Algorithms and Data Structures for Data Science

Trees 4

CS 277

October 20, 2021

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Exam 2 October 29th - October 31st

Exam will be combination multiple choice and coding

Covers all content up until October 20th

Sign up for registration as early as October 14th

Coding question will use new workspace setup.

Learning Objectives

Discuss efficiency of BSTs

Identify best, worst, and average case for BSTs

Calculate height balance

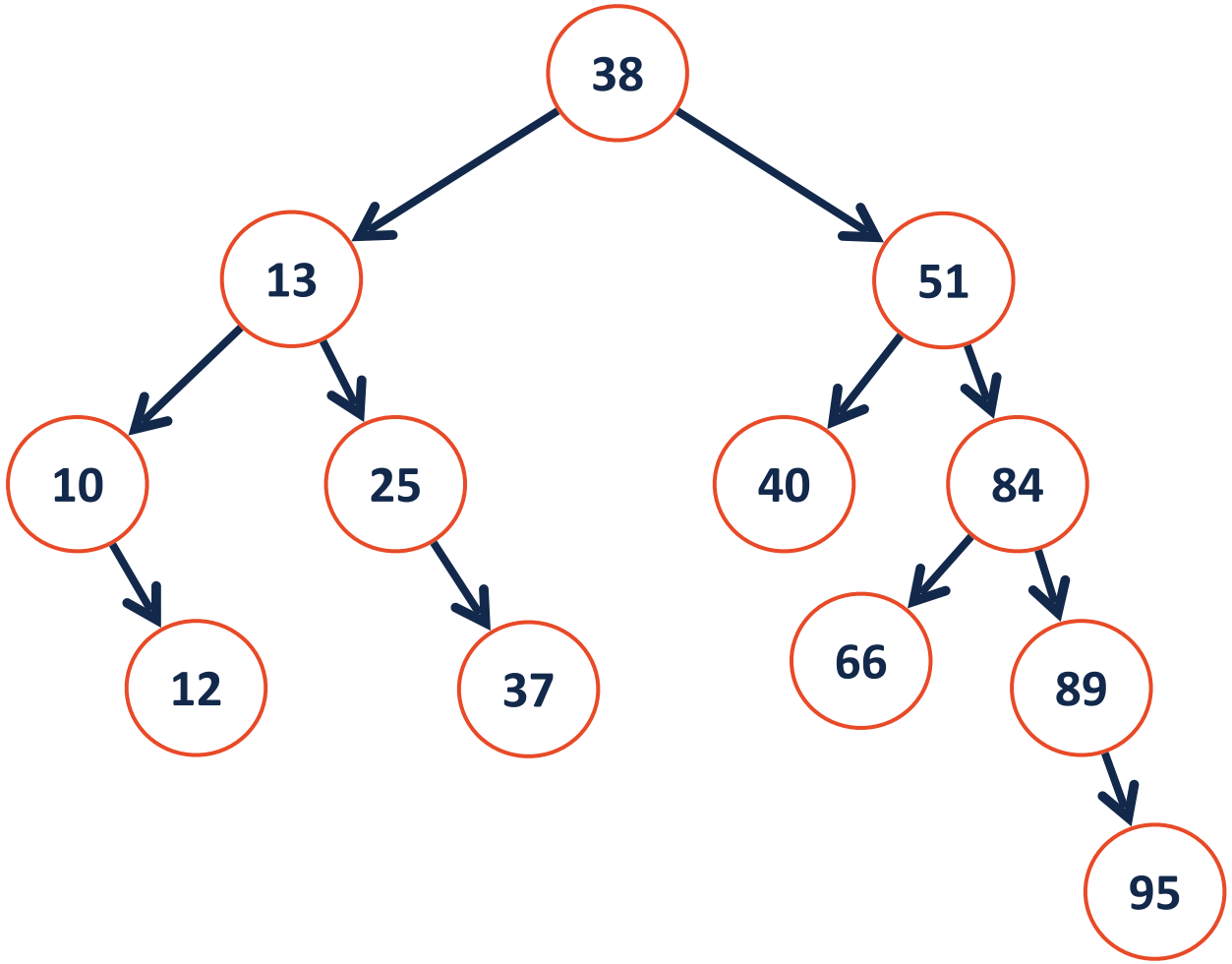
NOT ON EXAM: Introduce tree rotations

Binary Search Tree (BST)

A **BST** is a binary tree $T = treeNode(val, T_L, T_r)$ such that:

$$\forall n \in T_L,\ n.val < T.val$$

$$\forall n \in T_R,\ n.val > T.val$$



BST Analysis – Running Time

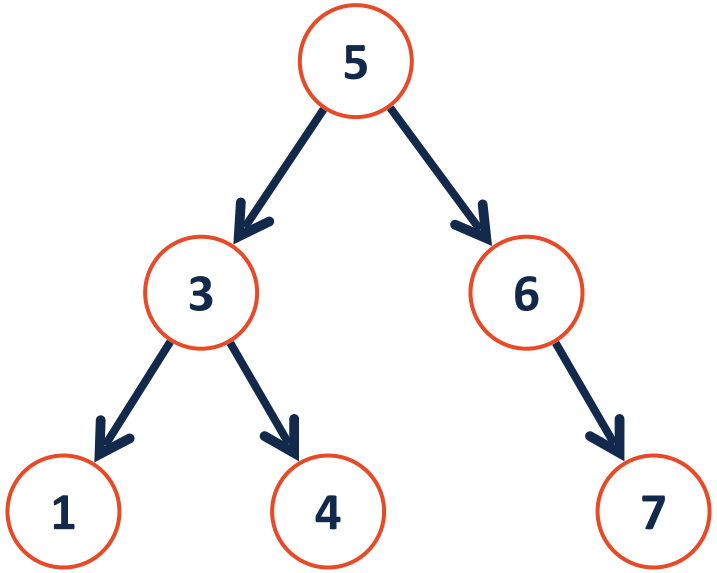
Operation	BST Worst Case
find	
insert	
remove	
traverse	

BST Find

```
def find(self, key):
    n = self.find_helper(self.root, key)
    if n:
        return n.val
    else:
        return None

def find_helper(self,node,key):
    nkey = node.key
    if nkey > key:
        if node.left:
            return self.find_helper(node.left, key)
        else:
            return None
    elif nkey < key:
        if node.right:
            return self.find_helper(node.right, key)
        else:
            return None
    else:
        return node


```

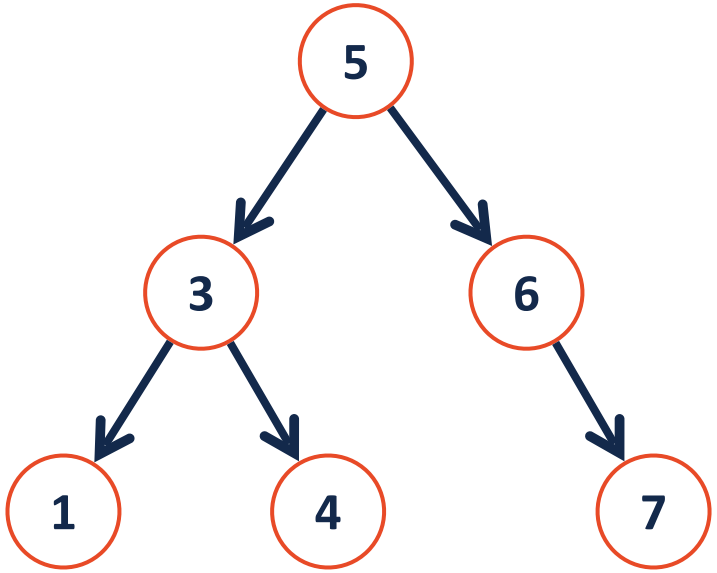


BST Insert

Base Case

Recursive Step

Combining



BST Remove

```
def remove(self, key):
    self.root = self.remove_helper(self.root, key)
```

```
def remove_helper(self, node, key):
    if node == None:
        return None

    if node.key > key:
        node.left = self.remove_helper(node.left, key)
    elif node.key < key:
        node.right = self.remove_helper(node.right, key)
    else:
        if node.left == None:
            temp = node.right
            return temp

        if node.right == None:
            temp = node.left
            return temp

        iop = self.findIOP(node)
        node.key = iop.key
        node.val = iop.val
        node.left = self.remove_helper(node.left, iop.key)

    return node
```

BST Analysis

Every operation on a BST depends on the **height** of the tree.

… how do we relate $O(h)$ to n, the size of our dataset?

BST Analysis

What is the max number of nodes in a tree of height h ?

BST Analysis

What is the min number of nodes in a tree of height h ?

BST Analysis

For all BSTs:

Lower-bound:

Upper-bound:

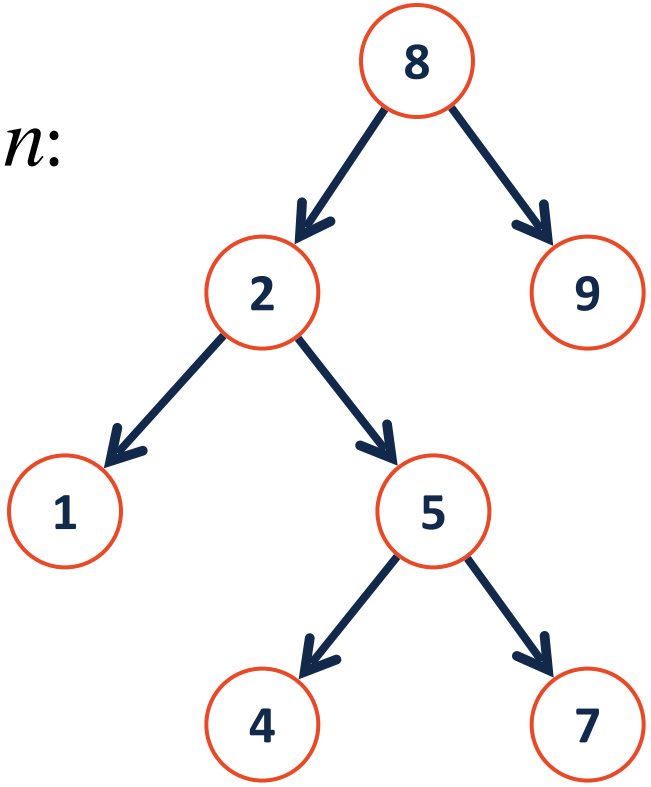
Tree Property: "Full"

A tree F is **full** if (and only if) for each node n:

n has zero children

OR

n has two children



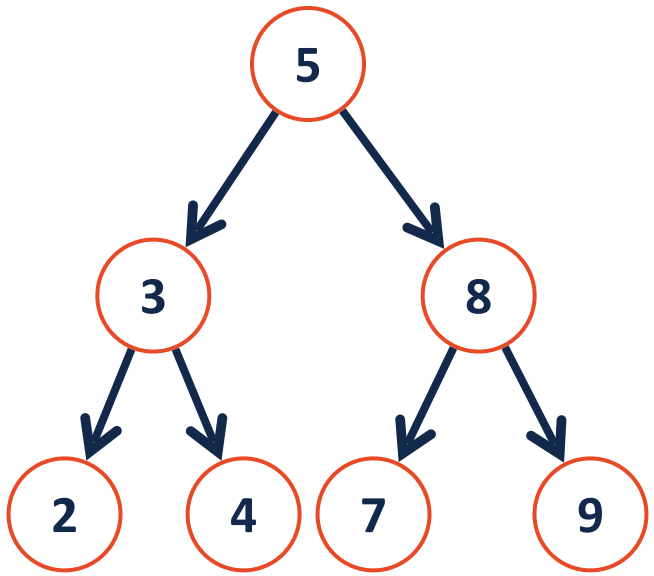
Tree Property: "Perfect"

A **perfect** tree P_h is a tree of height h where:

$$P_{-1} = \text{None}$$

AND

$$P_h = \{\text{key, value}, P_{h-1}, P_{h-1}\}$$



Tree Property: "Complete"

A **complete** tree C_h is a tree of height h where:

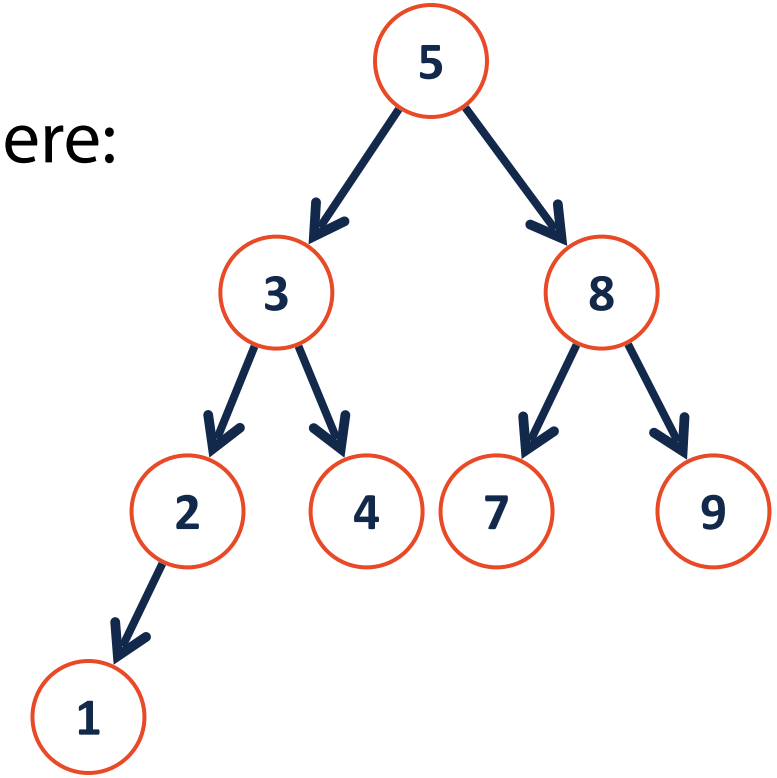
$$C_{-1} = \text{None}$$

AND

$$C_h = \{\text{key, value}, C_{h-1}, P_{h-2}\}$$

OR

$$C_h = \{\text{key, value}, P_{h-1}, P_{h-1}\}$$



Which tree properties are “good”?

BST Analysis

The height of a BST depends on the order in which the data was inserted

Insert Order: [1, 3, 2, 4, 5, 6, 7]

Insert Order: [4, 2, 3, 6, 7, 1, 5]

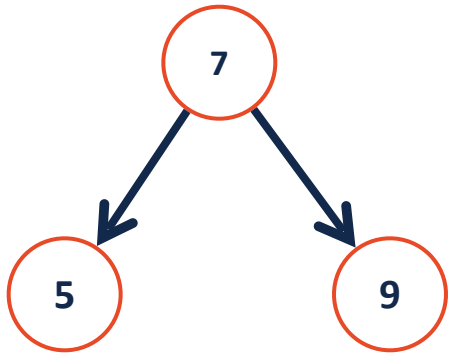
BST Analysis

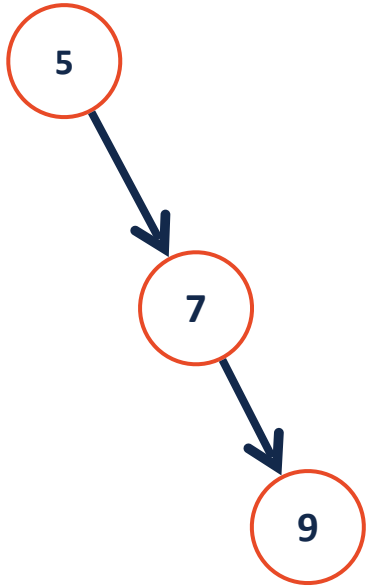
How many different ways are there to insert n keys into a BST?

Claim: The average height of all arrangements is $O(log\ n)$

Height-Balanced Tree

What tree is better?

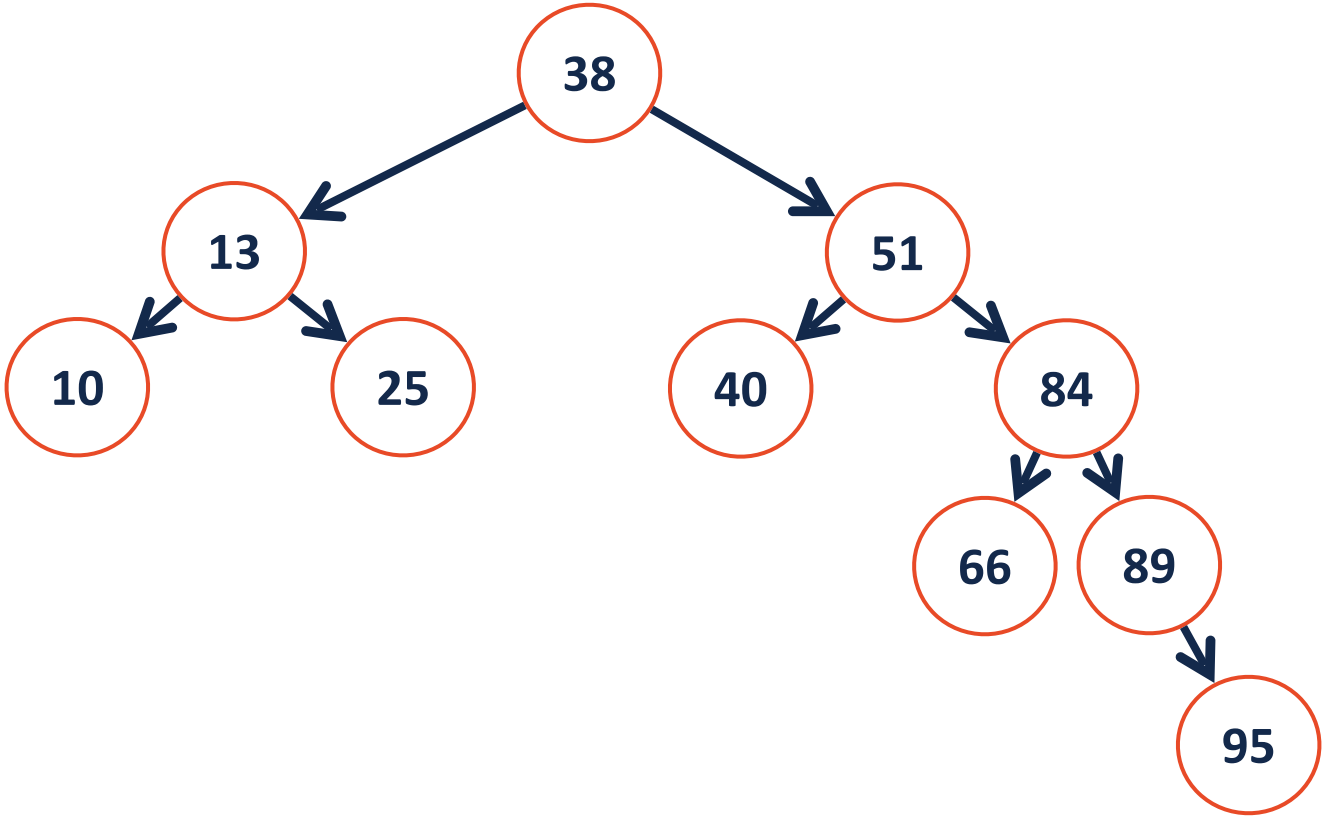




Height balance: $b = height(T_R) - height(T_L)$

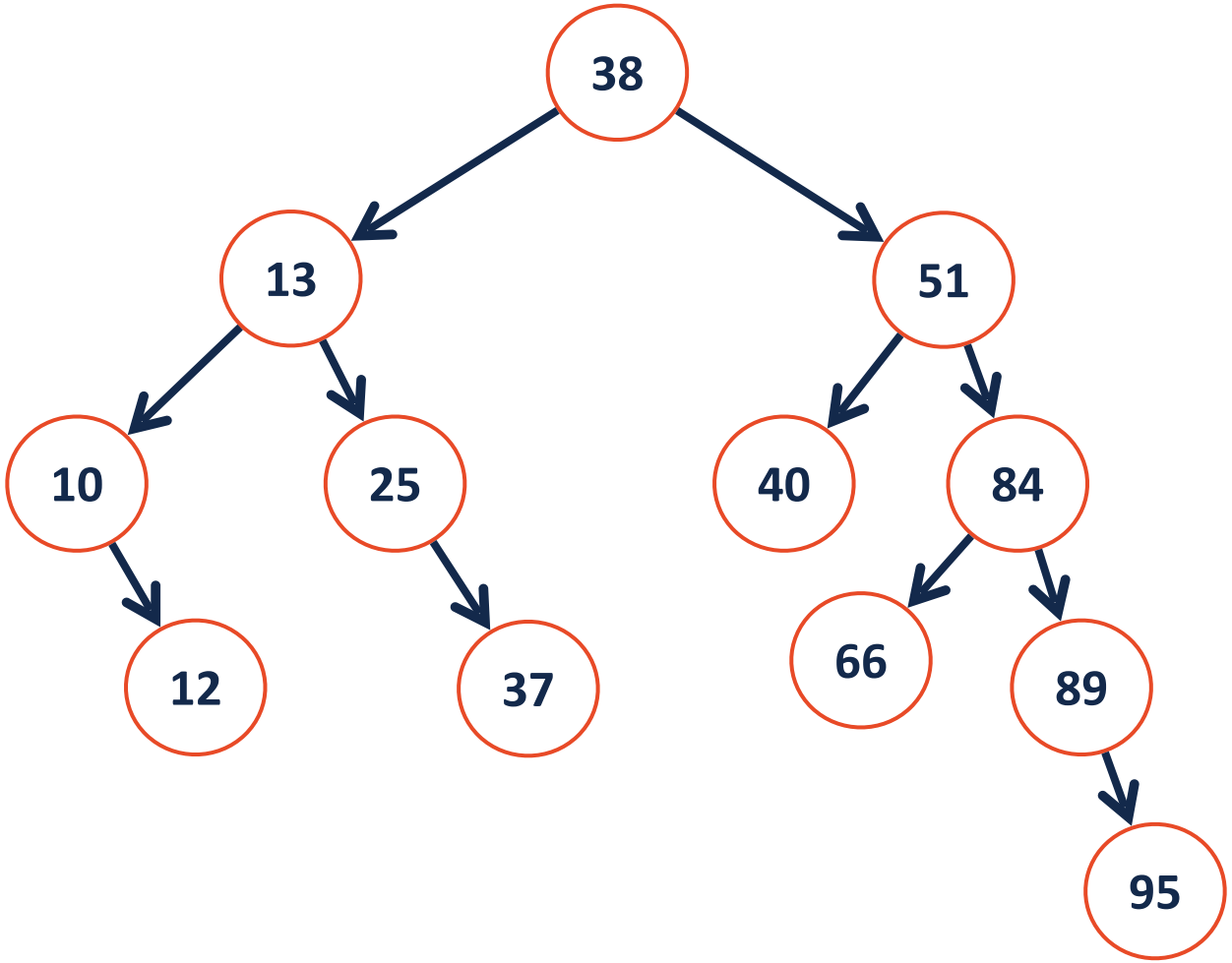
A tree is "balanced" if:

Recursive balance calculations



Modifying insert (for height)

`insert(98)`



BST Analysis — Running Time (n)

Operation	BST Average case	BST Worst case	Sorted array
find			
insert			
remove			
traverse			

End of Exam 2 Material!

Sorting

Search

Recursion

Trees

Sorting

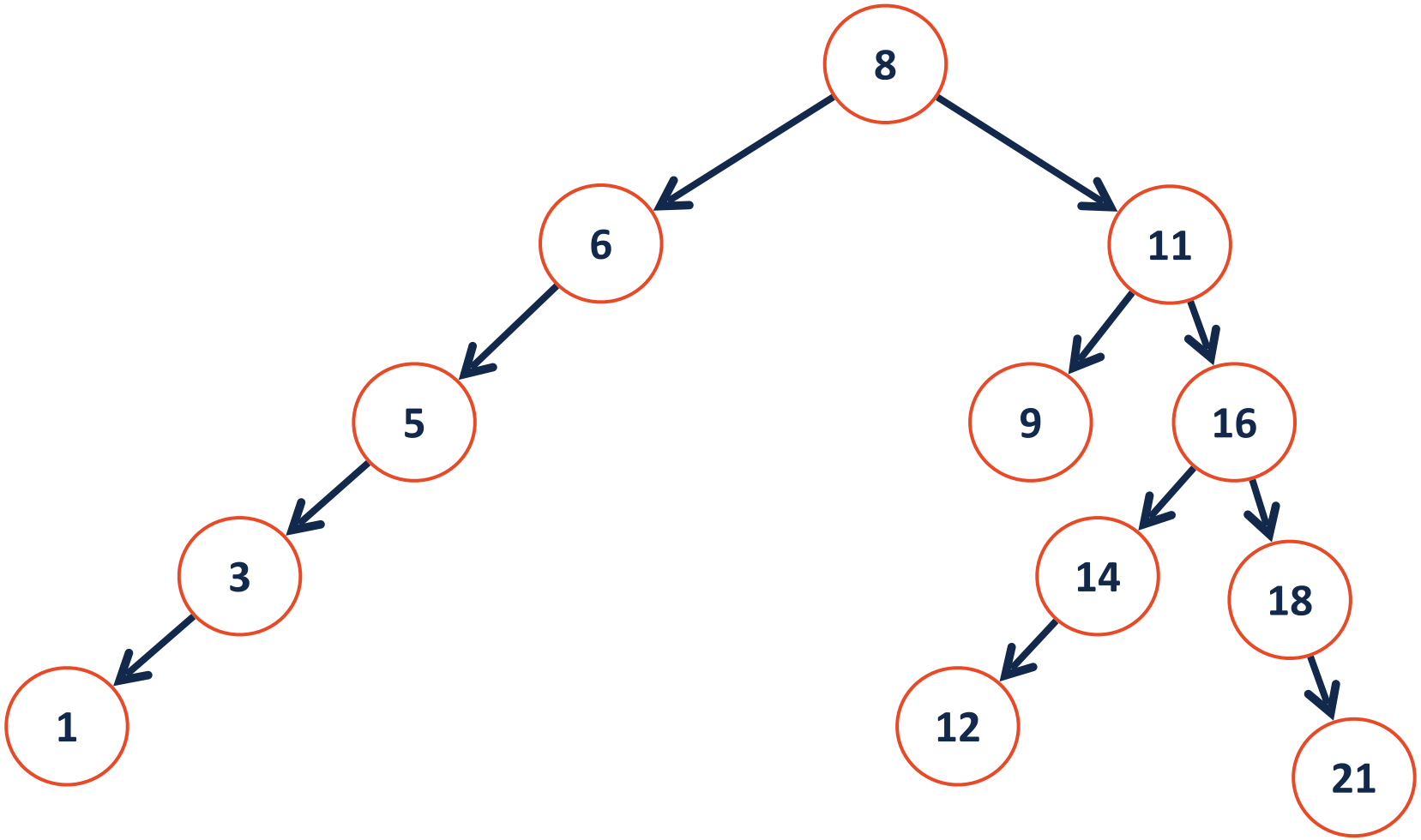
Search

Recursion

Trees

BST Remove

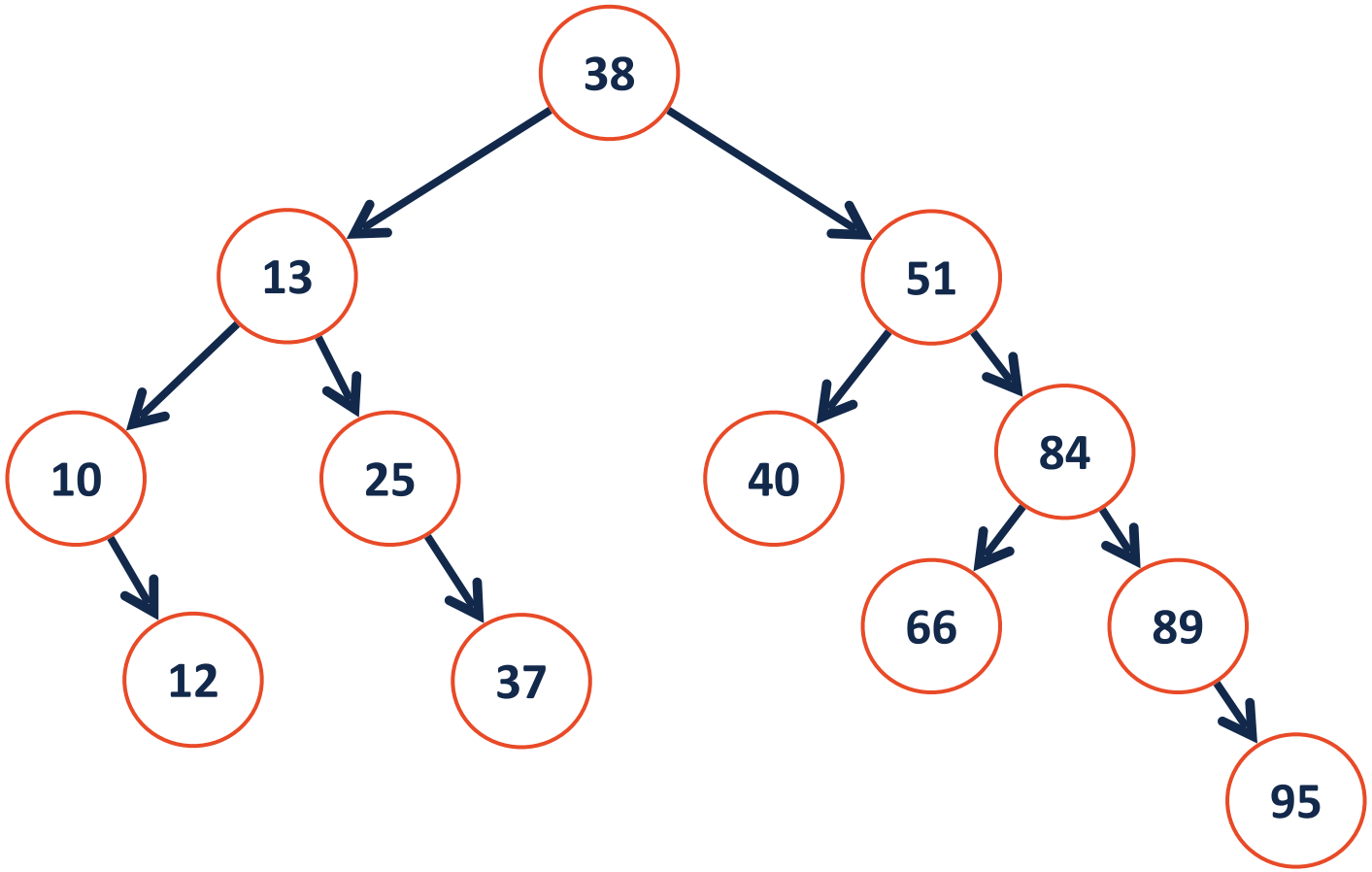
What will the tree structure look like if we remove node 16 using IOS?



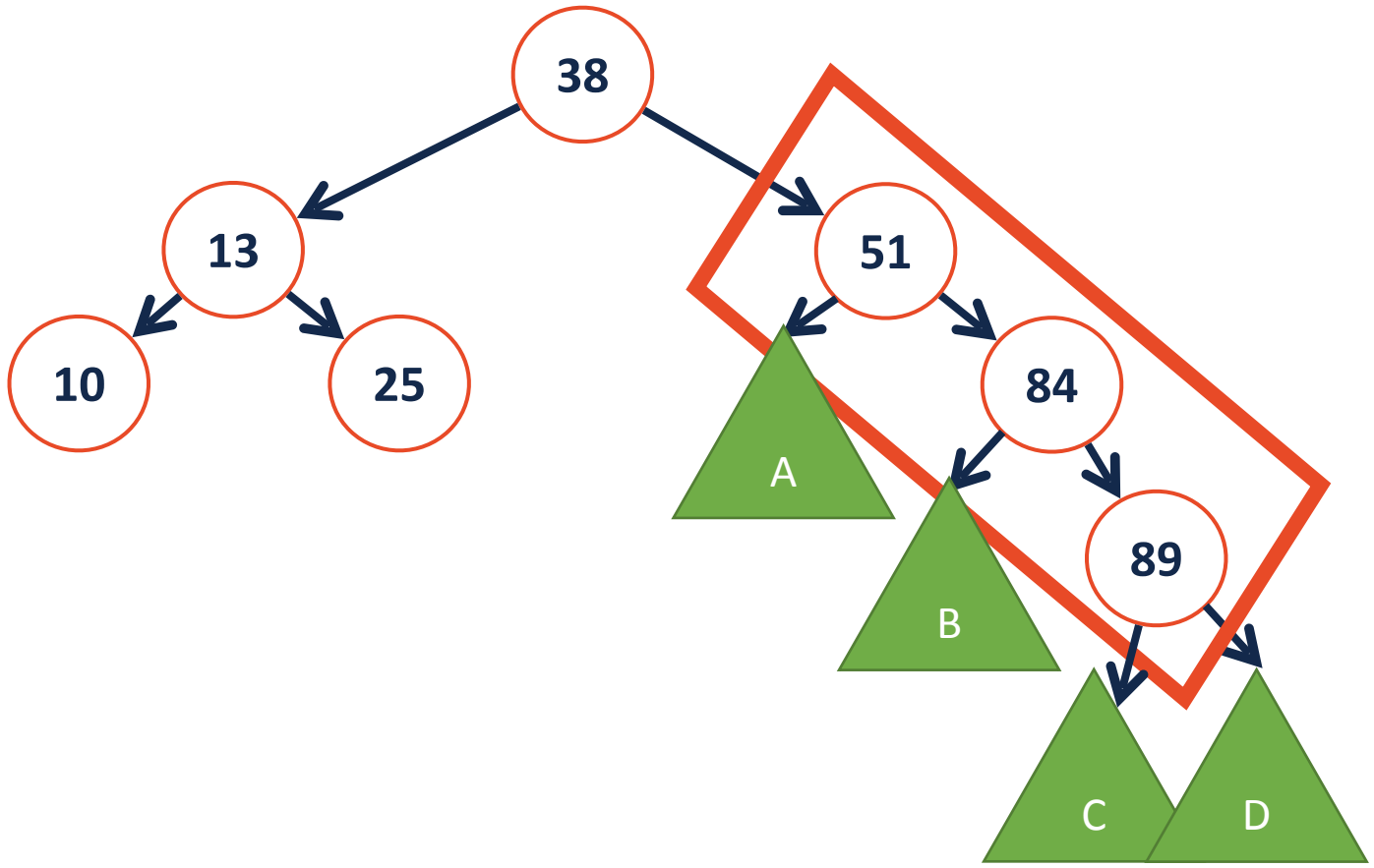
THIS MATERIAL IS NOT ON EXAM!

BST Rotations

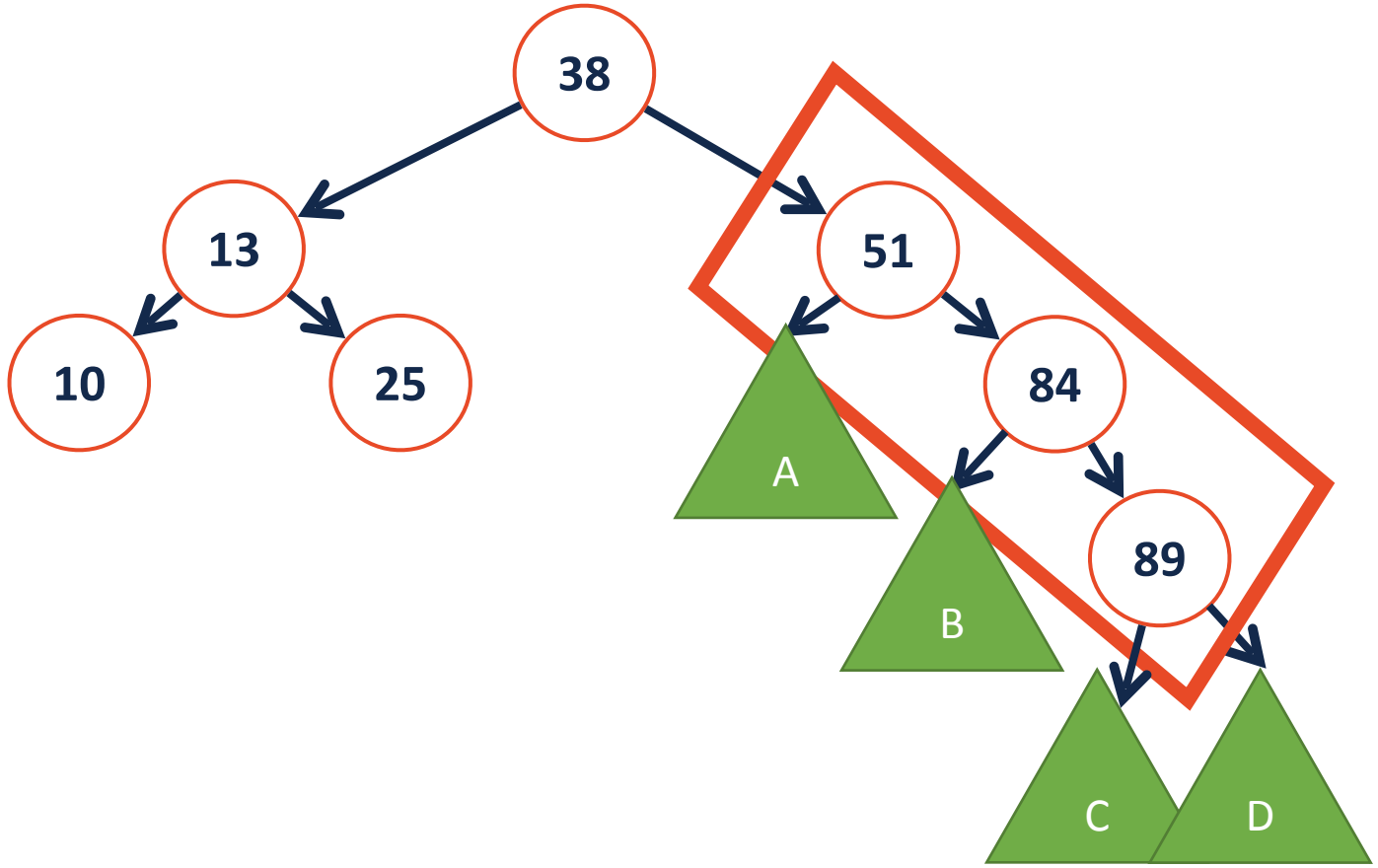
We can adjust the BST structure by performing **rotations**.

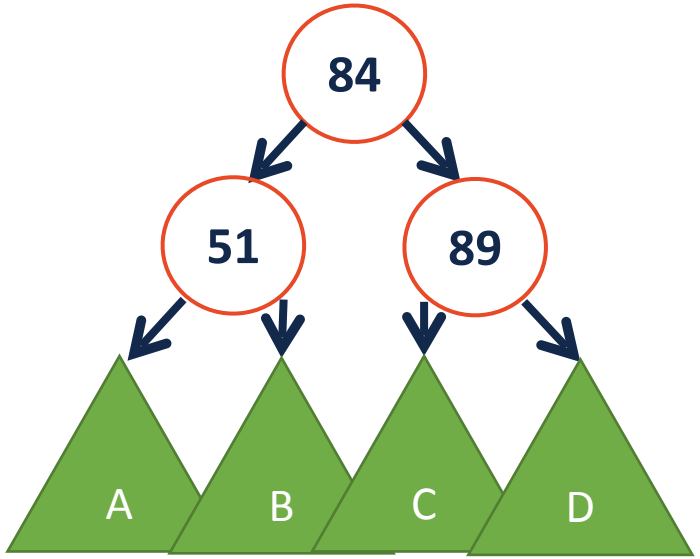


BST Rotations

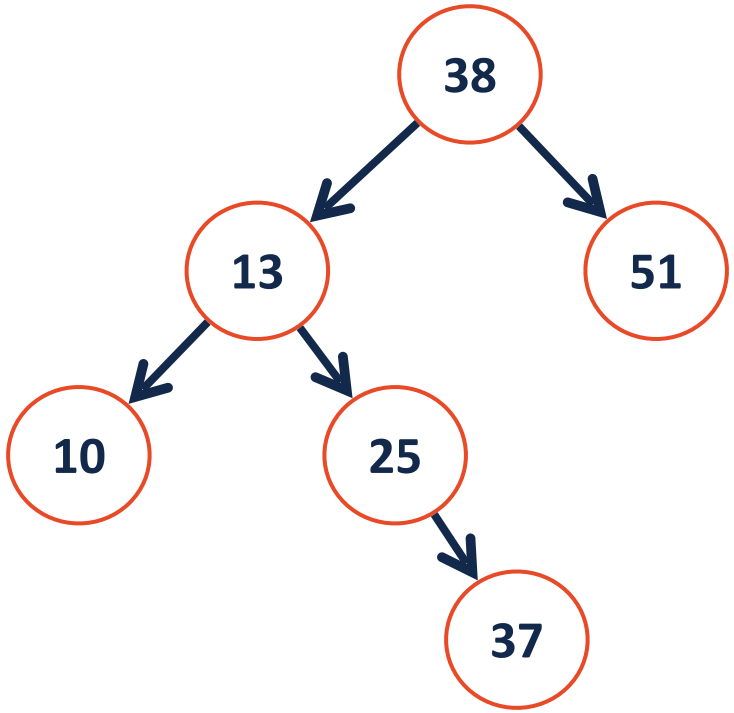


BST Rotations

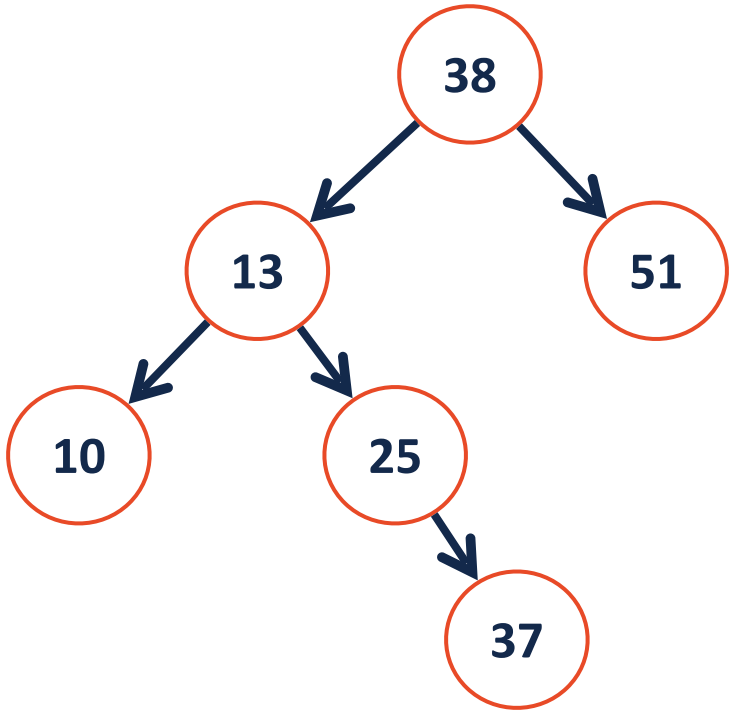




BST Rotations



BST Rotations



BST Rotations

Next week we will define four kinds of rotations (L, R, LR, RL)

We will see that:

1. All rotations are local (subtrees are not impacted)
2. The running time of rotations are constant
3. The rotations maintain BST property

Motivation for rotations:

We call these trees: