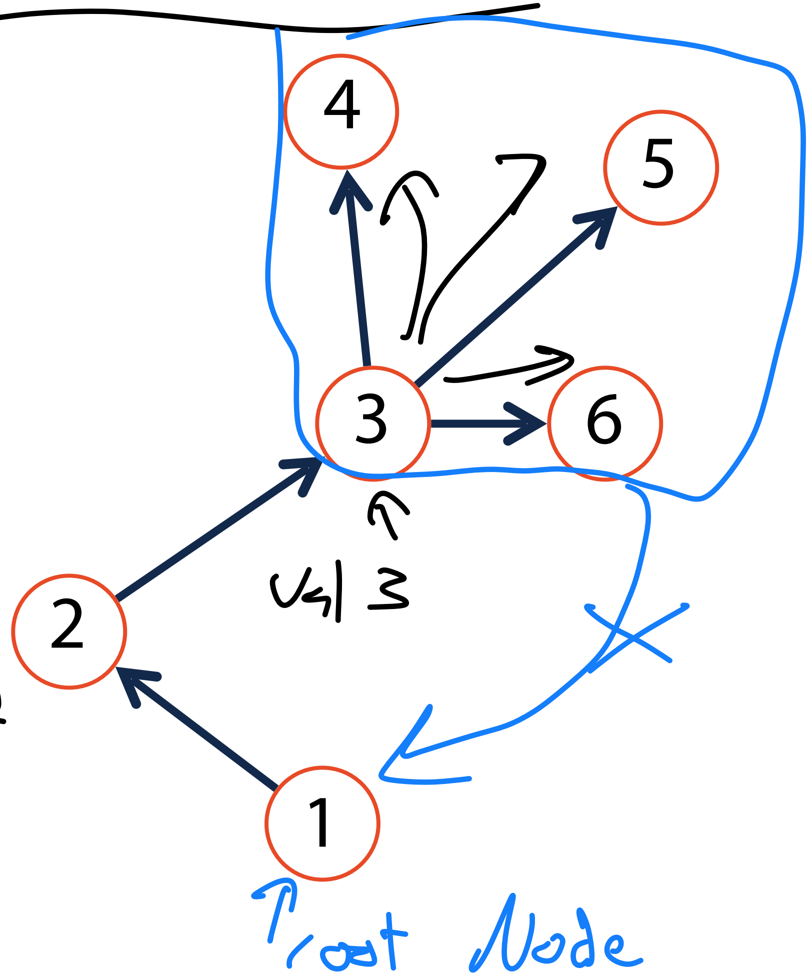
Trees

A non-linear data structure defined recursively as a collection of nodes where each node contains a value and zero or more connected nodes.

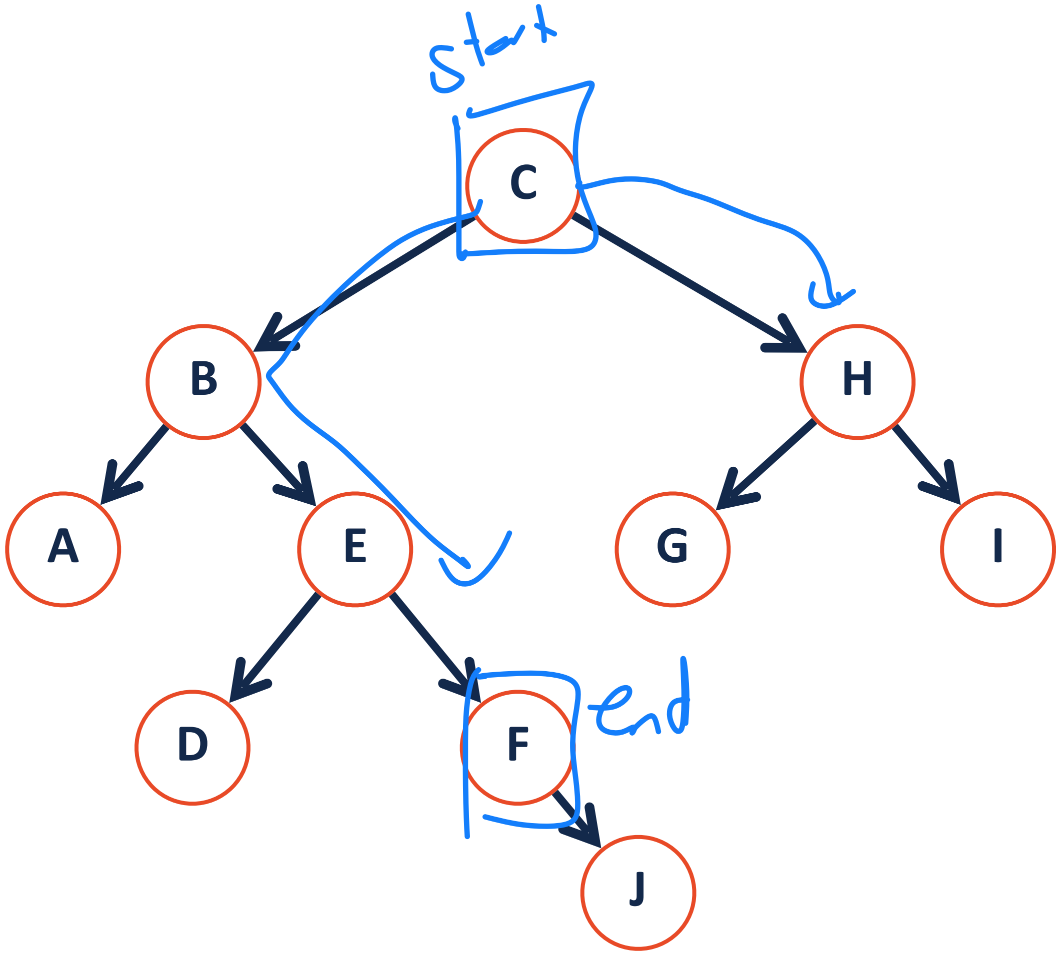
(In CS 277) a tree is also:

1) Acyclic – No cycles in edges
No path from Node to itself

2) Rooted – We have some labeled root Node
every node in tree can be reached by path from root



Tree Terminology

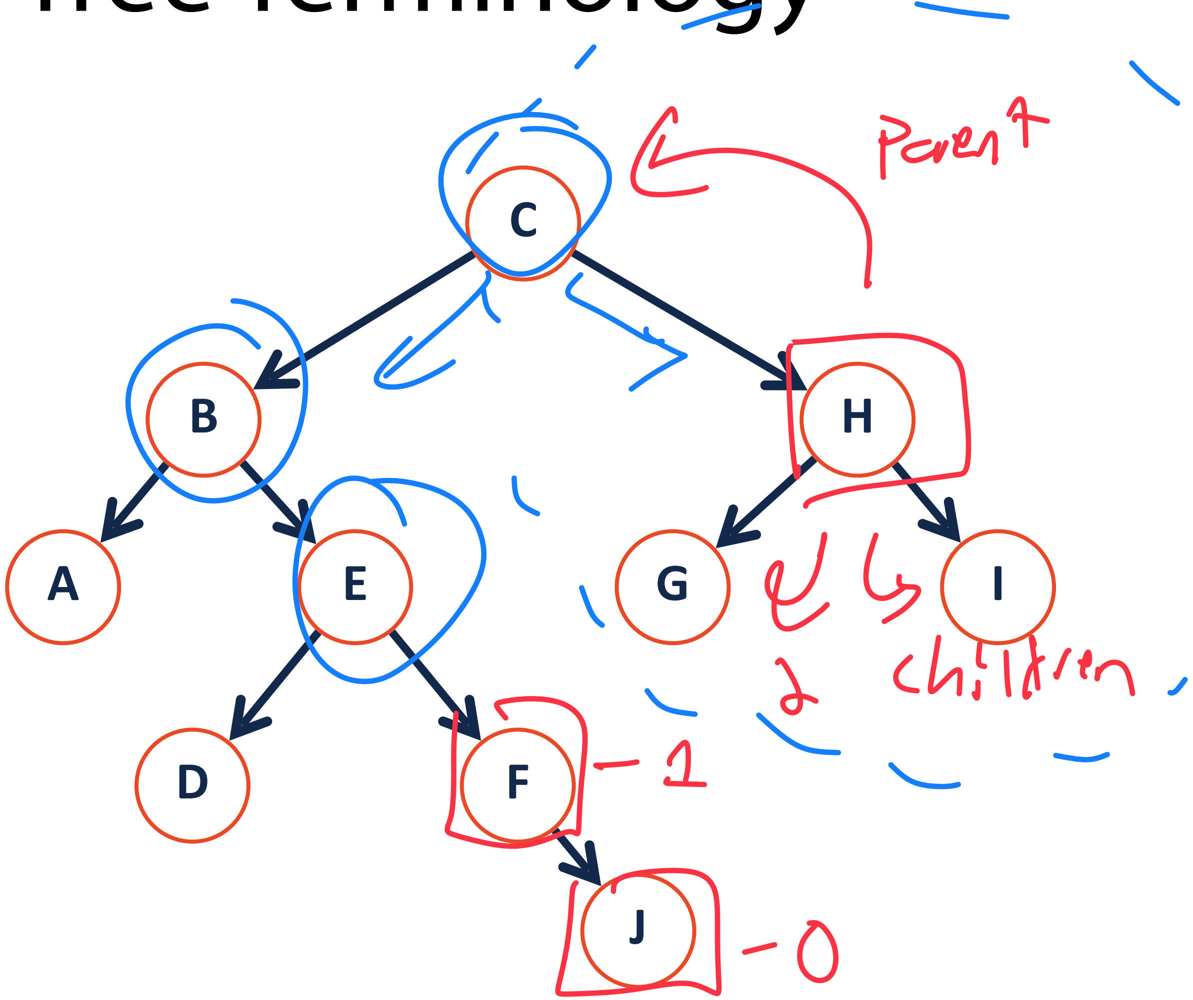


Node: The vertex of a tree

Edge: The [theoretical] connecting path between nodes

Path: A list of the edges (or nodes) traversed to go from node start to node end

Tree Terminology



At most degree 2

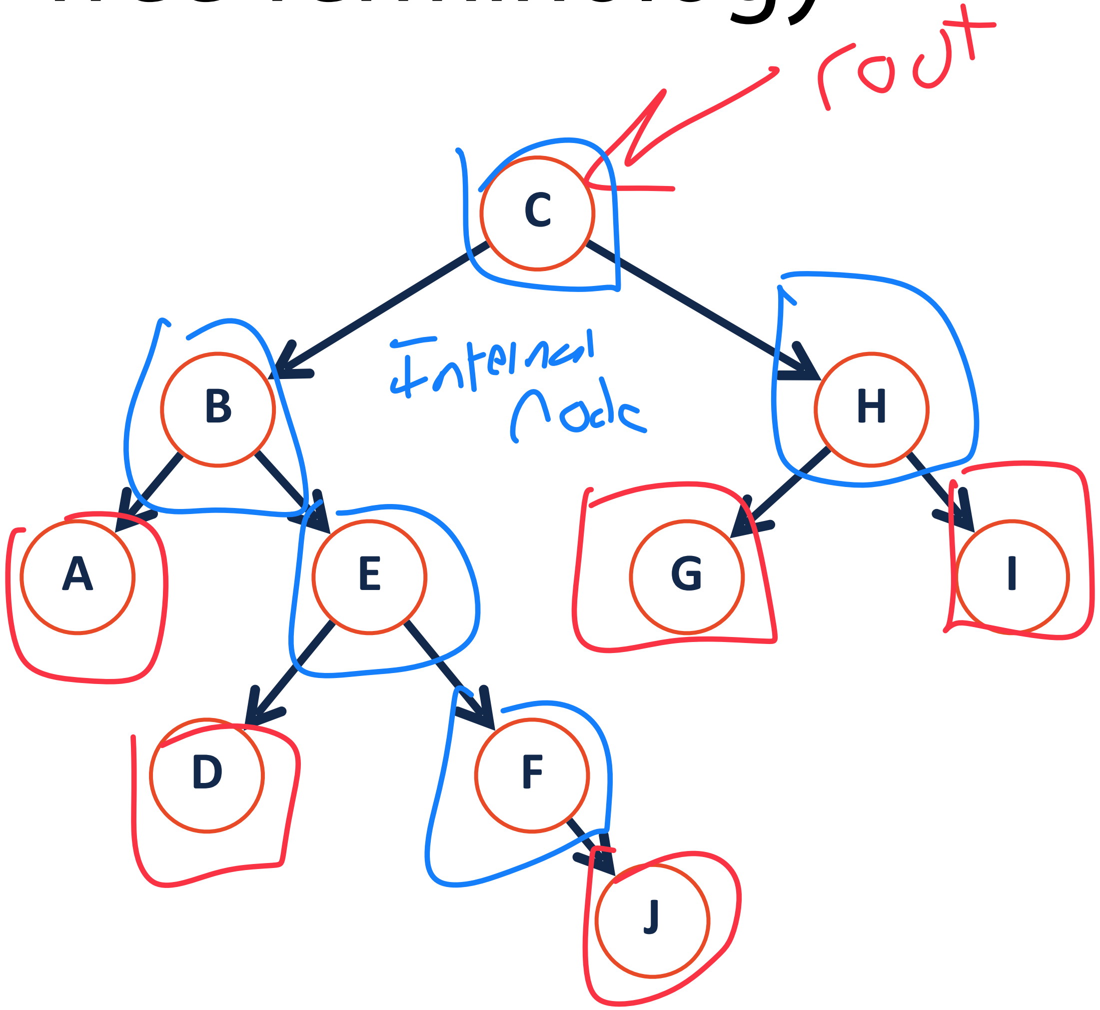
Parent: The precursor node to the current node is the 'parent'

Child: The nodes linked by the current node are it's 'children'

Neighbor: Parent or child

Degree: The number of children for a given node

Tree Terminology



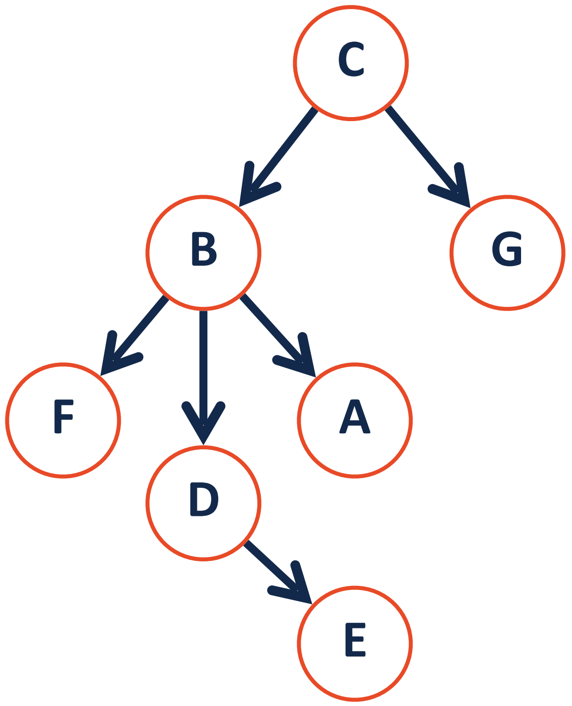
Root: The start of a tree (the only node with no parent).

Leaf: The terminating nodes of a tree (have no children)

↳ degree 0

Internal: A node with at least one child

Tree Terminology Practice



What is the longest path in the tree?

C - B - D - E

What is the neighbors of node B?

C, F, D, A

one parent, multiple children

How many leaves does this tree have?

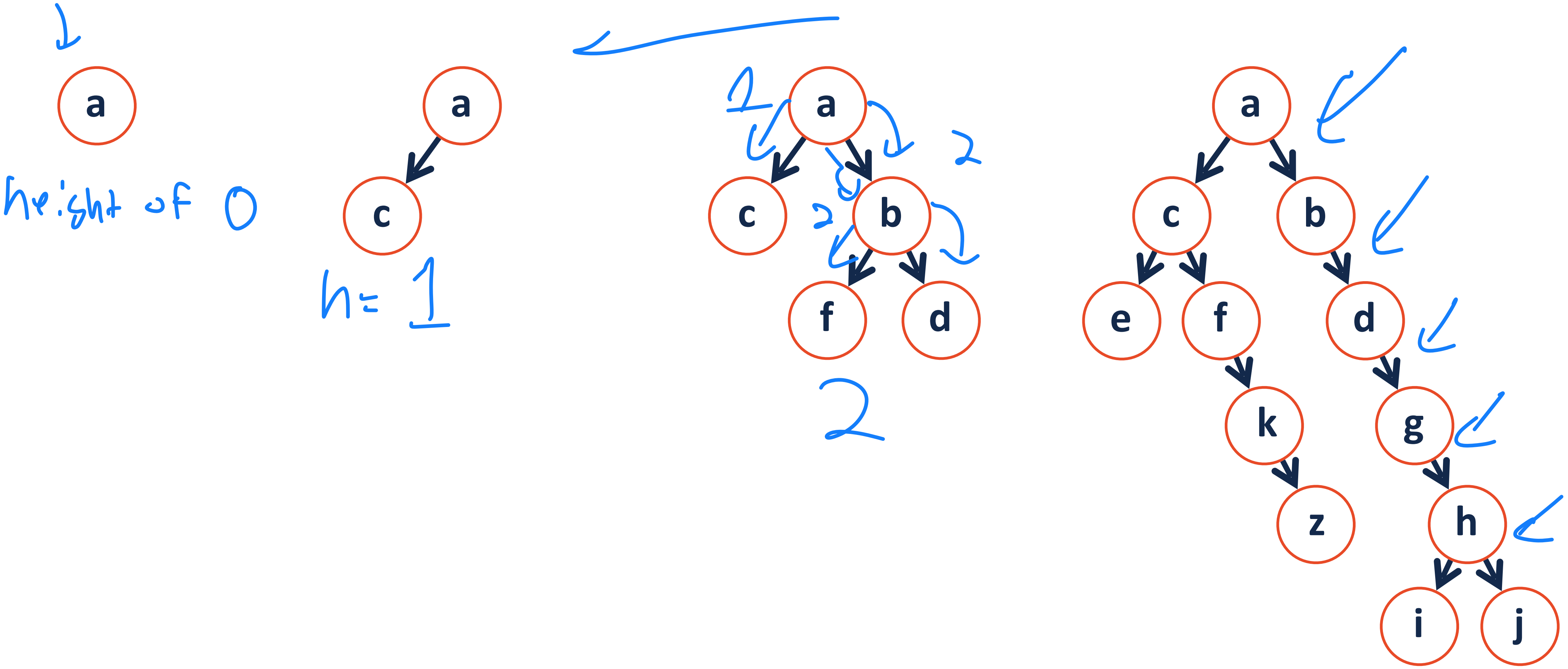
F, E, A, G are leaves (4)

What is the largest degree in the tree?

3 (Node B)

Tree Terminology

Height: the length of the longest path from the root to a leaf



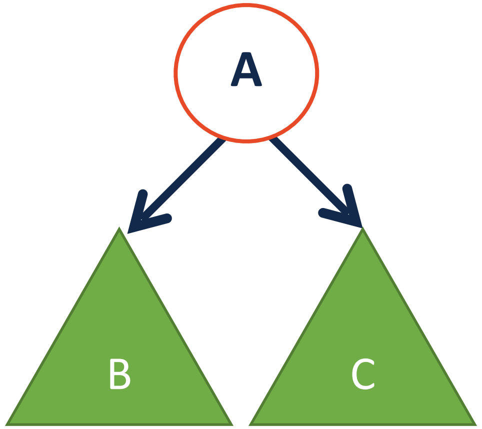
Tree Height Calculation Breakdown

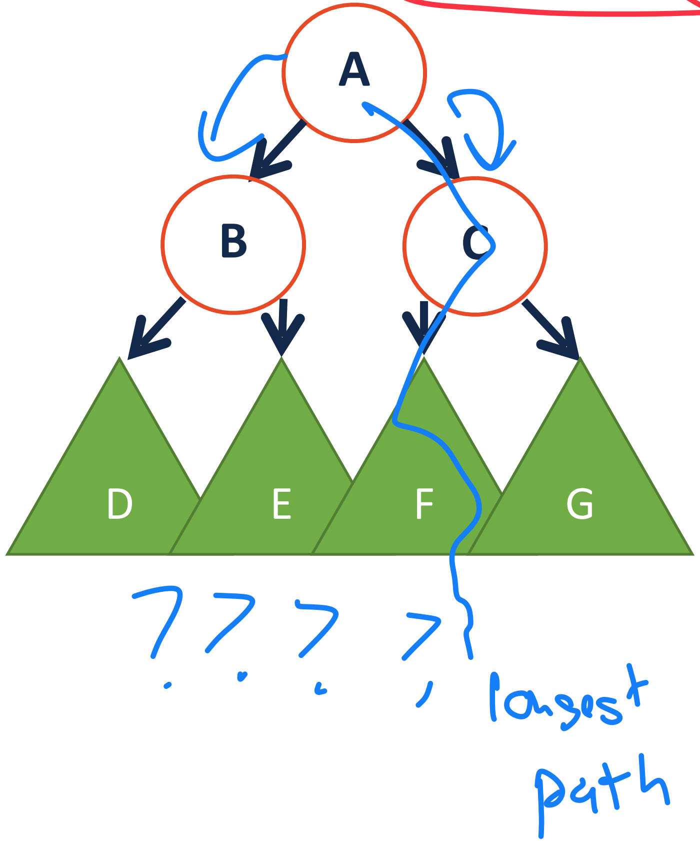
How does a *program* identify the height of a tree?

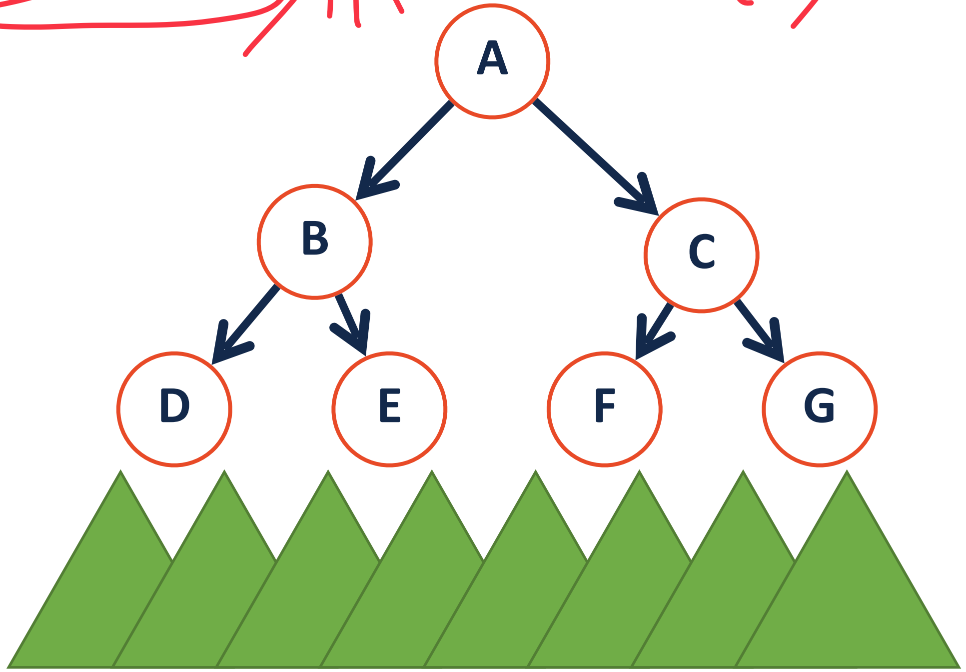
Height (root = A) = 1 + H(B) or H(C)

H(D) & H(E)

H(F) & H(G)







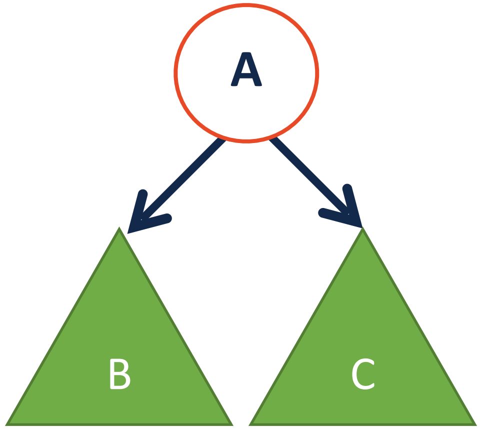
Tree Height Calculation Breakdown

def height(node):

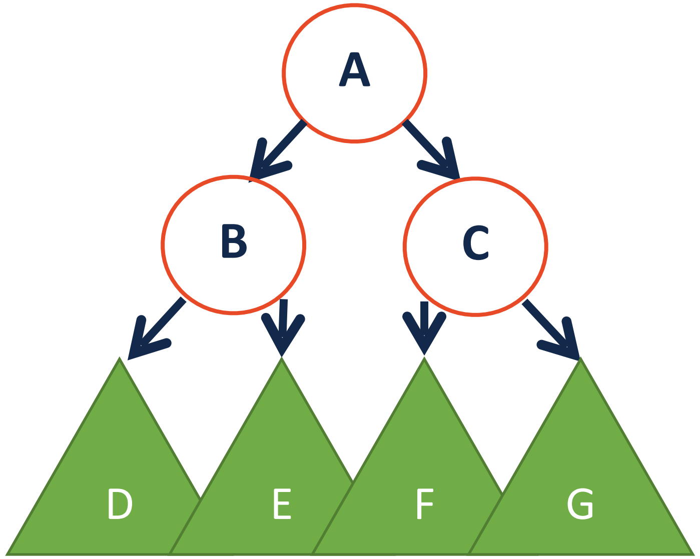
height( )

How does a *program* identify the height of a tree?

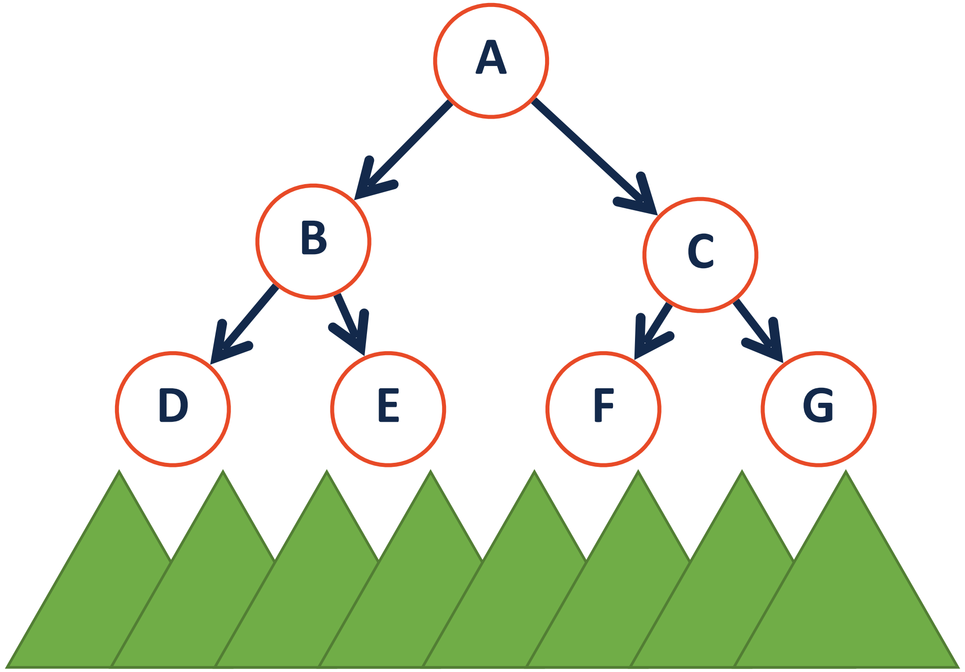
The height of my tree is 1 plus the height of my children!



To get H(A)



I need H(B) and H(C)



I need H(D) and H(E) and…

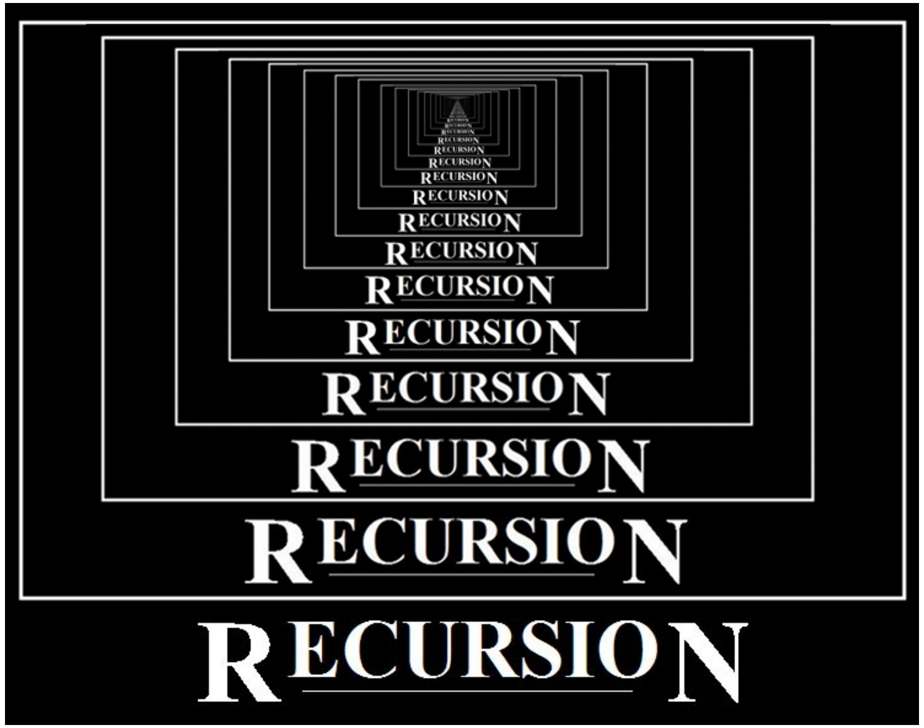
I need H(F) and H(G) and…

Leaf has height = 0

Tree of None nodes = -1

Programming Toolbox: Recursion

The process by which a function calls itself directly or indirectly is called **recursion**.



Don't panic — we've already used it before!

Linked List Recursion

A **linked list** is a list L such that:

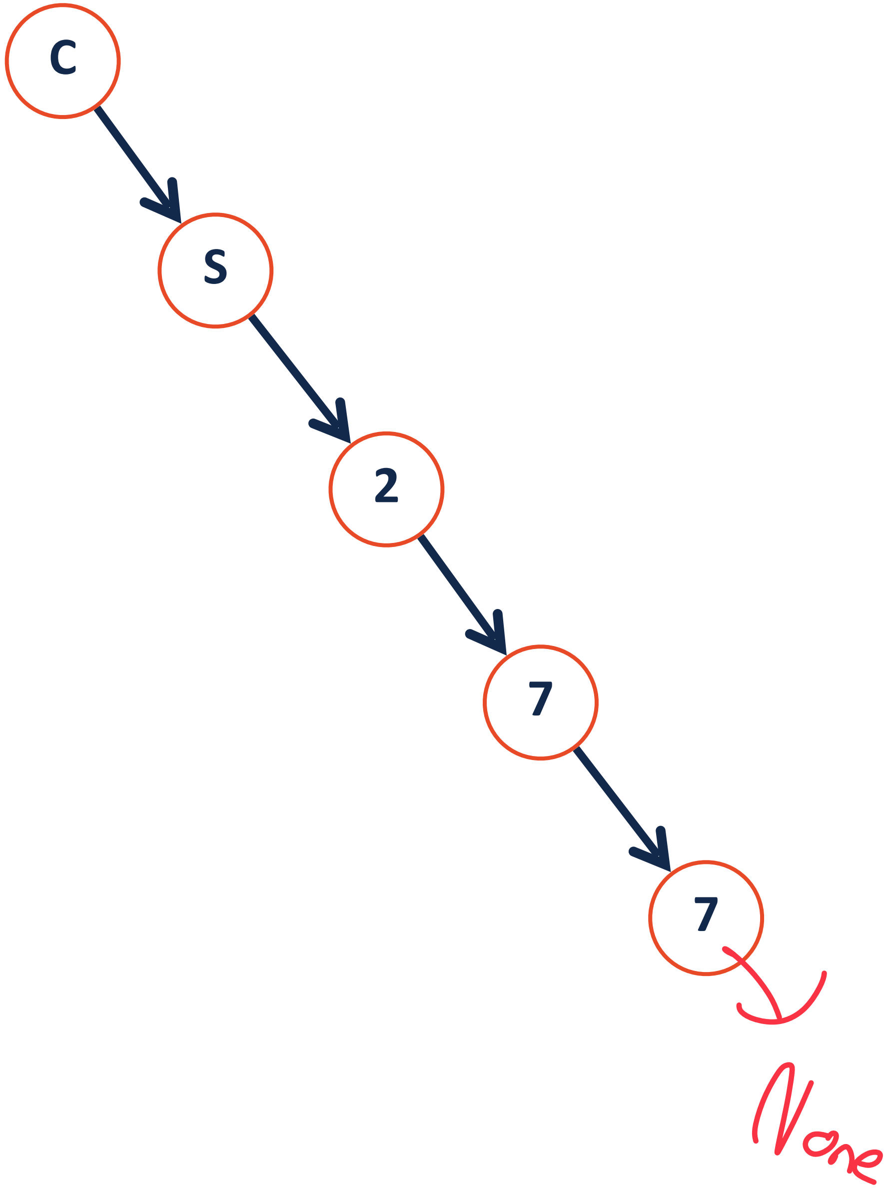
$$L = None$$

or

$$L = listNode(val,\ L_{next})$$

```
class listNode:
    def __init__(self, val, next=None):
        self.val = val
        self.next = next

```

a listNode



(Binary) Tree Recursion

A **binary tree** is a tree T such that:

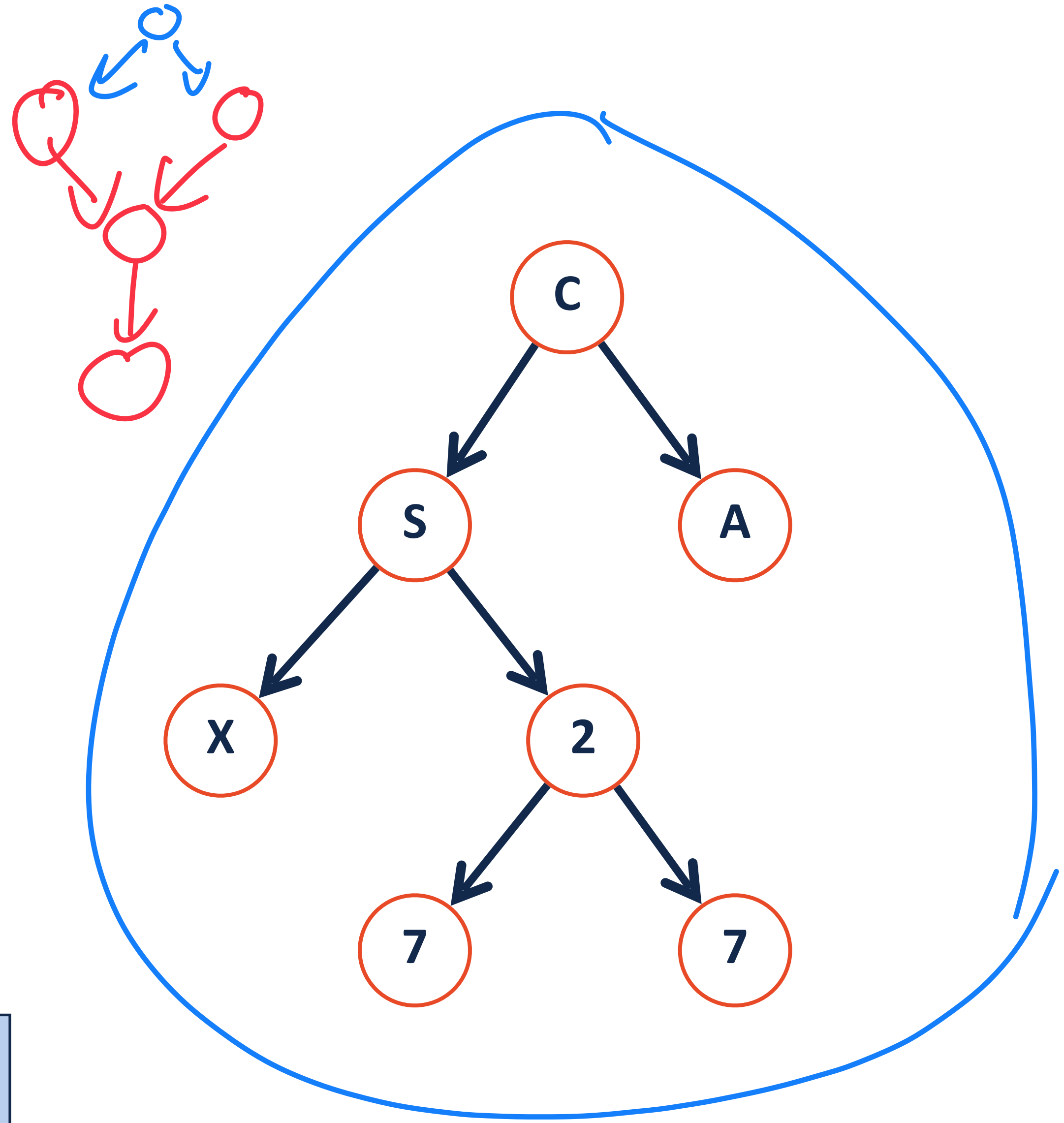
$$T = None$$

or

$$T = treeNode(val, T_L, T_R)$$

No next we have left & right instead

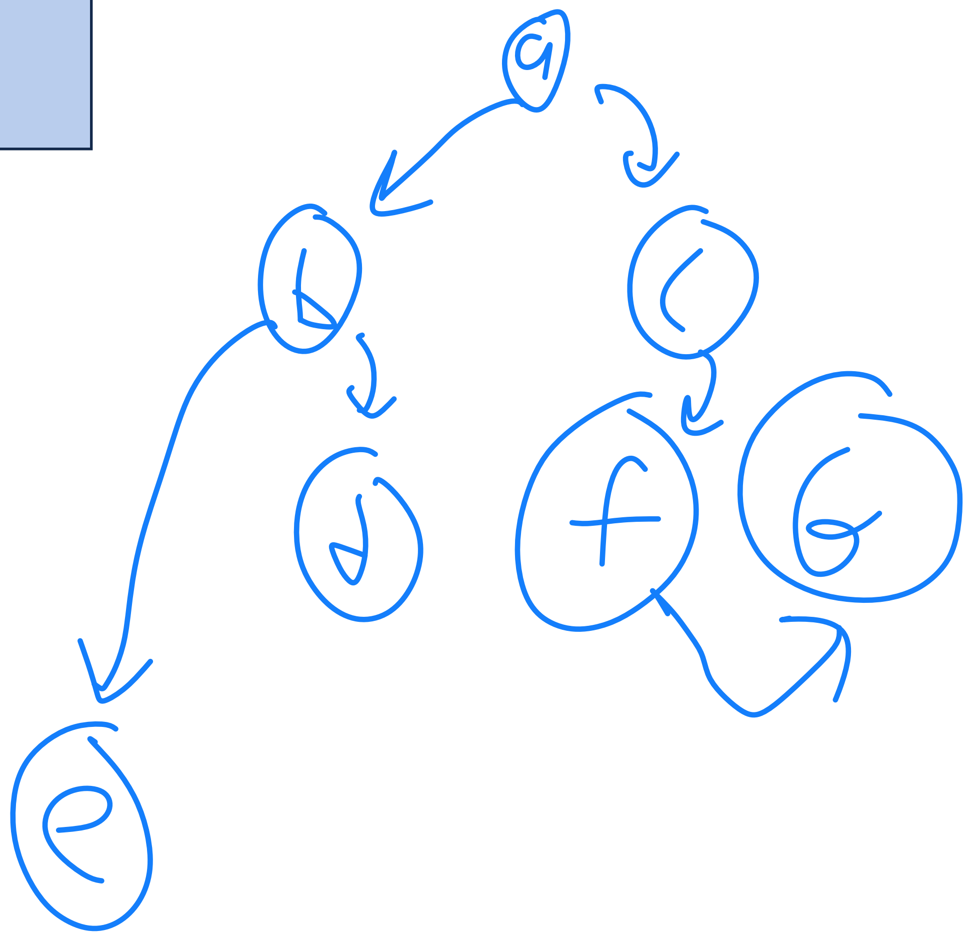
```
class treeNode:
    def __init__(self, val, left=None, right=None):
        self.val = val
        self.left = left
        self.right = right
```



Visualizing a binary tree

```
class treeNode:
    def __init__(self, val, left=None, right=None):
        self.val = val
        self.left = left
        self.right = right
```

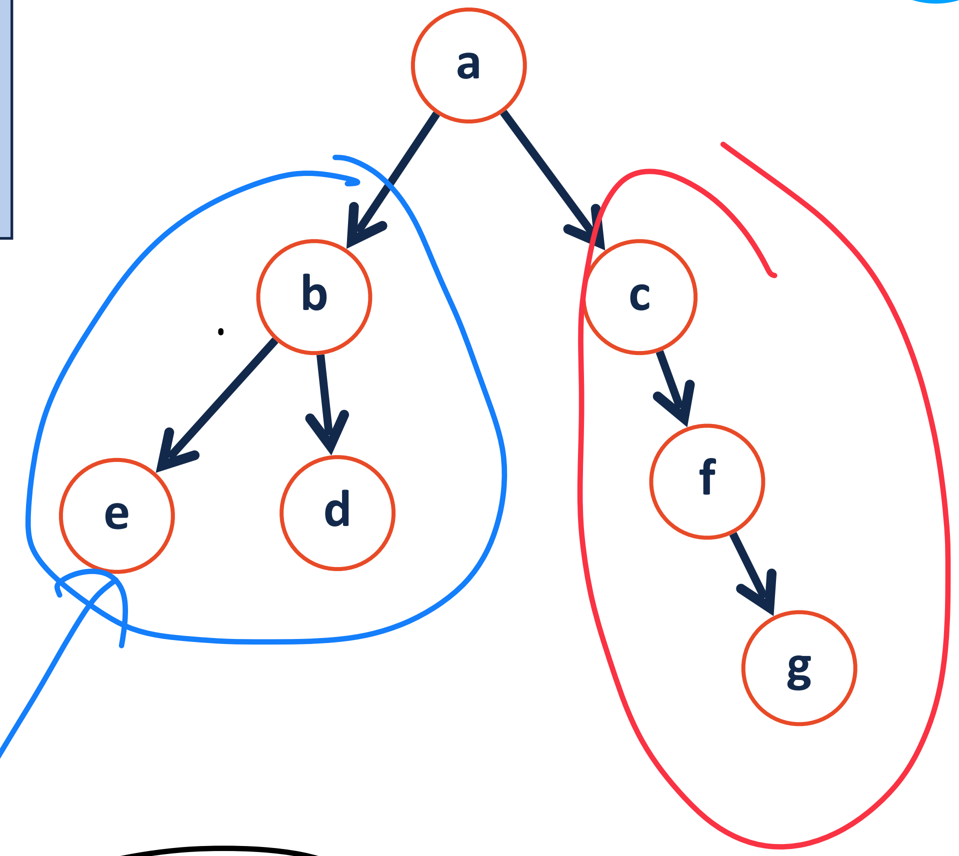
```
a = treeNode('a')
b = treeNode('b')
c = treeNode('c')
d = treeNode('d')
e = treeNode('e')
f = treeNode('f')
g = treeNode('g')

a.left = b
a.right= c
b.right = d
b.left = e
c.right = f
f.right = g
```



Visualizing a binary tree… recursively

```
class treeNode:
    def __init__(self, val, left=None, right=None):
        self.val = val
        self.left = left
        self.right = right
```

```
a = treeNode('a')
b = treeNode('b')
c = treeNode('c')
d = treeNode('d')
e = treeNode('e')
f = treeNode('f')
g = treeNode('g')

a.left = b
a.right= c
b.right = d
b.left = e
c.right = f
f.right = g
```



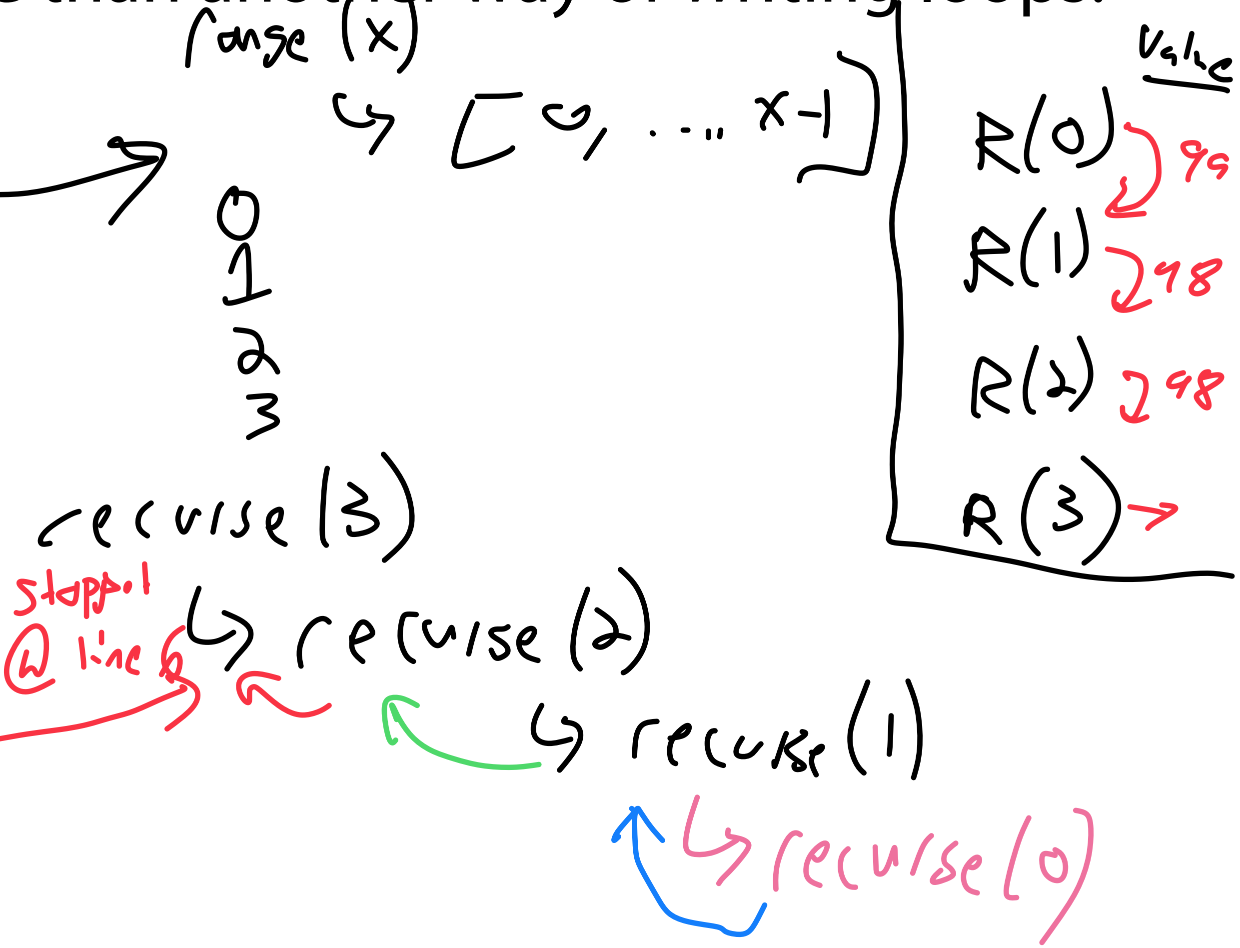

```
a = treeNode('a', treeNode('b',treeNode('e'),treeNode('d')),
treeNode('c', None, treeNode('f', None, treeNode('g'))))
```

Programming Toolbox: Recursion

At its core, recursion is nothing more than another way of writing loops:

```
for i in range(n+1):
    print(i)
```

```
def recursiveFor(n):
    if n == 0:
        print(n)
        return

    recursiveFor(n-1)

    print(n)
```



Programming Toolbox: Recursion

n = 2

Lets deep dive into whats actually happening here:

```
1  def recursiveFor(n):
2      if n == 0:
3          print(n)
4          return
5
6      recursiveFor(n-1)
7
8      print(n)
```

0, 1, 2

```
1  def recursiveFor(n):
2      if n == 0:
3          print(0)
4          return
5
6      print(n)
7
8      recursiveFor(n-1)
```

2, 1, 0

@ line 6 — call — Paused

recursiveFor(2)

print(2) @ line 8 — call

recursiveFor(1)

print(1)

@ line 6 — We continue where we left off

recursiveFor(0)

print(0) — return value

print(0)

Programming Practice: Recursive Code

What is the following code doing?

1) Give a specific (small) example

```
1 def recurse(i):
2     if i == 0:
3         return i
4     return recurse(i-1)+i
5
```

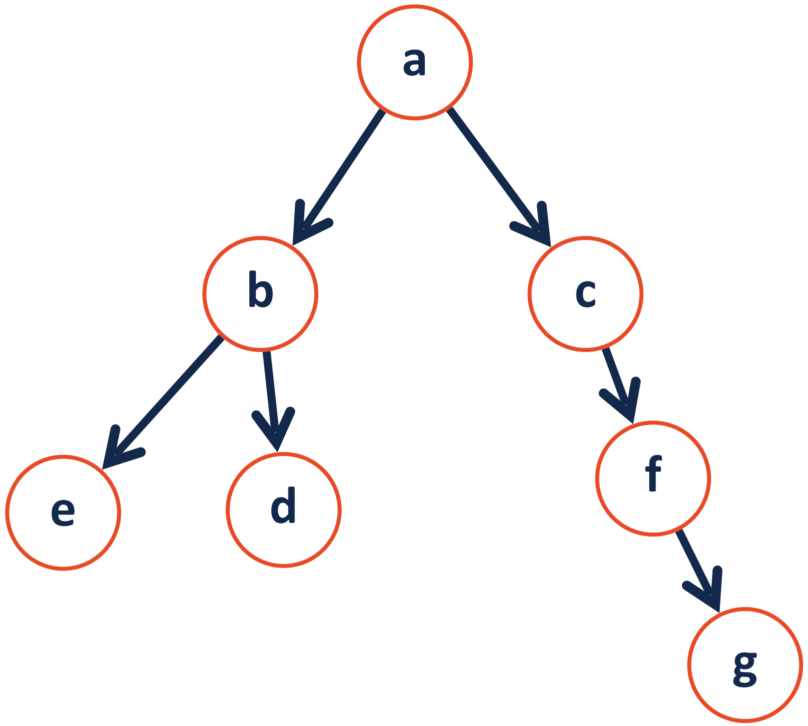
@ line 4: return recurse(0)+1

```
def recurse(inList):

    if len(inList)==0:
        return 0

    inList.pop()

    return recurse(inList)+1

```

Programming Toolbox: Recursion

Anything that can be solved with a loop can be solved with recursion

But sometimes its easier to code up a solution recursively



I can't loop through a tree with **for** or **while**…

But I can loop through the tree using recursion!

Programming Toolbox: Recursion

When thinking recursively, break the problem into parts:

Base Case: What is the smallest sub-problem? What is the trivial solution?

when (i = length)   ↳ when stop loop

Recursive Step: How can I reduce my problem to an easier one?

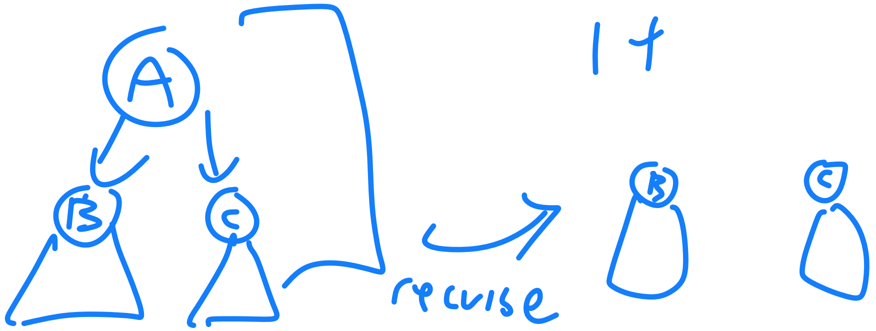
↳ i += 1   ↳ How I get to stop point

Combining: How can I build my solution from recursive pieces?

↳ How to I pass the value back

Recursive Tree Height

What is the height of my tree T?



Base Case: What is the smallest sub-problem? What is the trivial solution?

Tree is leaf (1 node case, no children) = height is 0

Tree is empty (0 node case) = height is -1

Recursive Step: How can I reduce my problem to an easier one?

Height(T is h3)

↳ Height(T_L = 2) → T_R

↳ Height(T_R

3

Combining: How can I build my solution from recursive pieces?

↳ 1 + max[height(T_{Left}), height(T_{Right})]

Recursive Sum

X min
X

[x, y, ...] —recurse→ [ ] [ ] [ ]

Given a list, sum all the items in the list ***using recursion***

Base Case: What is the smallest sub-problem? What is the trivial solution?

A list of length 0 → return 0

length 1 → return the one item

Recursive Step: How can I reduce my problem to an easier one?

↳ pop() or remove() one item (label it x)

A list of length n-1 is easier than a list of length n

Combining: How can I build my solution from recursive pieces?

↳ return x + recurseSum(list)

↳ list of n-1 items

Recursive Sum

Given a list, sum all the items in the list ***using recursion***

8	4	2	6	5

Recursive findMax

Given a list, find the max item in the list ***using recursion***

Base Case:

Recursive Step:

Combining:

Recursive findMax

Given a list, find the max item in the list ***using recursion***

8	4	2	6	5

Recursive Fibonacci

Given a number *n*, return the *nth* Fibonacci number:

$$Fib(n) = Fib(n-1) + Fib(n-2), \quad n > 1$$

Base Case:

Recursive Step:

Combining:

Recursive List Partitioning

Using all elements in a list, can we make two lists which have equal sums?

6	5	4	2	7

1	1	1	1	1

2	3	3	3	1

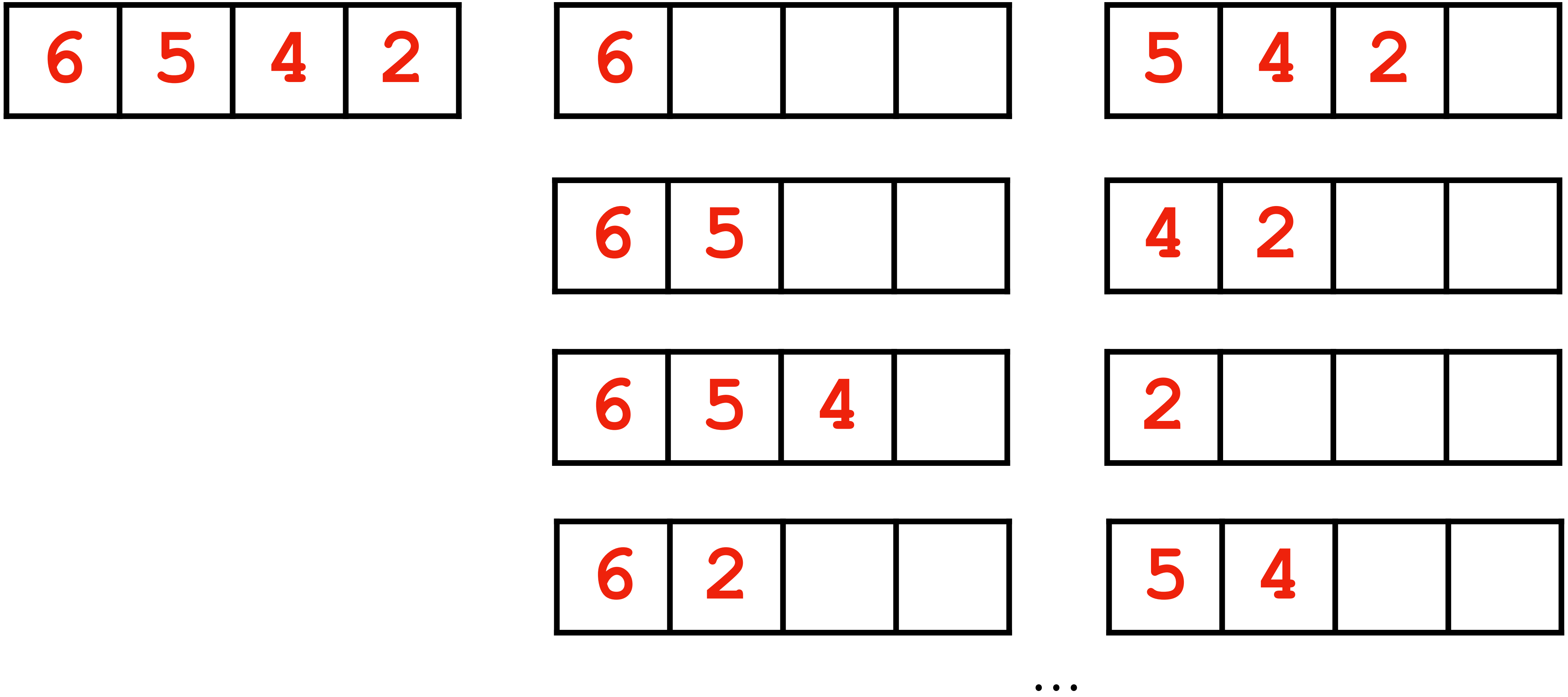
Recursive List Partitioning

How would a computer solve this problem?

6	5	4	2

Recursive List Partitioning

How would a computer solve this problem? **Compute every permutation!**



Recursive List Partitioning

Writing code to attempt every possible permutation is tricky with loops.

But its a great example of recursion in action!

Recursive Step: Given list L, pop() L[0] to left ***and*** right and recurse on both

Recursive List Partitioning

Recursive Step: Given list L, pop() L[0] to left ***and*** right and recurse on both

Input:	*Left*	*Right*
6 5 4 2		
Recursive Calls:		
5 4 2	6	
5 4 2		6

Recursive List Partitioning

Recursive Step: Given list L, pop() L[0] to left ***and*** right and recurse on both

Base Case:

Base Case: When my input list is empty, I have tried every permutation

Recursive Step: Given list L, pop() L[0] to left ***and*** right and recurse on both

```
[4, 3, 1]                      ([], [])

[3, 1]        ([4], [])                          ([], [4])

[1]  ([3, 4], [])  ([4], [3])        ([3], [4])   ([], [3, 4])

[]

([1, 3, 4], [])  ([1, 4], [3])  ([1, 3], [4])  ([1], [3, 4])

([3, 4], [1])    ([4], [1, 3])  ([3], [1, 4])  ([], [1, 3, 4])
```

Recursive List Partitioning

Base Case: When my input list is empty, I have tried every permutation

Recursive Step: Given list L, pop() L[0] to left ***and*** right and recurse on both

Combination Step:

Lab Recursion

Recursive List Partitioning is ***now extra credit*** on Fridays lab!

In preparation for Friday, consider how you might use recursion to solve:

- Computing the factorial of a number
- Counting the sum of all digits in a number
- Checking if a string is a palindrome