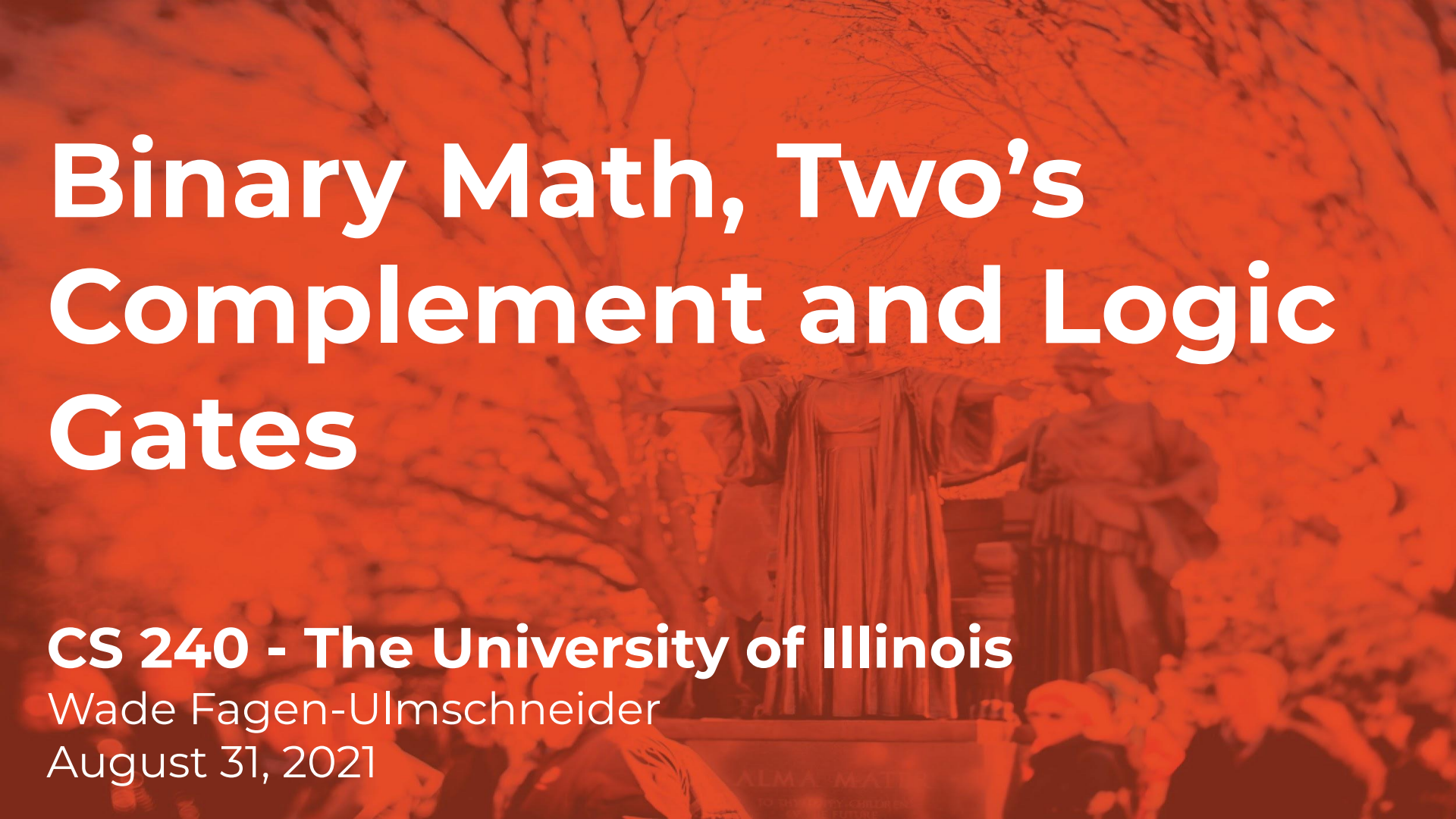
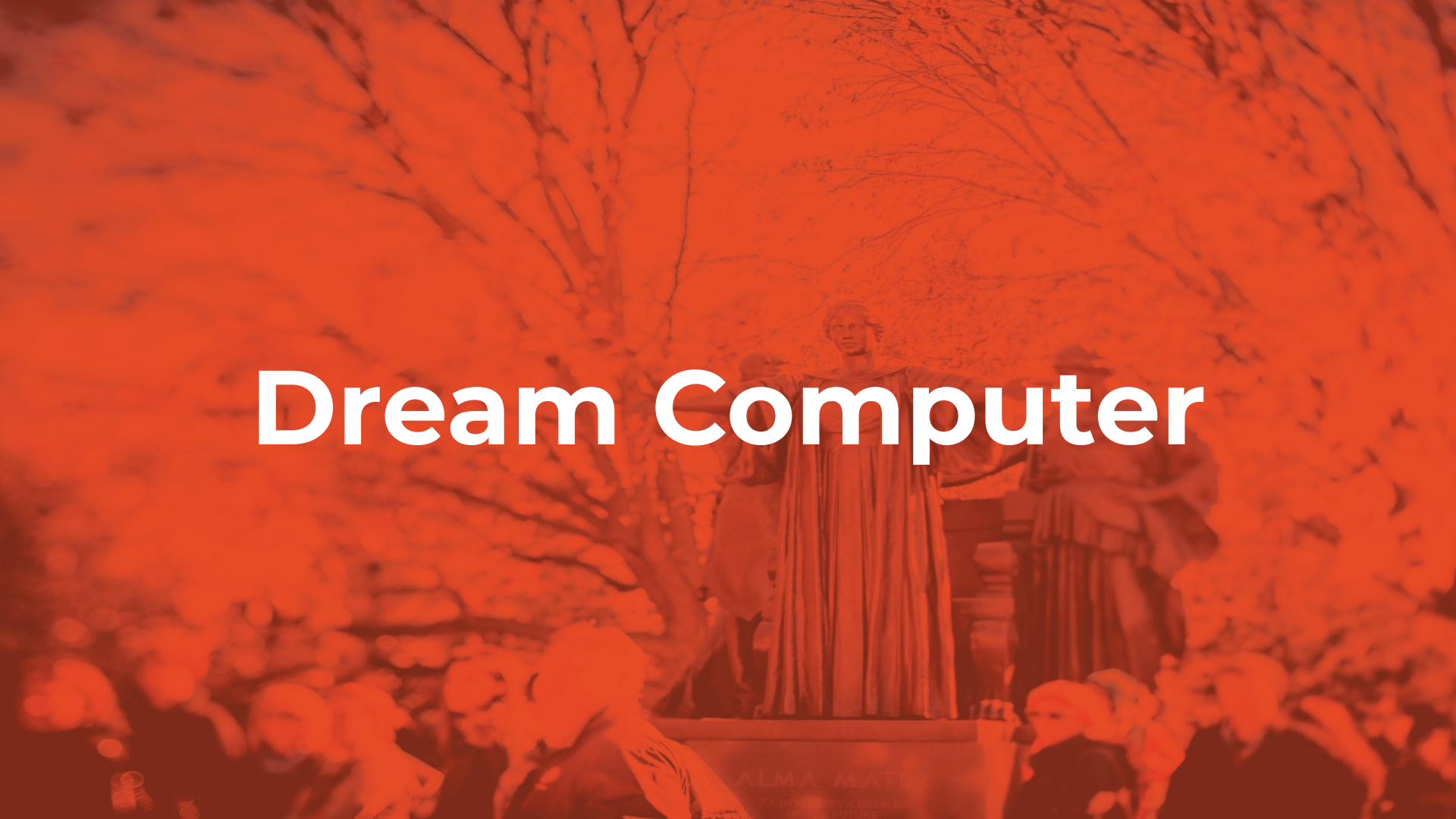


Binary Math, Two's Complement and Logic Gates

CS 240 - The University of Illinois

Wade Fagen-Ulmschneider

August 31, 2021



Dream Computer

Binary Addition

```
  0b 010011
+0b 001001
```

```
  0b 0011
+0b 0111
```

Negative Numbers

```
  0b 010011
- 0b 001001
```

```
  0b 0011
- 0b 0111
```

```
  0b 0011     ⇒     0011
-0b 0111     ⇒   + 1111
             ⇒    10010
```

Two’s Complement

Two's Complement

Big Idea: Represent numbers signed numbers in binary in a way that:

Two's Complement

Method:

$-17 =$

$-4 =$

$-1 =$

$$\begin{array}{r} 42 \\ -\underline{18} \end{array}$$

$$\begin{array}{r} 18 \\ -42 \\ \hline \end{array}$$

$$
\begin{array}{r}
-42 \\
-\ \underline{32}
\end{array}
$$

Overflow Detection

$$
\begin{array}{r}
-31 \\
- \quad 42 \\
\hline
\end{array}
$$

Towards Multiplication

$$10 \times 2 =$$

$$10 \times 4 =$$

$$10 \times 9 =$$

Left Shift:

Right Shift:

Logic Gates

Logic Gate #1

Logic Gate #2

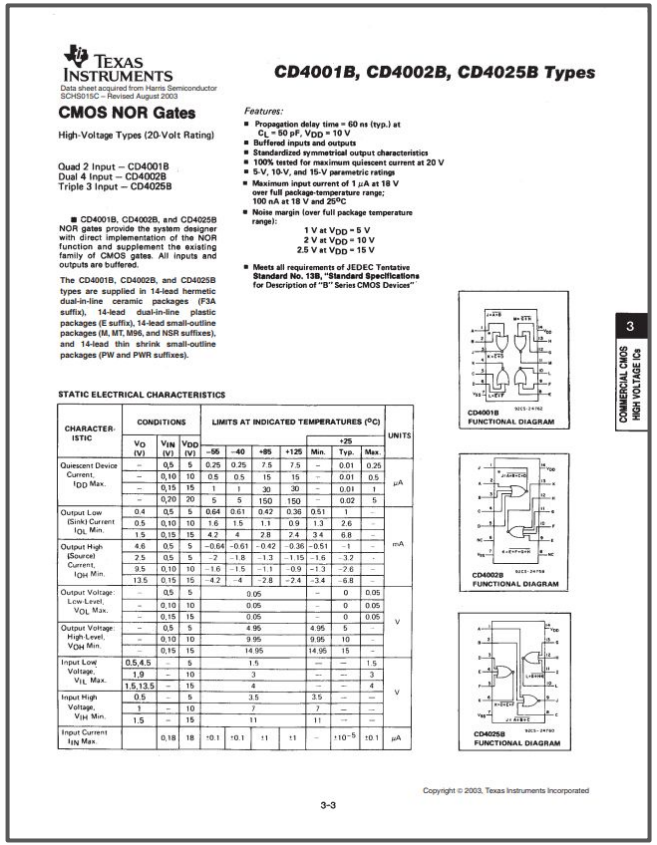
Logic Gate #3

Logic Gate #4

Functionally Complete Gate?

A	B	A&B	A|B	A^B	
0	0	0	0	0	
0	1	0	1	1	
1	0	0	1	1	
1	1	1	1	0	

Hardware Schematics



TEXAS INSTRUMENTS

Data sheet acquired from Harris Semiconductor
SCHS015C – Revised August 2003

CD4001B, CD4002B, CD4025B Types

CMOS NOR Gates

High-Voltage Types (20-Volt Rating)

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- Noise margin (over full package temperature range):
  1 V at V_{DD} = 5 V
  2 V at V_{DD} = 10 V
  2.5 V at V_{DD} = 15 V
- Meets all requirements of JEDEC Tentative Standard No. 13B, "Standard Specifications for Description of "B" Series CMOS Devices"

CD4001B FUNCTIONAL DIAGRAM

CD4002B FUNCTIONAL DIAGRAM

CD4025B FUNCTIONAL DIAGRAM

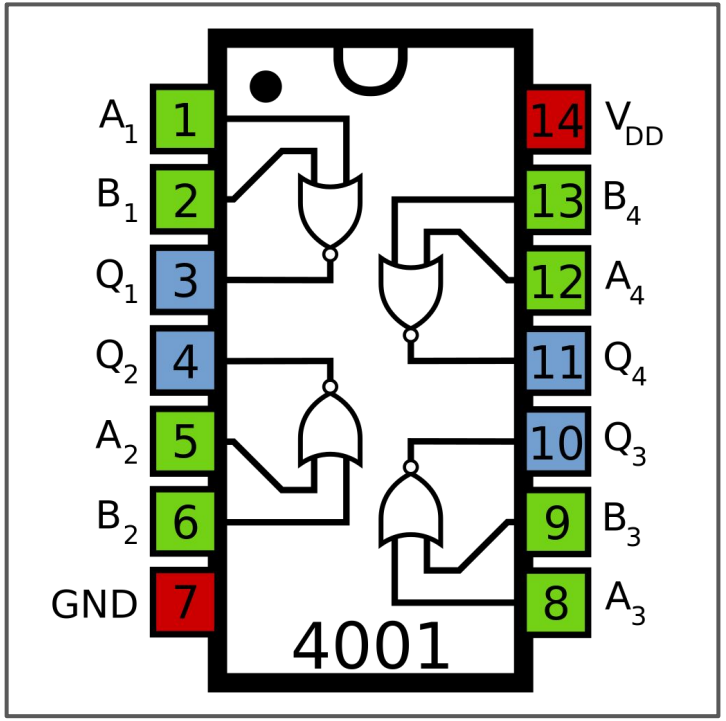
3 COMMERCIAL CMOS HIGH VOLTAGE ICs

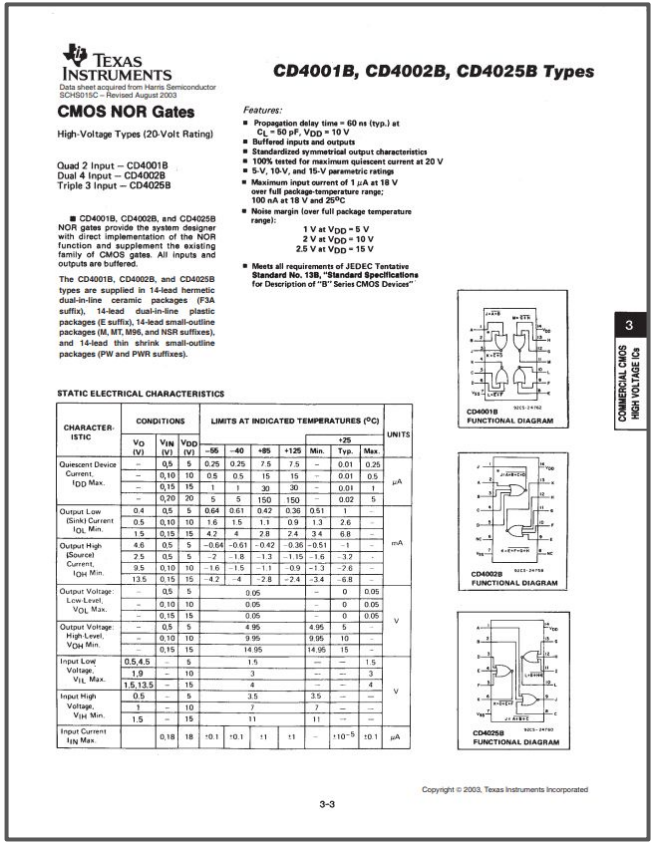
STATIC ELECTRICAL CHARACTERISTICS

CHARACTERISTIC	CONDITIONS			LIMITS AT INDICATED TEMPERATURES (°C)							UNITS
	V_O (V)	V_{IN} (V)	V_{DD} (V)	−55	−40	+85	+125	+25 Min.	+25 Typ.	+25 Max.	
Quiescent Device Current, I_{DD} Max.	–	0,5	5	0.25	0.25	7.5	7.5	–	0.01	0.25	μA
	–	0,10	10	0.5	0.5	15	15	–	0.01	0.5	
	–	0,15	15	1	1	30	30	–	0.01	1	
	–	0,20	20	5	5	150	150	–	0.02	5	
Output Low (Sink) Current I_{OL} Min.	0.4	0,5	5	0.64	0.61	0.42	0.36	0.51	1	–	mA
	0.5	0,10	10	1.6	1.5	1.1	0.9	1.3	2.6	–	
	1.5	0,15	15	4.2	4	2.8	2.4	3.4	6.8	–	
Output High (Source) Current, I_{OH} Min.	4.6	0,5	5	−0.64	−0.61	−0.42	−0.36	−0.51	−1	–	
	2.5	0,5	5	−2	−1.8	−1.3	−1.15	−1.6	−3.2	–	
	9.5	0,10	10	−1.6	−1.5	−1.1	−0.9	−1.3	−2.6	–	
	13.5	0,15	15	−4.2	−4	−2.8	−2.4	−3.4	−6.8	–	
Output Voltage: Low-Level, V_{OL} Max.	–	0,5	5	0.05				–	0	0.05	V
	–	0,10	10	0.05				–	0	0.05	
	–	0,15	15	0.05				–	0	0.05	
Output Voltage: High-Level, V_{OH} Min.	–	0,5	5	4.95				4.95	5	–	
	–	0,10	10	9.95				9.95	10	–	
	–	0,15	15	14.95				14.95	15	–	
Input Low Voltage, V_{IL} Max.	0.5,4.5	–	5	1.5				–	–	1.5	V
	1,9	–	10	3				–	–	3	
	1.5,13.5	–	15	4				–	–	4	
Input High Voltage, V_{IH} Min.	0.5	–	5	3.5				3.5	–	–	
	1	–	10	7				7	–	–	
	1.5	–	15	11				11	–	–	
Input Current I_{IN} Max.		0,18	18	±0.1	±0.1	±1	±1	–	$±10^{-5}$	±0.1	μA

https://www.ti.com/lit/ds/symlink/cd4025b.pdf

Hardware Schematics





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- Noise margin (over full package temperature range):
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CD4001B FUNCTIONAL DIAGRAM

CD4002B FUNCTIONAL DIAGRAM

CD4025B FUNCTIONAL DIAGRAM

3 COMMERCIAL CMOS HIGH VOLTAGE ICs

STATIC ELECTRICAL CHARACTERISTICS

CHARACTERISTIC	CONDITIONS			LIMITS AT INDICATED TEMPERATURES (°C)							UNITS
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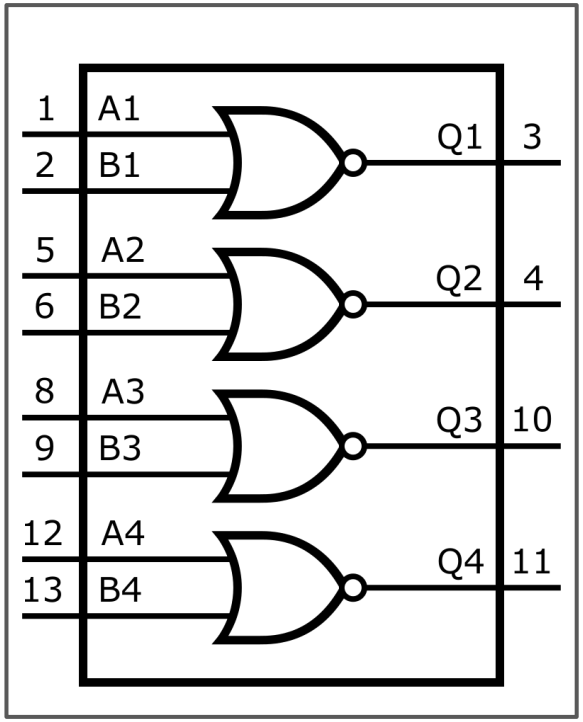
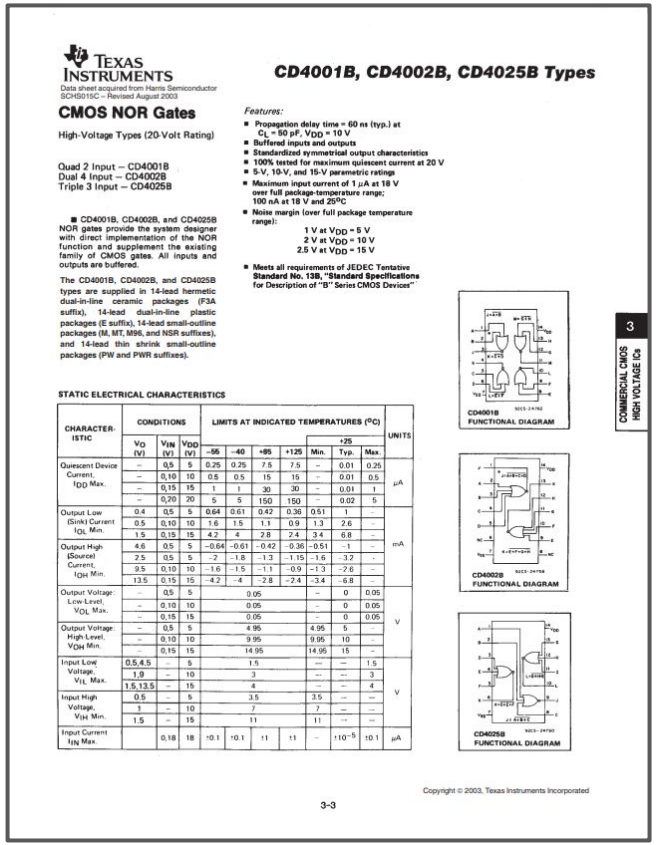
Quiescent Device Current, I_{DD} Max.	–	0,5	5	0.25	0.25	7.5	7.5	–	0.01	0.25	μA
	–	0,10	10	0.5	0.5	15	15	–	0.01	0.5	
	–	0,15	15	1	1	30	30	–	0.01	1	
	–	0,20	20	5	5	150	150	–	0.02	5	
Output Low (Sink) Current I_{OL} Min.	0.4	0,5	5	0.64	0.61	0.42	0.36	0.51	1	–	mA
	0.5	0,10	10	1.6	1.5	1.1	0.9	1.3	2.6	–	
	1.5	0,15	15	4.2	4	2.8	2.4	3.4	6.8	–	
Output High (Source) Current, I_{OH} Min.	4.6	0,5	5	−0.64	−0.61	−0.42	−0.36	−0.51	−1	–	
	2.5	0,5	5	−2	−1.8	−1.3	−1.15	−1.6	−3.2	–	
	9.5	0,10	10	−1.6	−1.5	−1.1	−0.9	−1.3	−2.6	–	
	13.5	0,15	15	−4.2	−4	−2.8	−2.4	−3.4	−6.8	–	
Output Voltage: Low-Level, V_{OL} Max.	–	0,5	5	0.05				–	0	0.05	V
	–	0,10	10	0.05				–	0	0.05	
	–	0,15	15	0.05				–	0	0.05	
Output Voltage: High-Level, V_{OH} Min.	–	0,5	5	4.95				4.95	5	–	
	–	0,10	10	9.95				9.95	10	–	
	–	0,15	15	14.95				14.95	15	–	
Input Low Voltage, V_{IL} Max.	0.5,4.5	–	5	1.5				–	–	1.5	V
	1,9	–	10	3				–	–	3	
	1.5,13.5	–	15	4				–	–	4	
Input High Voltage, V_{IH} Min.	0.5	–	5	3.5				3.5	–	–	
	1	–	10	7				7	–	–	
	1.5	–	15	11				11	–	–	
Input Current I_{IN} Max.		0,18	18	±0.1	±0.1	±1	±1	–	$±10^{-5}$	±0.1	μA

Hardware Schematics





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CD4001B FUNCTIONAL DIAGRAM

CD4002B FUNCTIONAL DIAGRAM

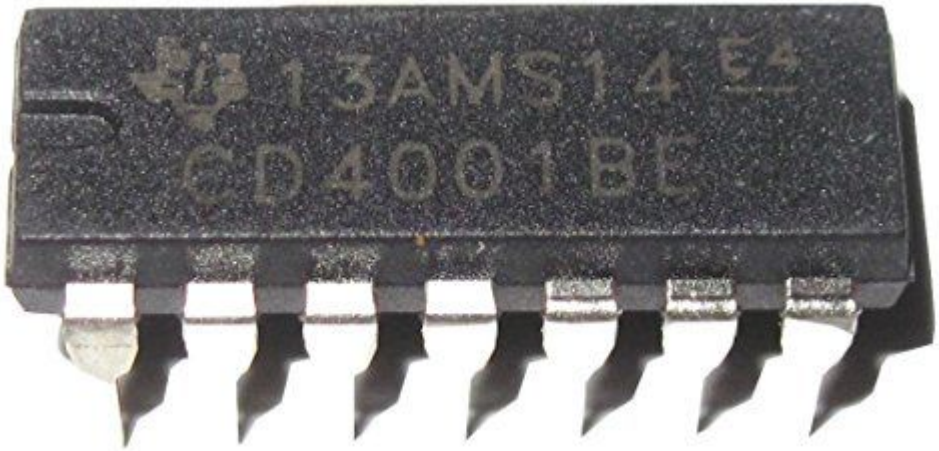
CD4025B FUNCTIONAL DIAGRAM

3 COMMERCIAL CMOS HIGH VOLTAGE ICs

STATIC ELECTRICAL CHARACTERISTICS

CHARACTERISTIC	CONDITIONS V_O (V)	V_{IN} (V)	V_{DD} (V)	LIMITS AT INDICATED TEMPERATURES (°C) −55	−40	+85	+125	+25 Min.	+25 Typ.	+25 Max.	UNITS
Quiescent Device Current, I_{DD} Max.	–	0,5	5	0.25	0.25	7.5	7.5	–	0.01	0.25	μA
	–	0,10	10	0.5	0.5	15	15	–	0.01	0.5	
	–	0,15	15	1	1	30	30	–	0.01	1	
	–	0,20	20	5	5	150	150	–	0.02	5	
Output Low (Sink) Current I_{OL} Min.	0.4	0,5	5	0.64	0.61	0.42	0.36	0.51	1	–	mA
	0.5	0,10	10	1.6	1.5	1.1	0.9	1.3	2.6	–	
	1.5	0,15	15	4.2	4	2.8	2.4	3.4	6.8	–	
Output High (Source) Current, I_{OH} Min.	4.6	0,5	5	−0.64	−0.61	−0.42	−0.36	−0.51	−1	–	
	2.5	0,5	5	−2	−1.8	−1.3	−1.15	−1.6	−3.2	–	
	9.5	0,10	10	−1.6	−1.5	−1.1	−0.9	−1.3	−2.6	–	
	13.5	0,15	15	−4.2	−4	−2.8	−2.4	−3.4	−6.8	–	
Output Voltage: Low-Level, V_{OL} Max.	–	0,5	5	0.05				–	0	0.05	V
	–	0,10	10	0.05				–	0	0.05	
	–	0,15	15	0.05				–	0	0.05	
Output Voltage: High-Level, V_{OH} Min.	–	0,5	5	4.95				4.95	5	–	
	–	0,10	10	9.95				9.95	10	–	
	–	0,15	15	14.95				14.95	15	–	
Input Low Voltage, V_{IL} Max.	0.5,4.5	–	5	1.5				–	–	1.5	V
	1,9	–	10	3				–	–	3	
	1.5,13.5	–	15	4				–	–	4	
Input High Voltage, V_{IH} Min.	0.5	–	5	3.5				3.5	–	–	
	1	–	10	7				7	–	–	
	1.5	–	15	11				11	–	–	
Input Current I_{IN} Max.		0,18	18	±0.1	±0.1	±1	±1	–	$±10^{-5}$	±0.1	μA

Hardware Schematics



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CD4002B FUNCTIONAL DIAGRAM

CD4025B FUNCTIONAL DIAGRAM

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	–	0,10	10	0.5	0.5	15	15	–	0.01	0.5	
	–	0,15	15	1	1	30	30	–	0.01	1	
	–	0,20	20	5	5	150	150	–	0.02	5	
Output Low (Sink) Current I_{OL} Min.	0.4	0,5	5	0.64	0.61	0.42	0.36	0.51	1	–	mA
	0.5	0,10	10	1.6	1.5	1.1	0.9	1.3	2.6	–	
	1.5	0,15	15	4.2	4	2.8	2.4	3.4	6.8	–	
Output High (Source) Current, I_{OH} Min.	4.6	0,5	5	−0.64	−0.61	−0.42	−0.36	−0.51	−1	–	
	2.5	0,5	5	−2	−1.8	−1.3	−1.15	−1.6	−3.2	–	
	9.5	0,10	10	−1.6	−1.5	−1.1	−0.9	−1.3	−2.6	–	
	13.5	0,15	15	−4.2	−4	−2.8	−2.4	−3.4	−6.8	–	
Output Voltage: Low-Level, V_{OL} Max.	–	0,5	5	0.05				–	0	0.05	V
	–	0,10	10	0.05				–	0	0.05	
	–	0,15	15	0.05				–	0	0.05	
Output Voltage: High-Level, V_{OH} Min.	–	0,5	5	4.95				4.95	5	–	
	–	0,10	10	9.95				9.95	10	–	
	–	0,15	15	14.95				14.95	15	–	
Input Low Voltage, V_{IL} Max.	0.5,4.5	–	5	1.5				–	–	1.5	V
	1,9	–	10	3				–	–	3	
	1.5,13.5	–	15	4				–	–	4	
Input High Voltage, V_{IH} Min.	0.5	–	5	3.5				3.5	–	–	
	1	–	10	7				7	–	–	
	1.5	–	15	11				11	–	–	
Input Current I_{IN} Max.		0,18	18	±0.1	±0.1	±1	±1	–	$\pm 10^{-5}$	±0.1	μA

https://www.ti.com/lit/ds/symlink/cd4025b.pdf

Binary Addition

Binary Addition

Half Adder:

A	B	A + B	SUM	CARRY

Half Adder Circuit Diagram

Binary Addition

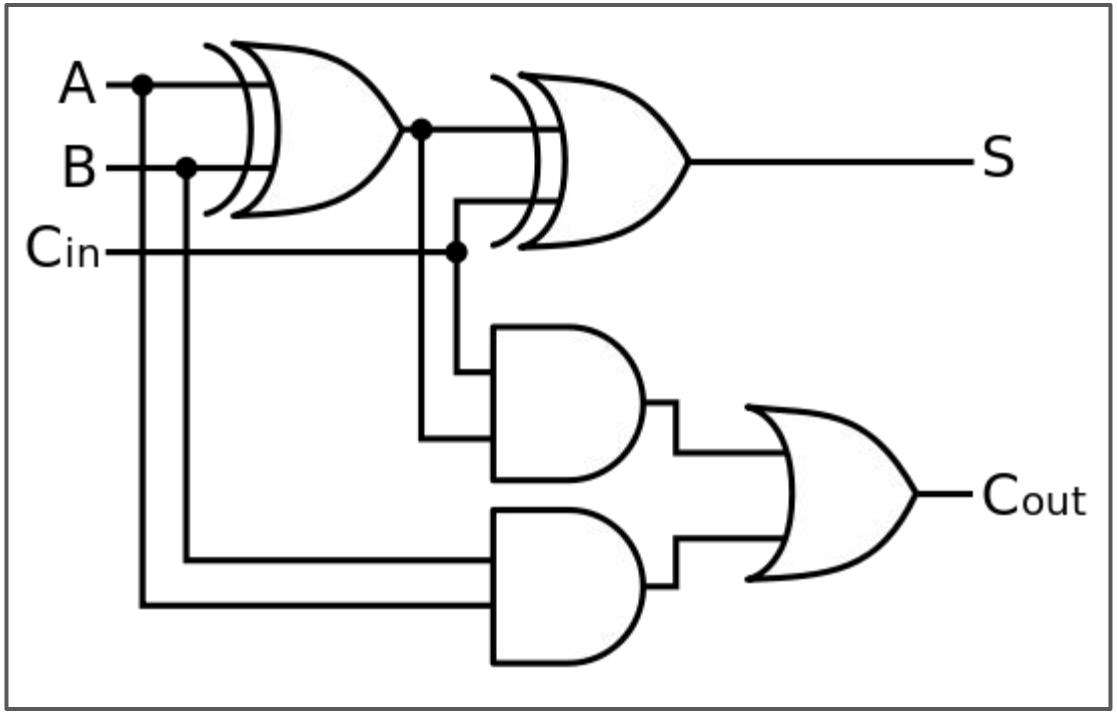
Full Adder:

A	B	C_{IN}			SUM	C_{OUT}

Full Adder Circuit Diagram

Full Adder Circuit Diagram

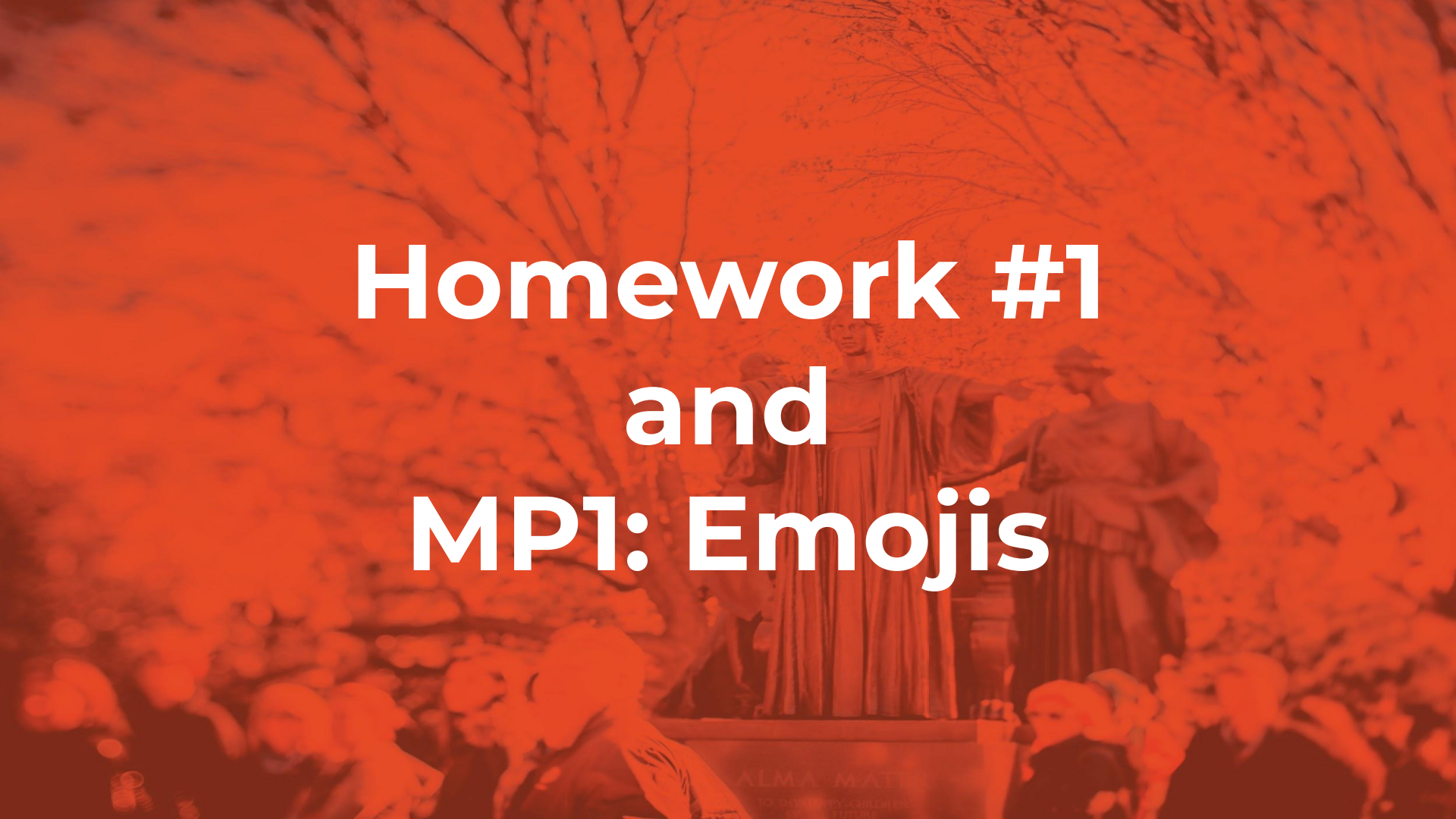
...or, with only "simple gates":



What more do we need?

Ripple Carry Adder (RCA)

Disadvantages



Homework #1
and
MP1: Emojis