Data Structures

Graph Implementations 2

CS 225

April 1, 2026

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Department of Computer Science

Exam 4 on 4/6 — 4/8

Autograded MC and one coding question

Manually graded short answer prompt

Practice exam releases this week on PL

Topics covered can be found on website

Register now!

https://courses.engr.illinois.edu/cs225/exams/

Lab this week is exam 4 review session

Like exam 2, some new questions will be released for lab

A great opportunity to go over practice exam too!

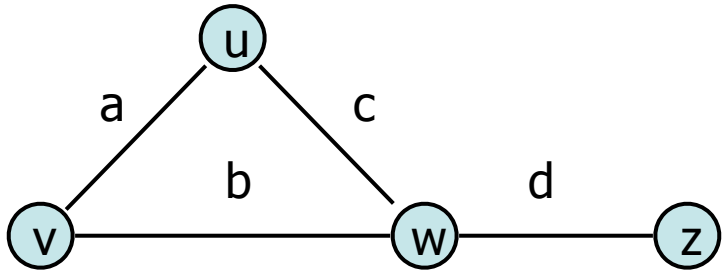
Form a study group to peer grade FRQs

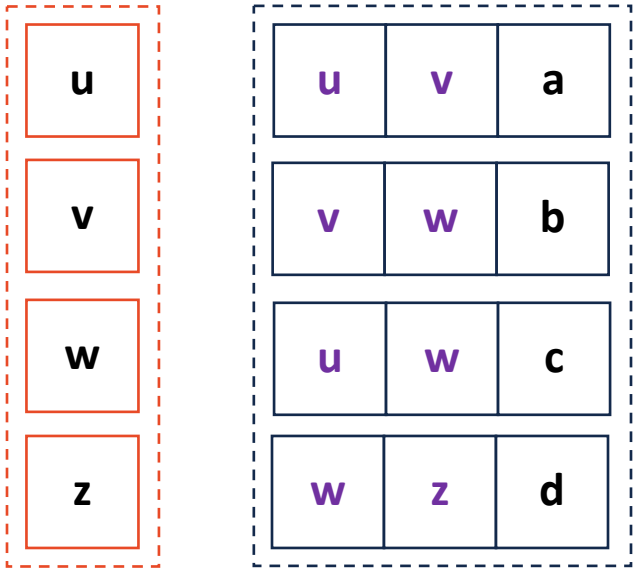
Learning Objectives

Discuss graph implementation and storage strategies

Introduce graph traversals

Graph Implementation: Edge List |V|= n, |E|= m





O(1)*

insertVertex(K key):

insertEdge(Vertex v1, Vertex v2, K key):

O(m)

removeVertex(Vertex v):

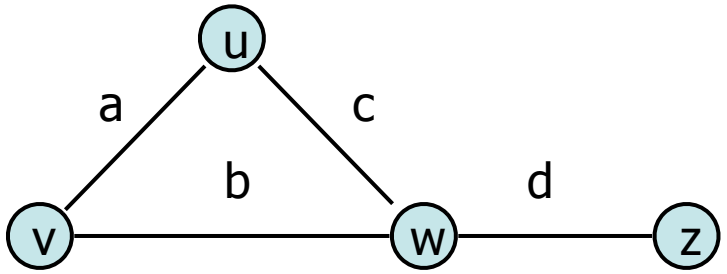
removeEdge(Vertex v1, Vertex v2, K key):

incidentEdges(Vertex v):

areAdjacent(Vertex v1, Vertex v2):

Graph Implementation: Adjacency Matrix

|V|= n, |E|= m



u	0
v	1
w	2
z	3

	0	1	2	3
0	-	a	c	0
1		-	b	0
2			-	d
3				-

Vertex Storage:

A hash table of vertices

Implicitly or explicitly store index

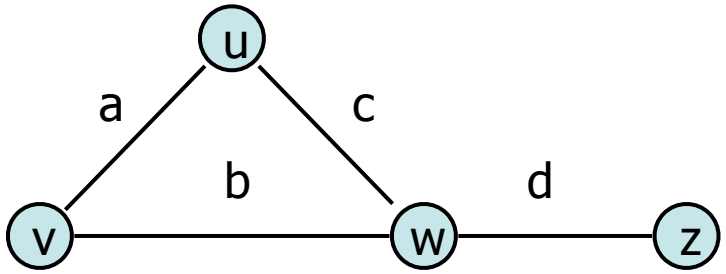
Edge Storage:

A |V| x |V| matrix of edges

Weight is stored at position (u, v)

Graph Implementation: Adjacency Matrix

|V|= n, |E|= m



removeVertex(Vertex v):

u	0
v	1
w	2
z	3

	0	1	2	3
0	-	a	c	0
1		-	b	0
2			-	d
3				-

Amortized Removal with Tombstoning

Remove an item by replacing its value or flipping a flag indicating 'deletion'

2	7	5	9	7	14	1	0	8	3

When there are enough deleted elements to merit resize, do it all at once!

u	0
v	1
w	2
z	3

	0	1	2	3
0	-	a	c	0
1		-	b	0
2			-	d
3				-

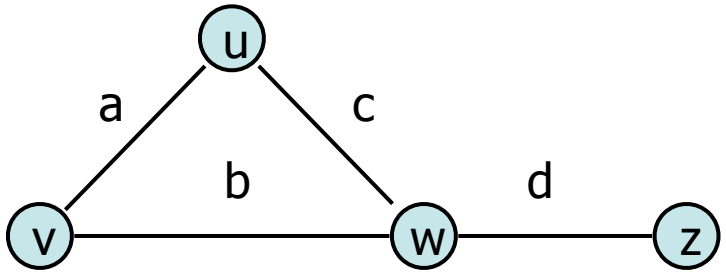
Adjacency Matrix:

removeVertex() by tombstoning |V| values

Resize when needed or by request

Graph Implementation: Adjacency Matrix

|V|= n, |E|= m



	u	v	w	z
u	-	a	c	0
v		-	b	0
w			-	d
z				-

u
v
w
z

O(1)

insertEdge(Vertex v1, Vertex v2, K key):

removeEdge(Vertex v1, Vertex v2, K key):

areAdjacent(Vertex v1, Vertex v2):

O(n)

incidentEdges(Vertex v):

O(n)—O(n^2)

insertVertex(K key):

removeVertex(Vertex v):

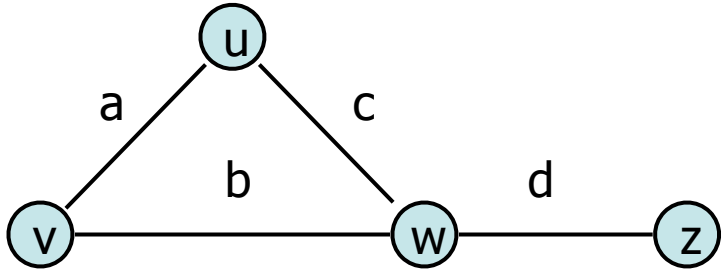
Graph Implementation Brainstorming

We want something…

Faster than an edge list

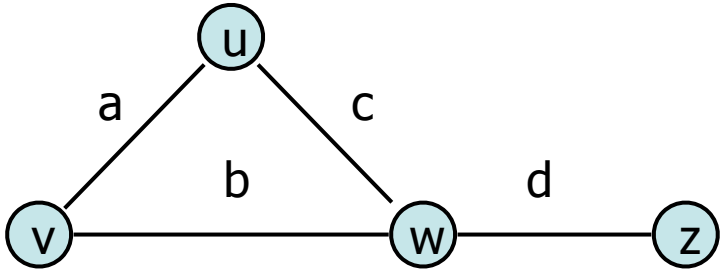
Less space than an adjacency matrix

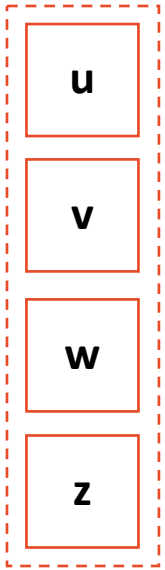
Particularly good at **finding adjacent elements**

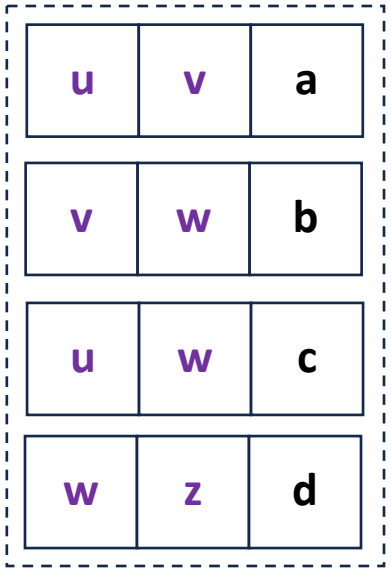


Graph Implementation: Edge List + ?

|V|= n,|E|= m

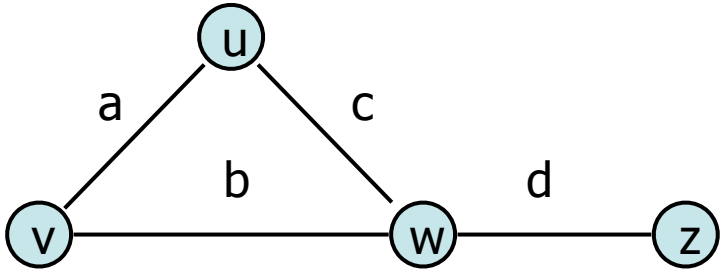


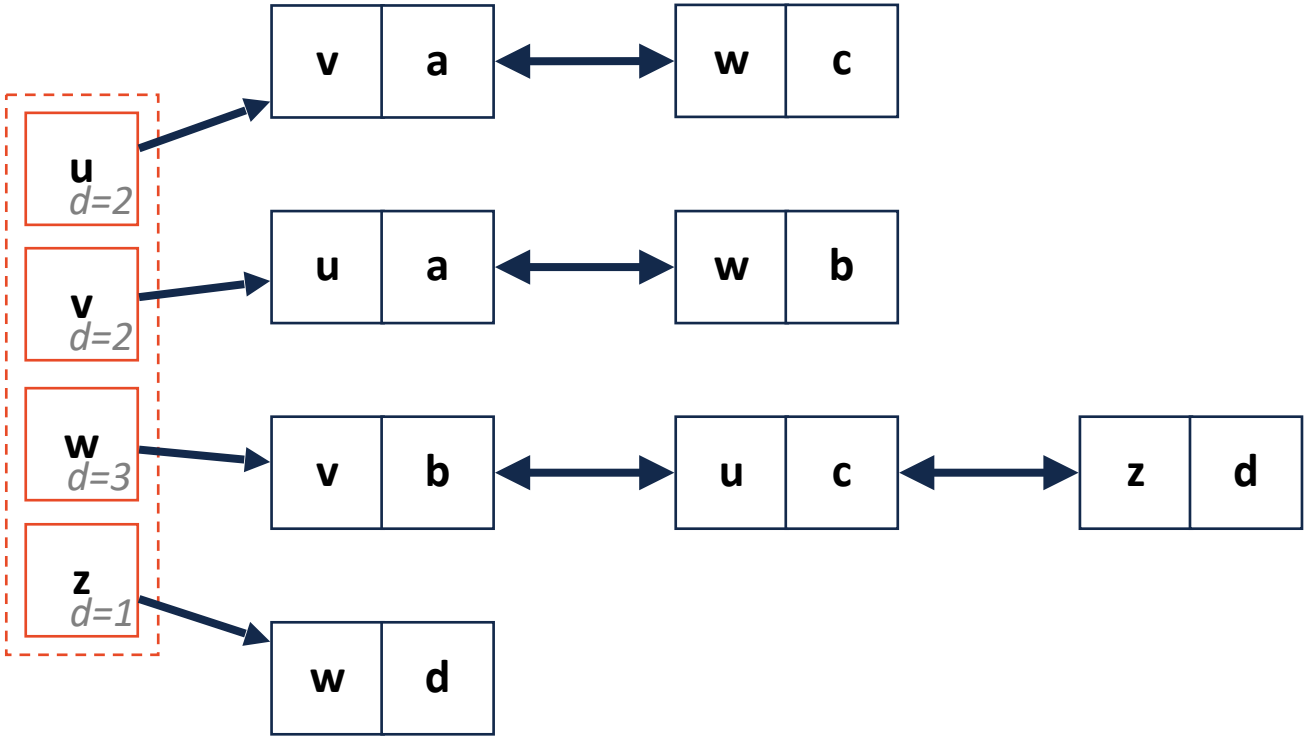




Naive Adjacency List

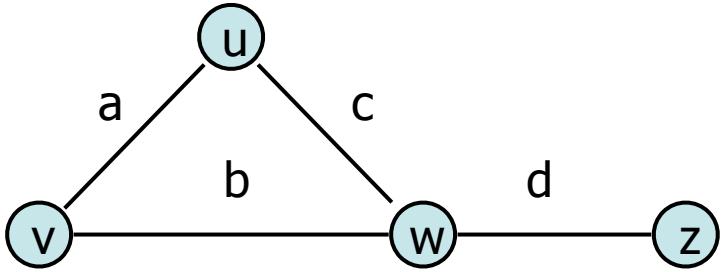
|V|= n,|E|= m



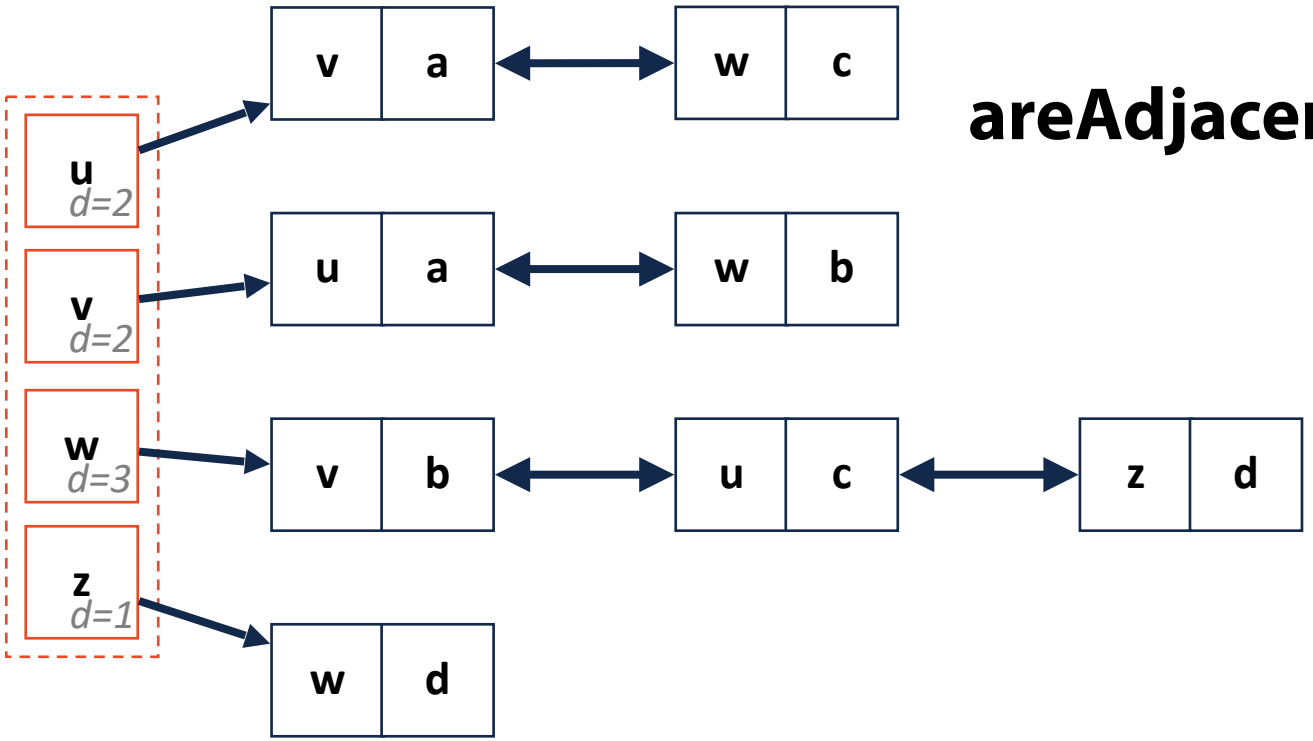


Naive Adjacency List

|V|= n, |E|= m



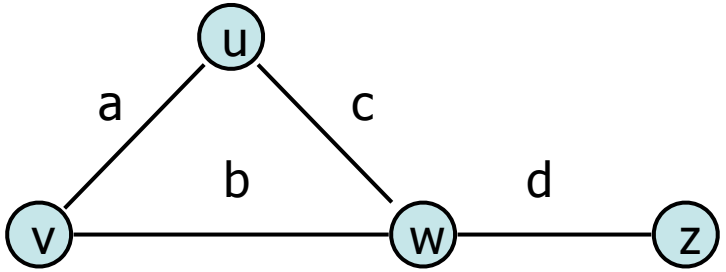
incidentEdges(Vertex v):

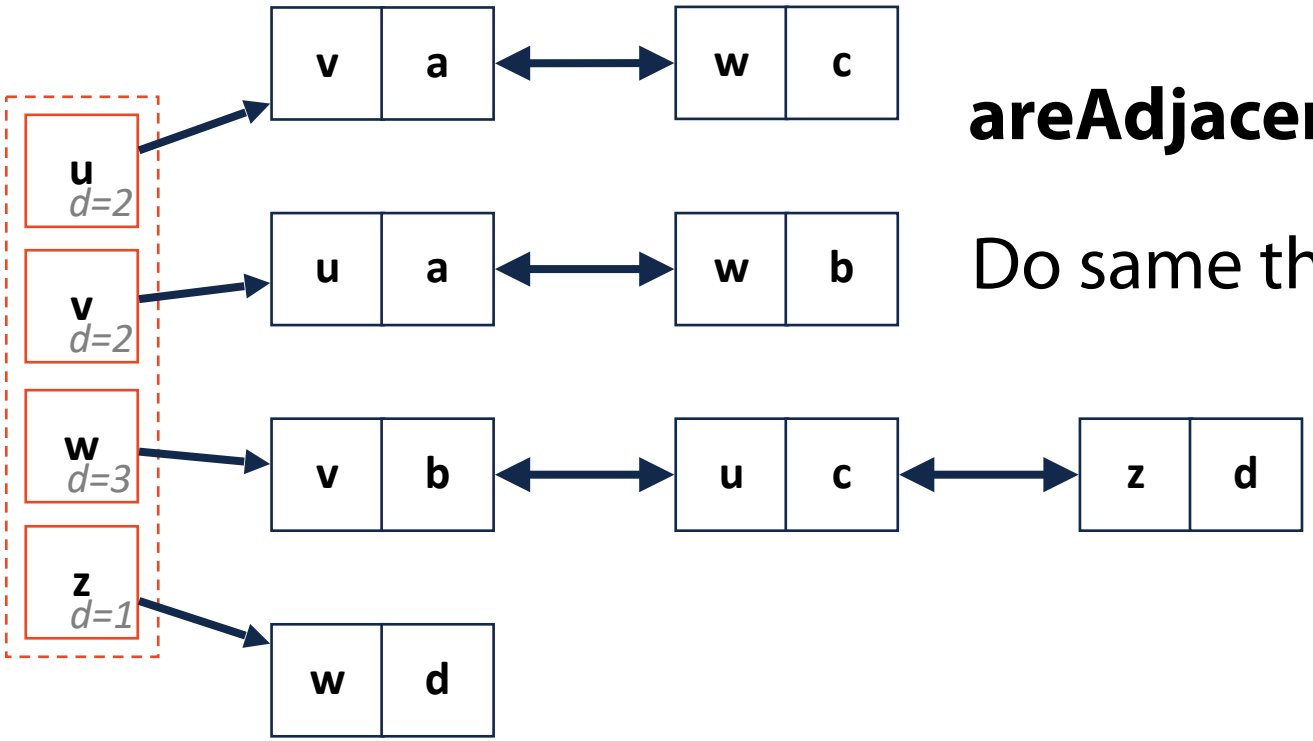


areAdjacent(Vertex v1, Vertex v2):

Naive Adjacency List

|V|= n, |E|= m





incidentEdges(Vertex v):

Lookup linked list for v

Walk down list! Every edge is incident.

areAdjacent(Vertex v1, Vertex v2):

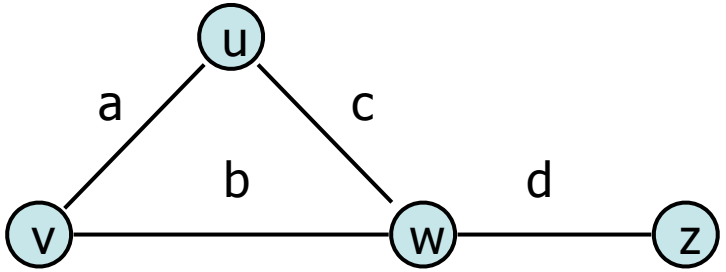
Do same thing but look for specific v2

Join Code: 225

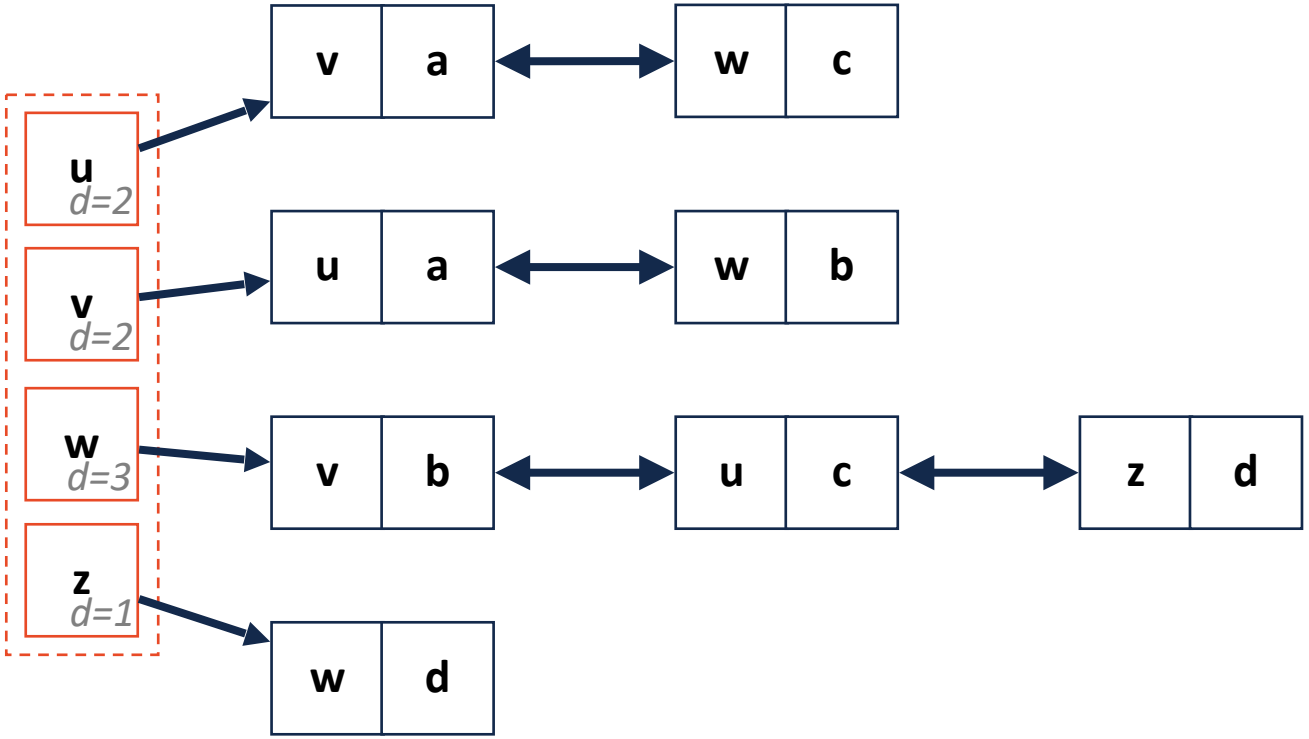
What are my Big Os?

Naive Adjacency List

|V|= n,|E|= m

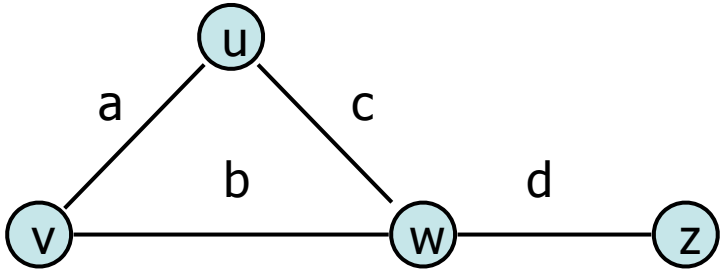


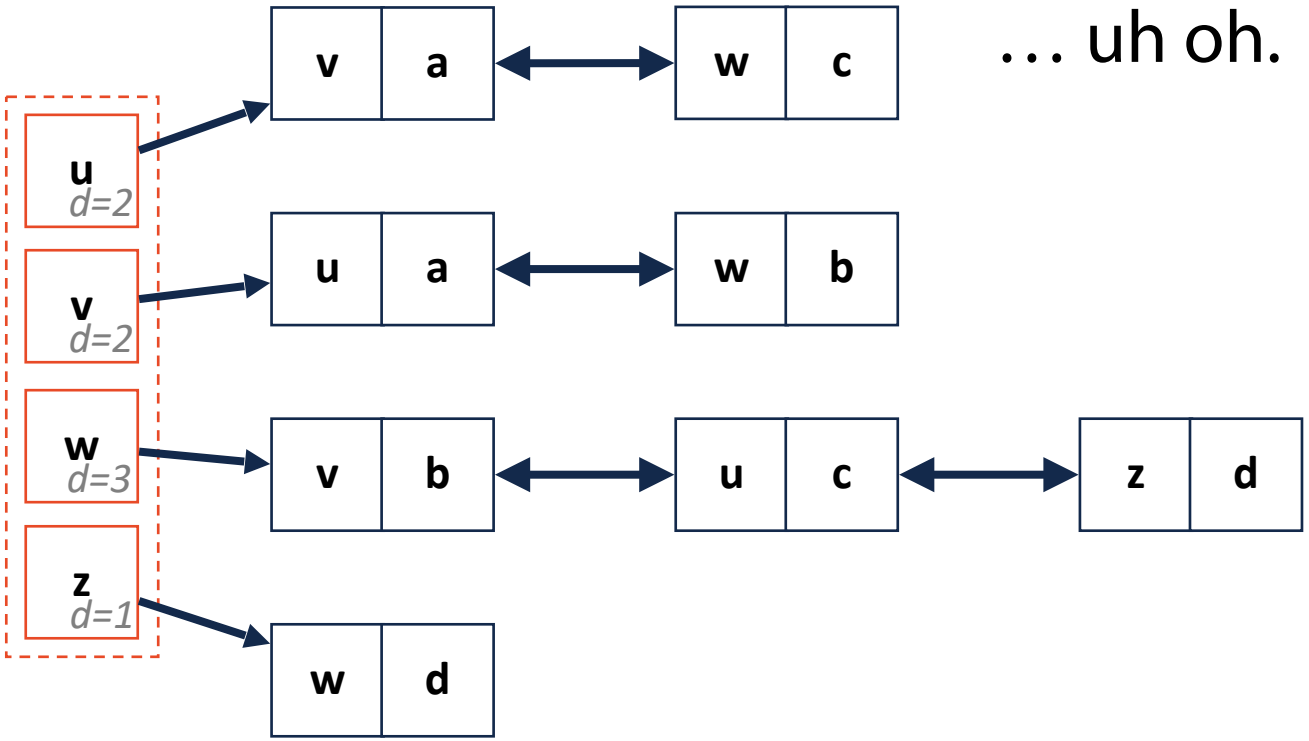
removeVertex(Vertex v):



Naive Adjacency List

|V|= n, |E|= m





removeVertex(Vertex v):

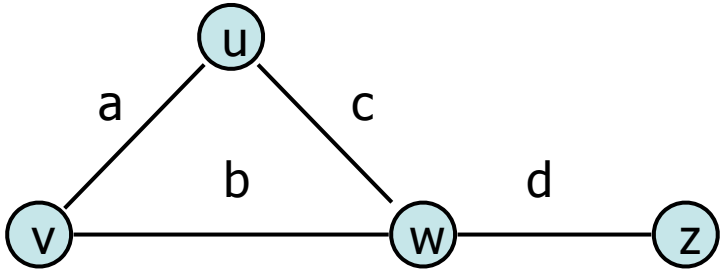
Look up V in our list and remove all entries.

… And then look for V in every other list

… uh oh.

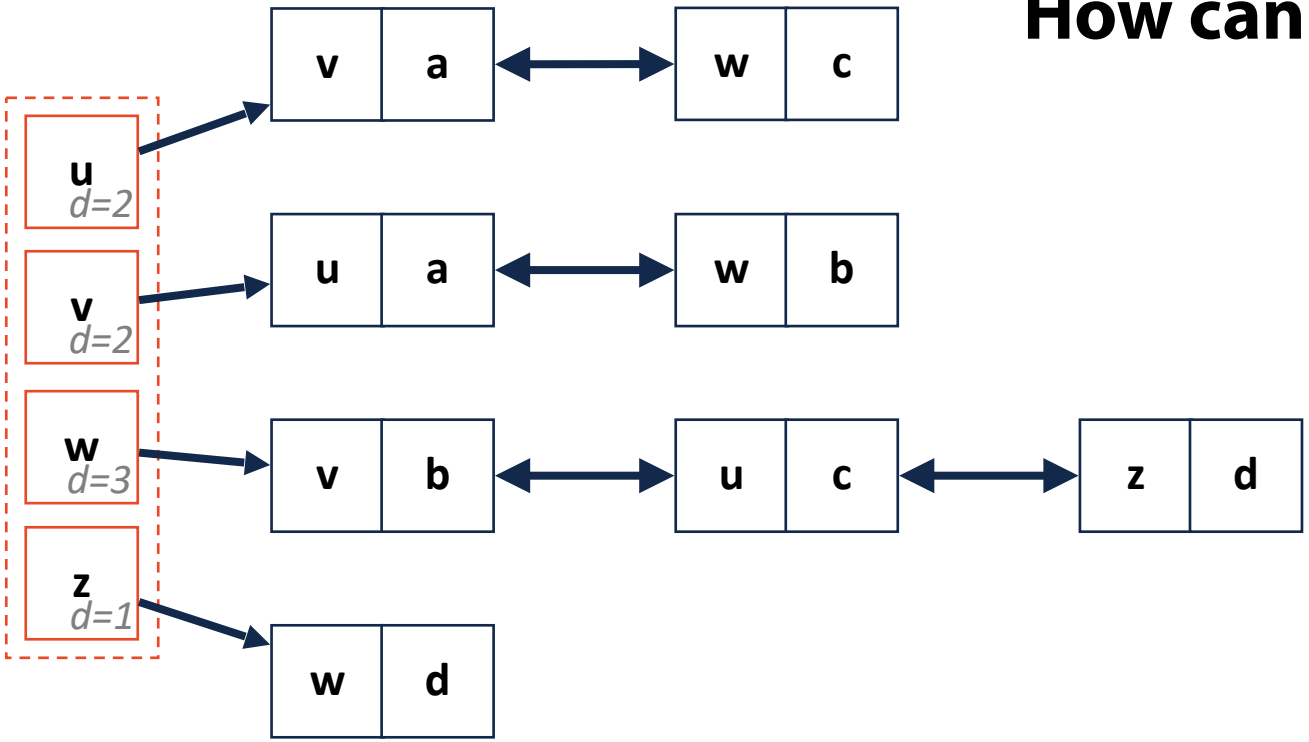
Naive Adjacency List

|V|= n,|E|= m



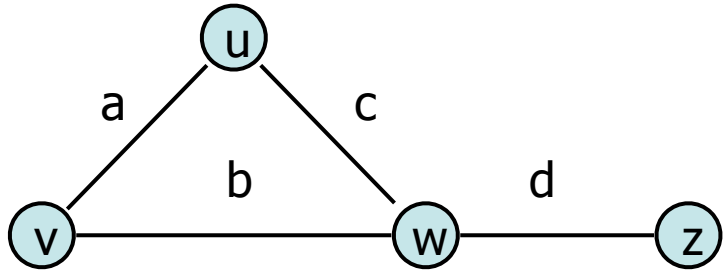
What's wrong with our implementation?

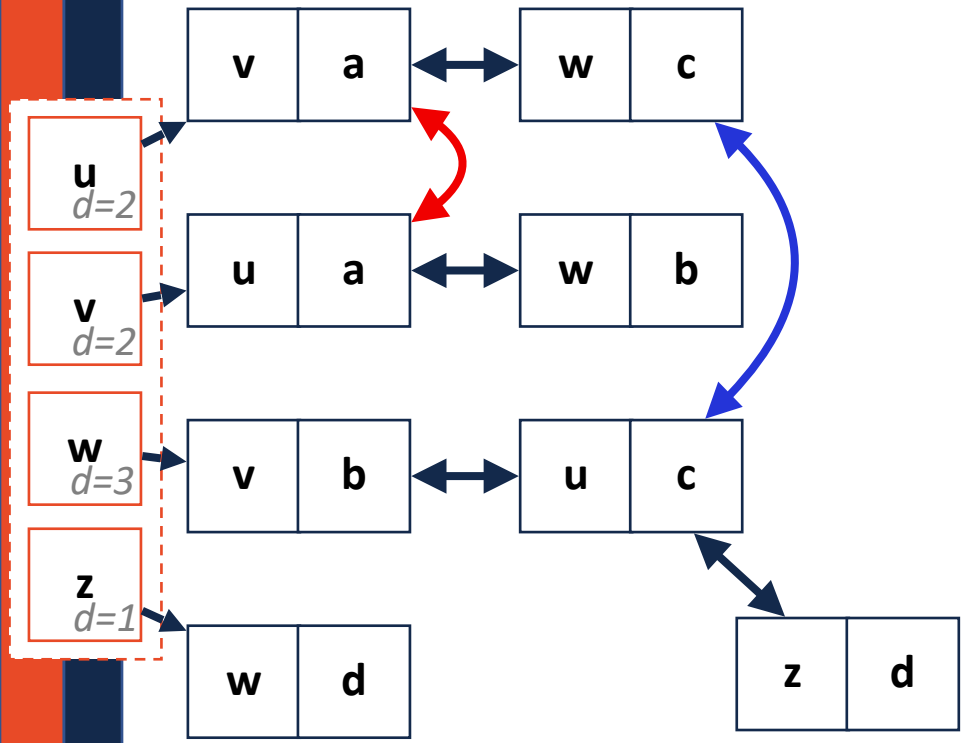
How can we fix it?



Adjacency List

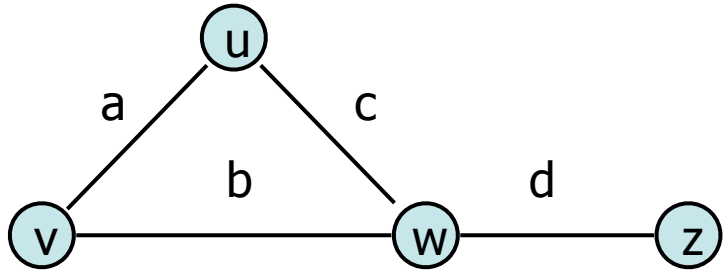
|V|= n, |E|= m

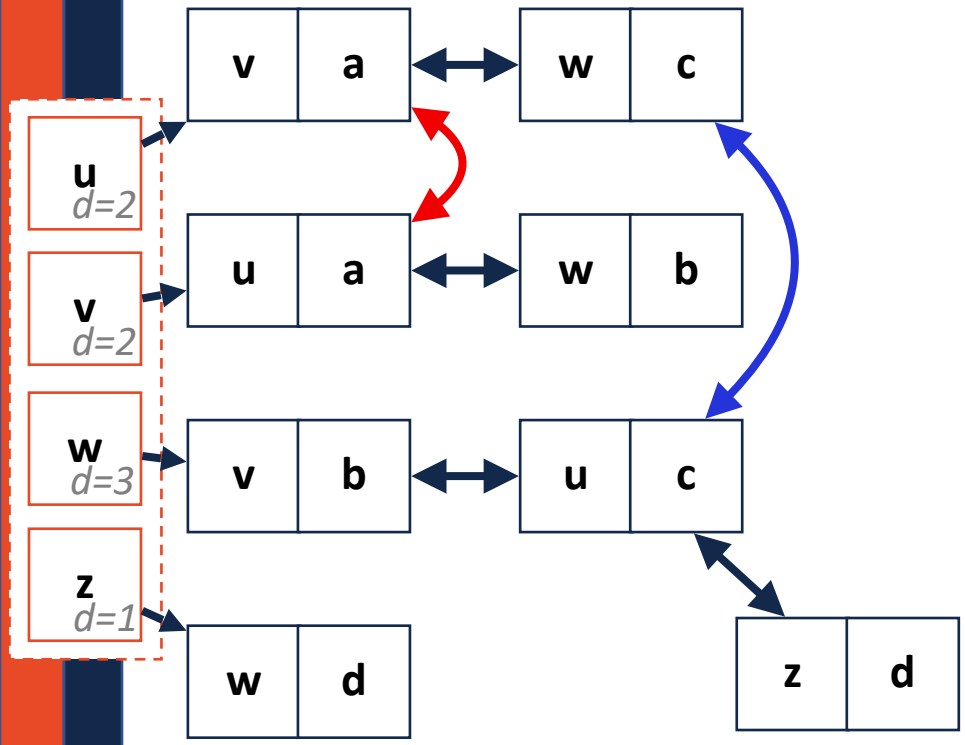


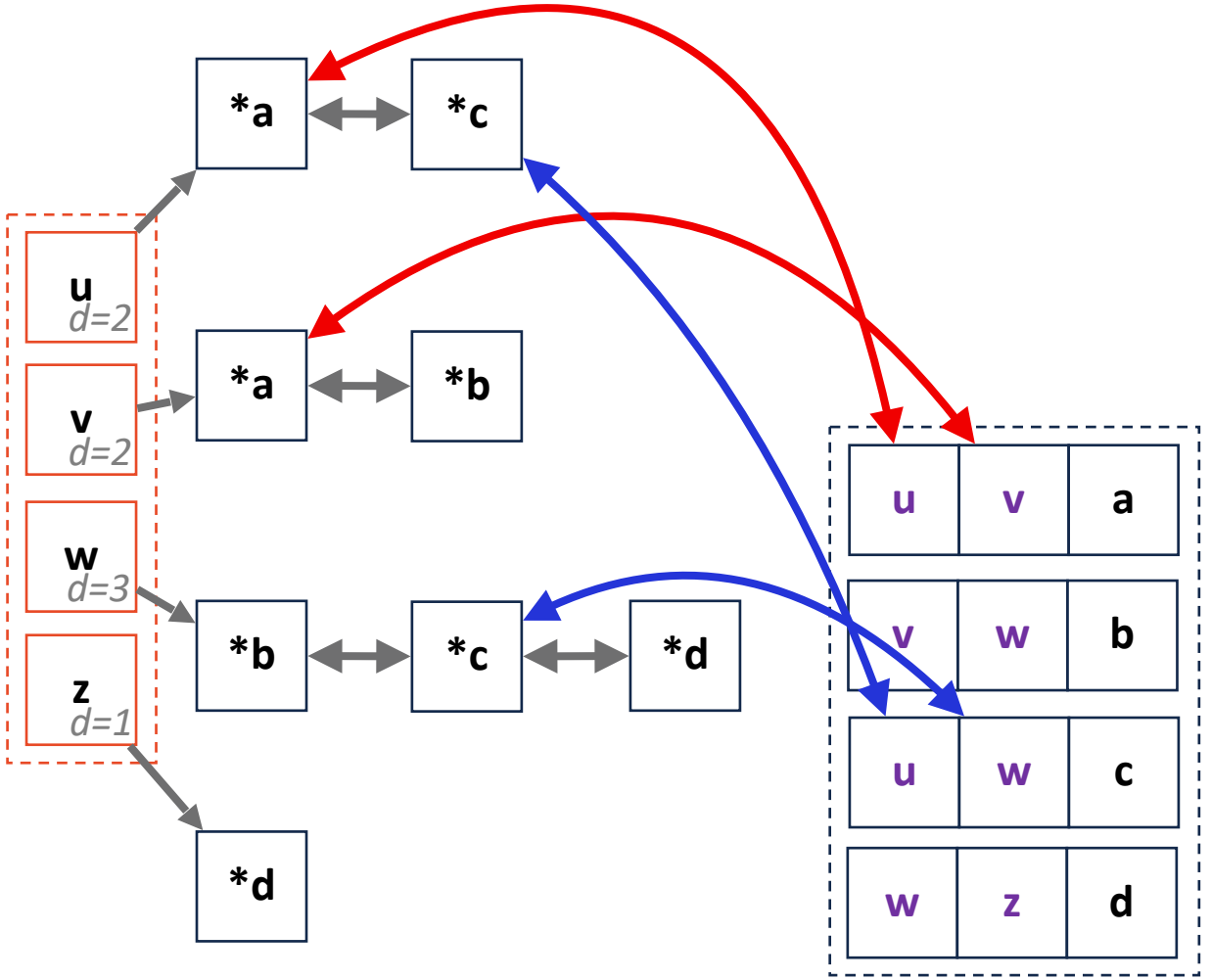


Adjacency List

|V|= n, |E|= m







Adjacency List

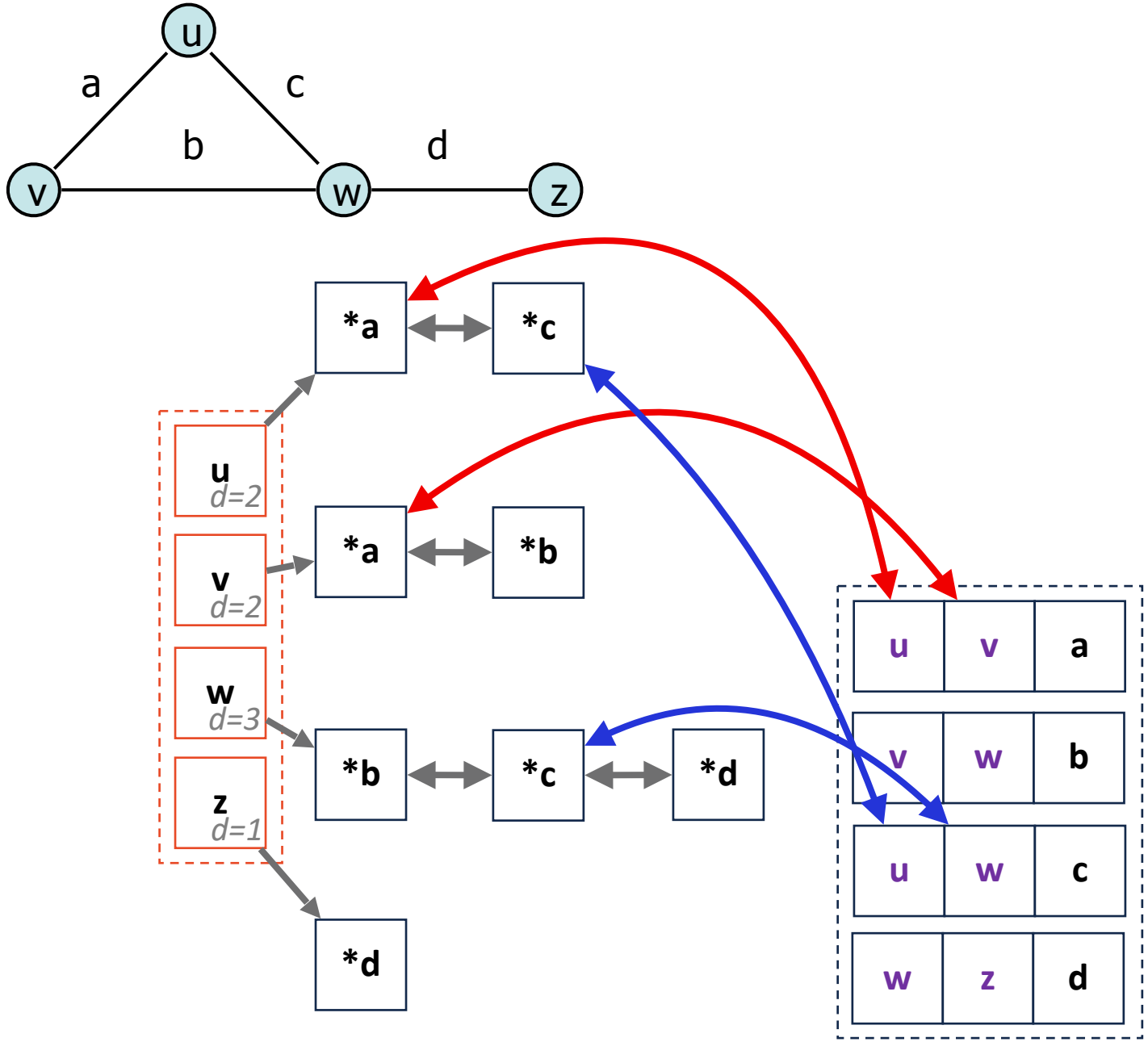
|V|= n, |E|= m

Adj List Node:

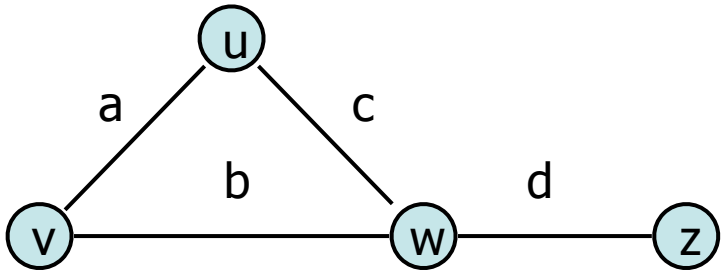
*Prev	*Edge	*Next

Edge List:

V1	V2	Weight
*V1	*V2	



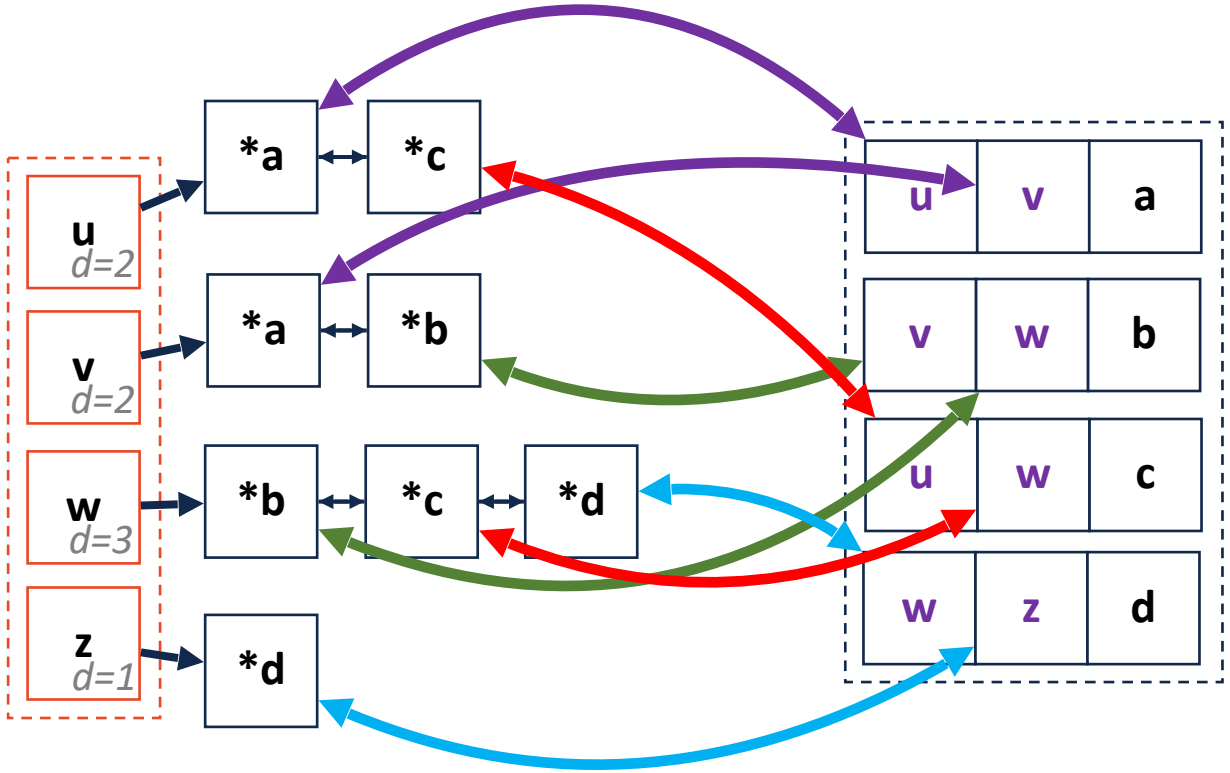
Adjacency List



Vertex Storage:

A bidirectional linked list with size variable

Each node is a pointer to edge in edge list

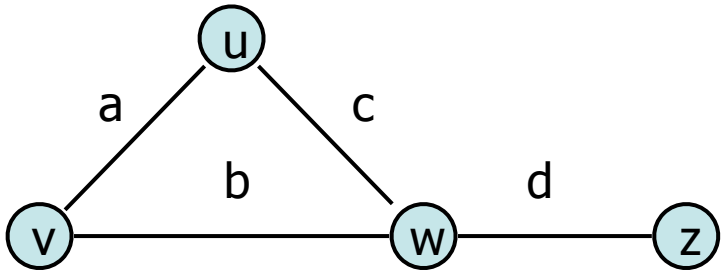


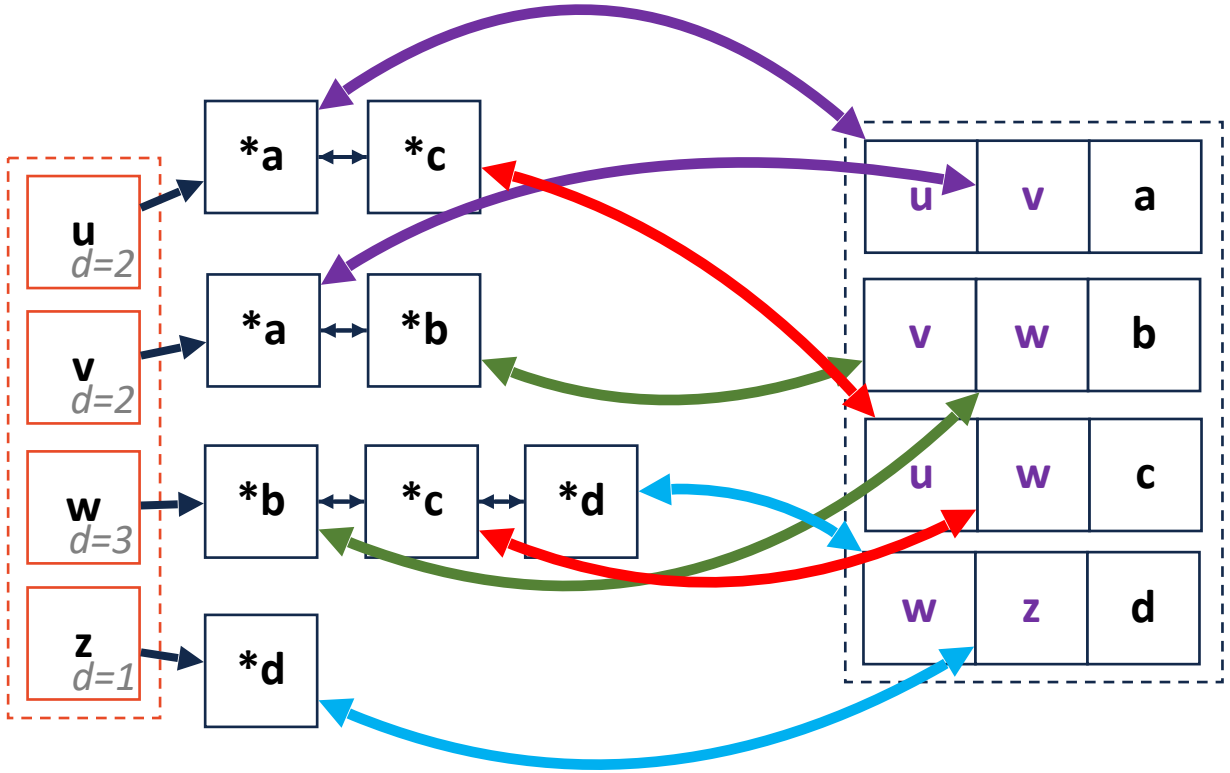
Edge Storage:

A list of (v1, v2, weight) edges

Also store pointers back to nodes

Adjacency List

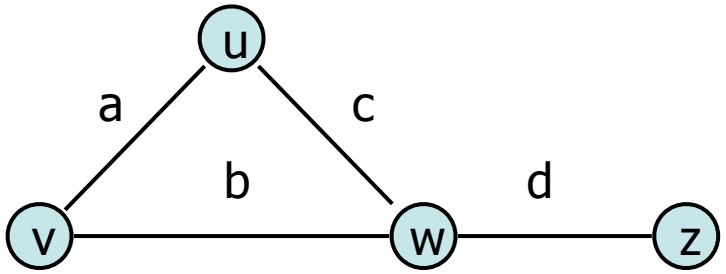




incidentEdges(Vertex v):

Look up vertex list (and walk across it)

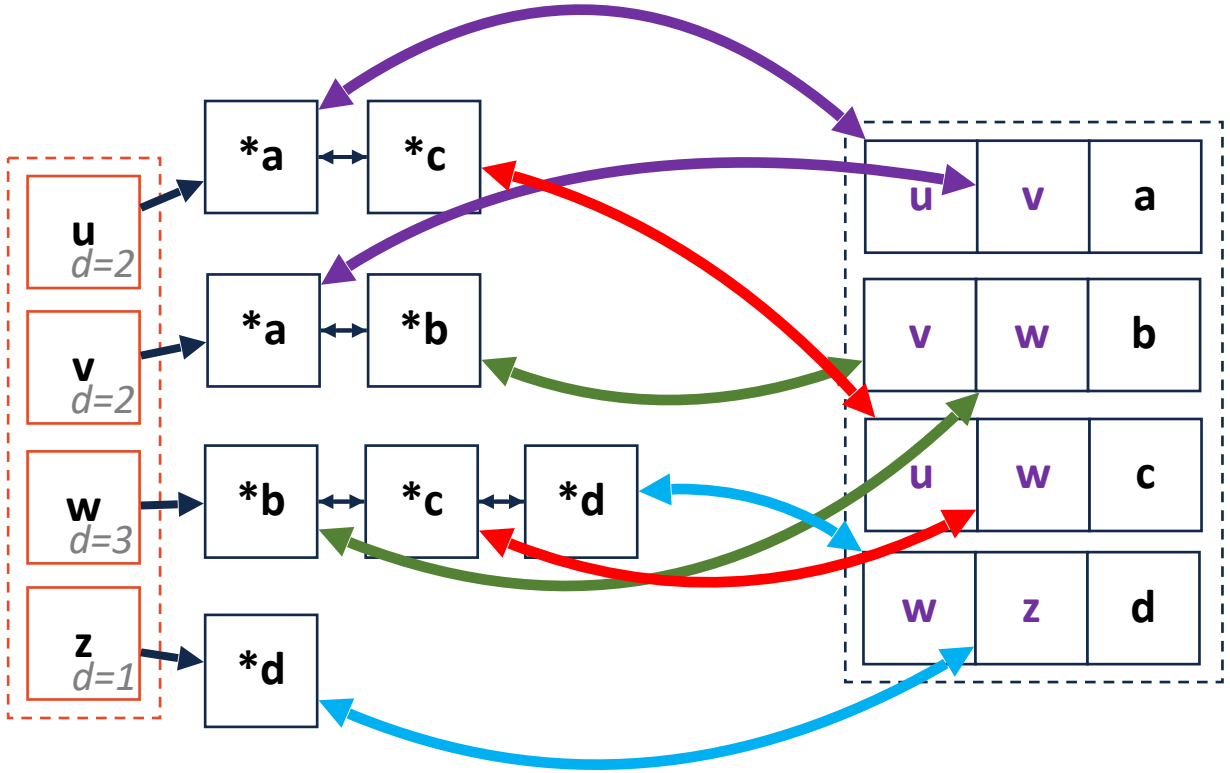
Adjacency List



areAdjacent(Vertex v1, Vertex v2):

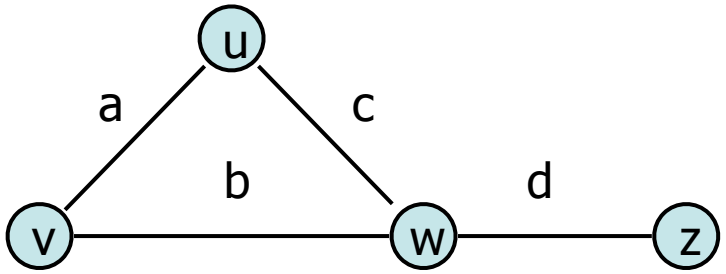
Look up min-degree vertex list

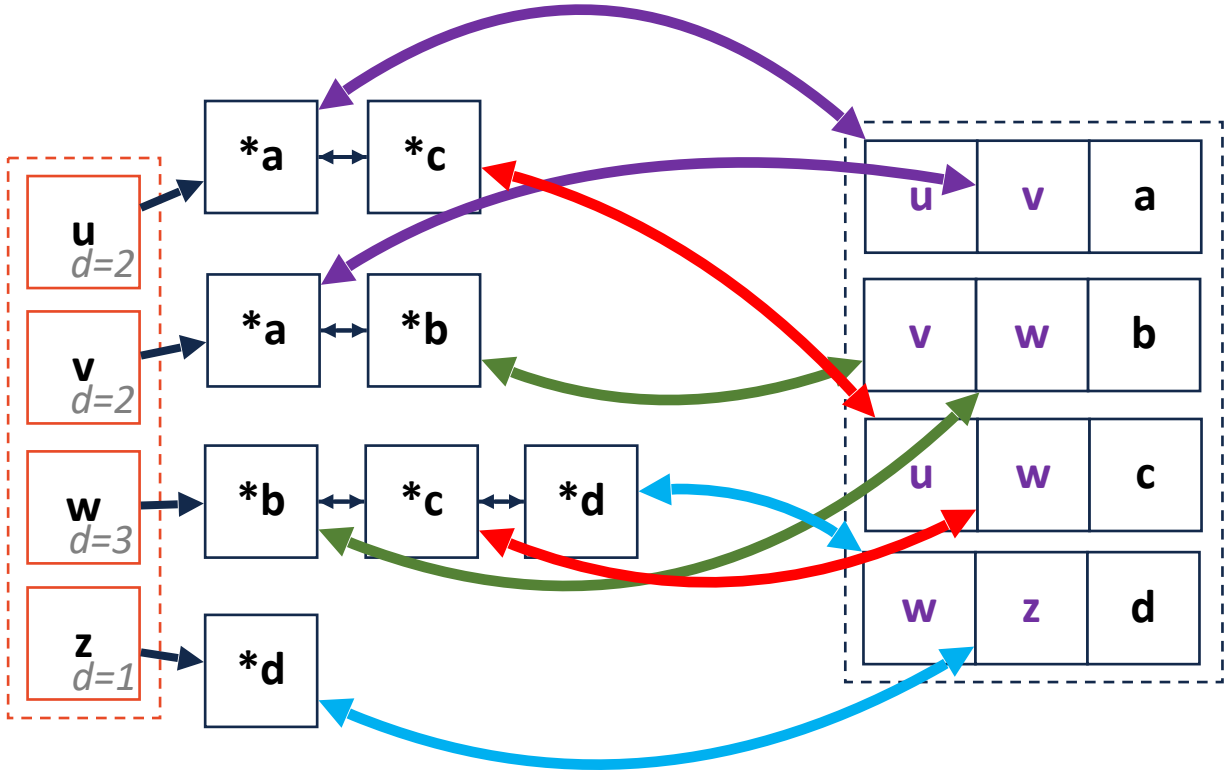
Search for other vertex across list



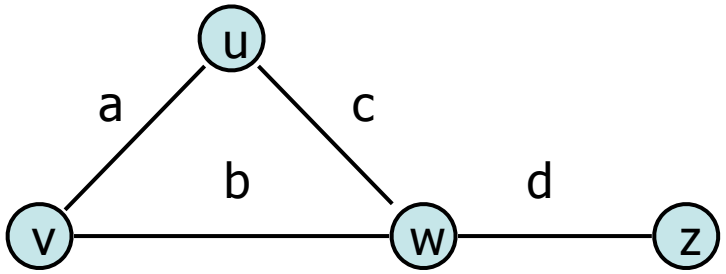
Adjacency List

insertEdge(Vertex v1, Vertex v2, K key):





Adjacency List

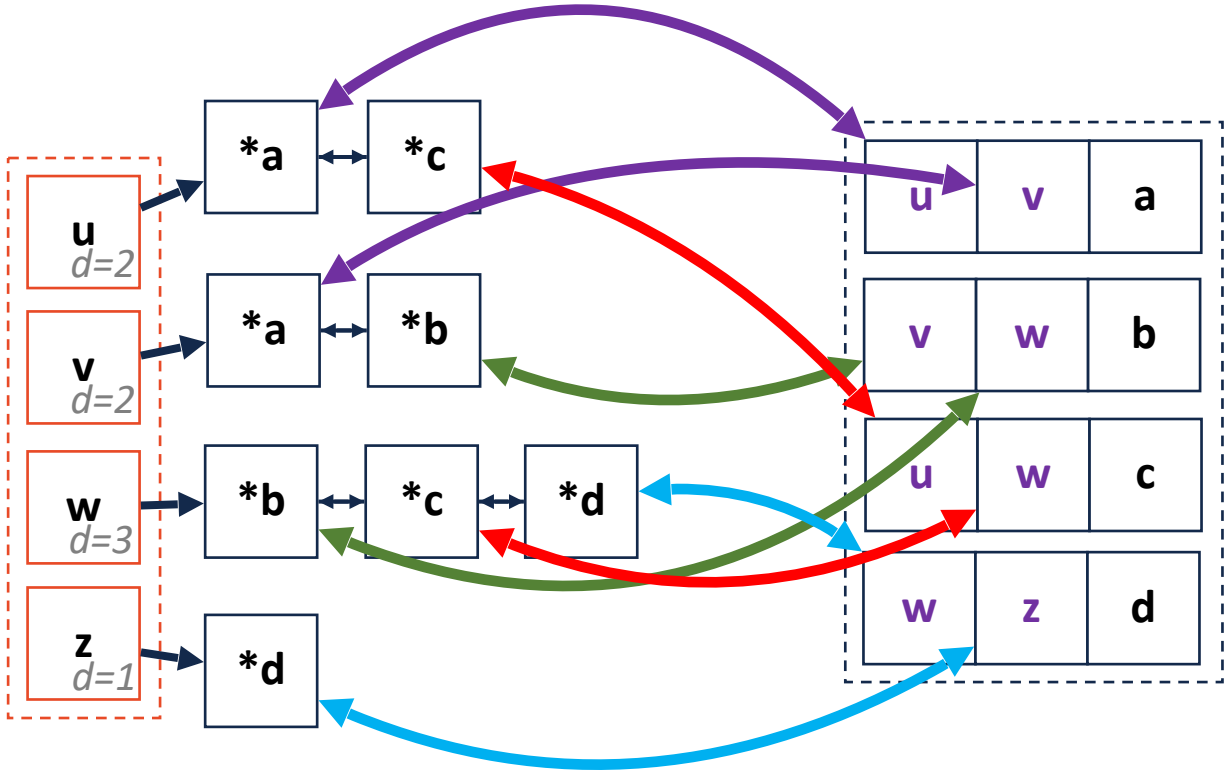


insertEdge(Vertex v1, Vertex v2, K key):

Add edge to edge list

Add node to each vertex list

Connect all three with pointers

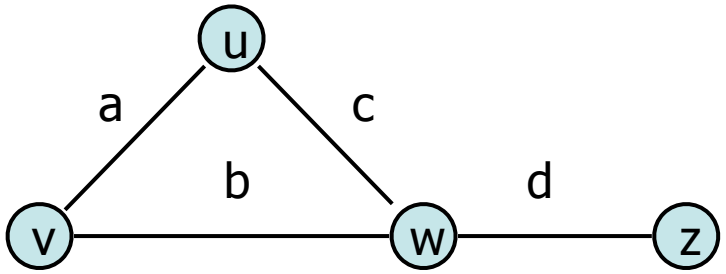


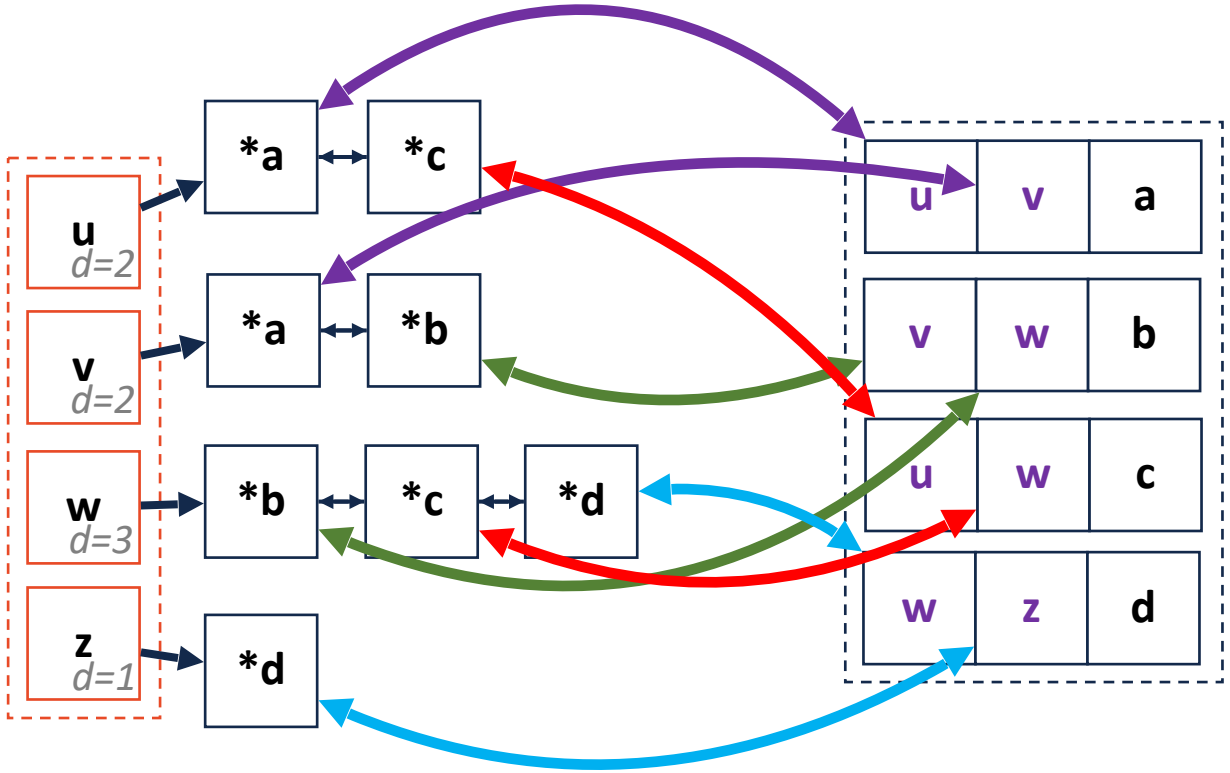
Join Code: 225

What are my Big Os?

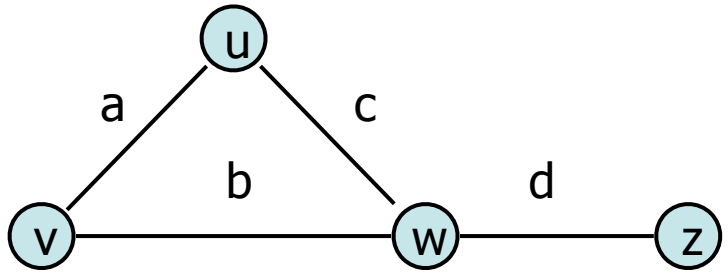
Adjacency List

removeEdge(Vertex v1, Vertex v2, K key):





Adjacency List



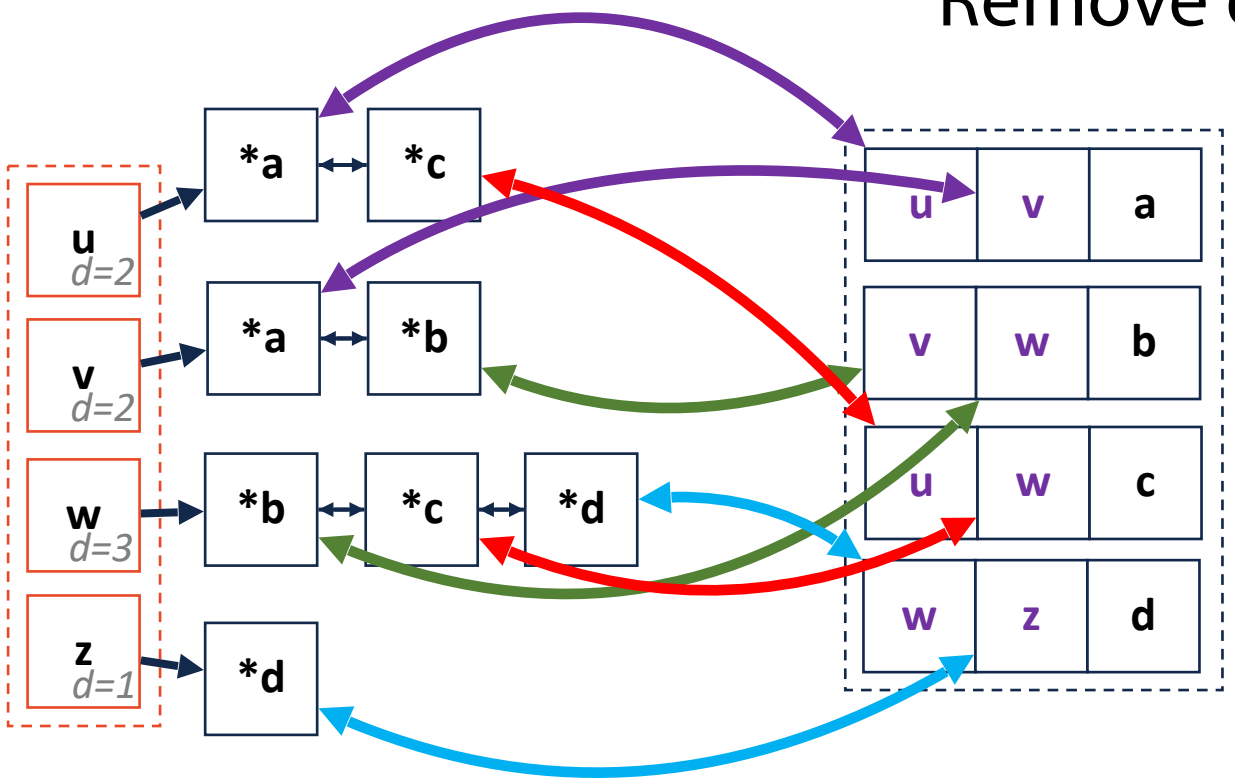
removeEdge(Vertex v1, Vertex v2, K key):

Search min-degree vertex list

Remove mirrored entry using pointers

Remove edge from edge list

Remove entry from vertex list

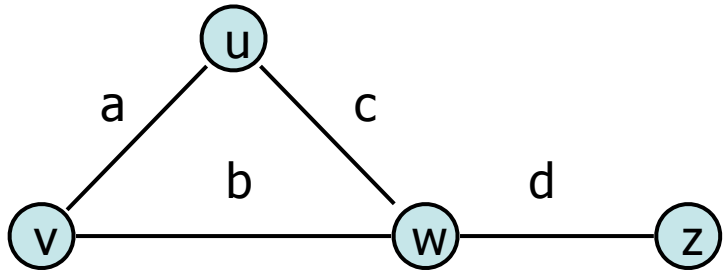


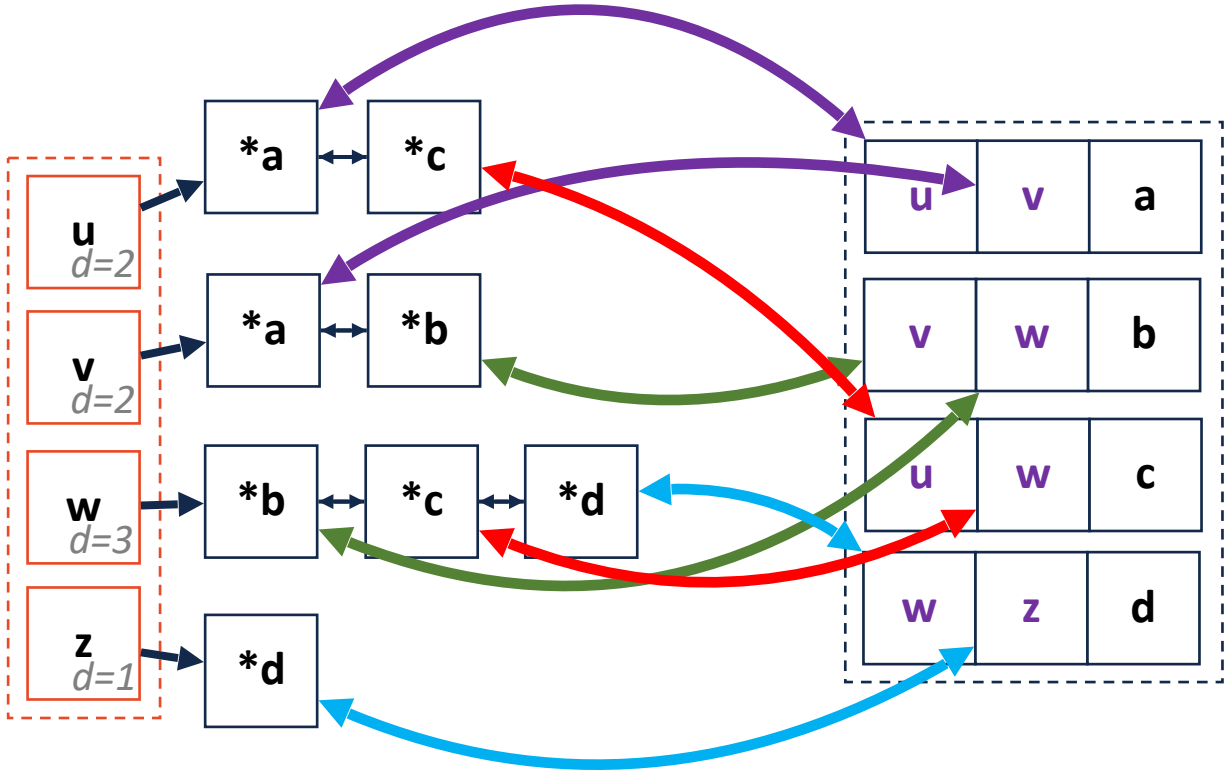
Join Code: 225

What are my Big Os?

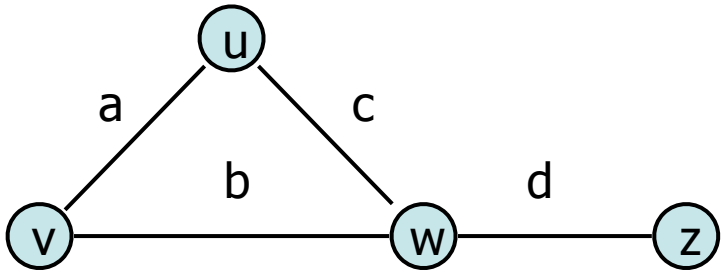
Adjacency List

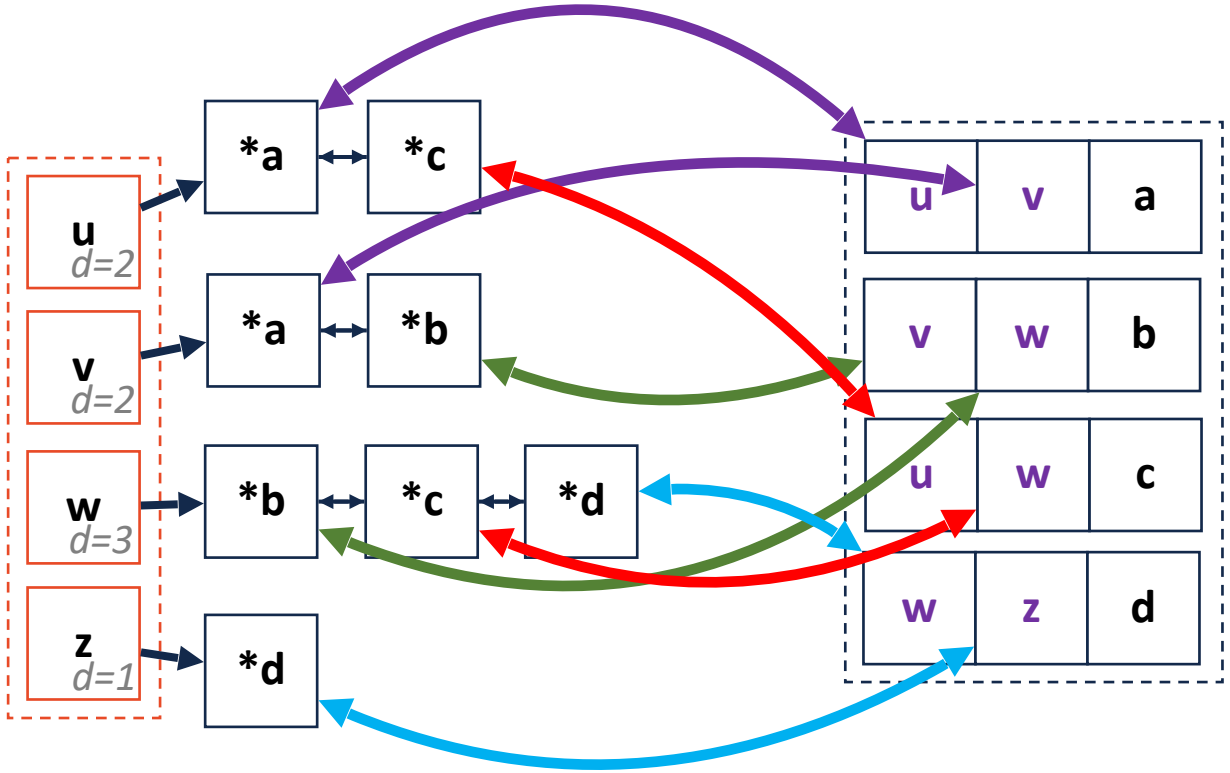
insertVertex(K key):





Adjacency List

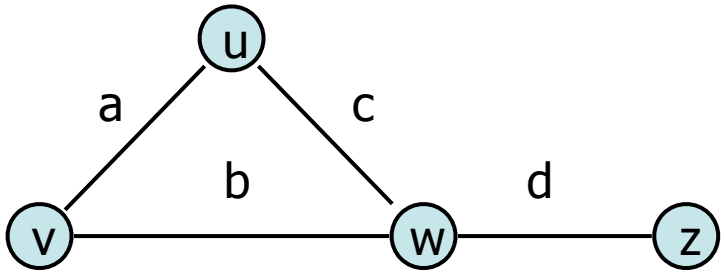


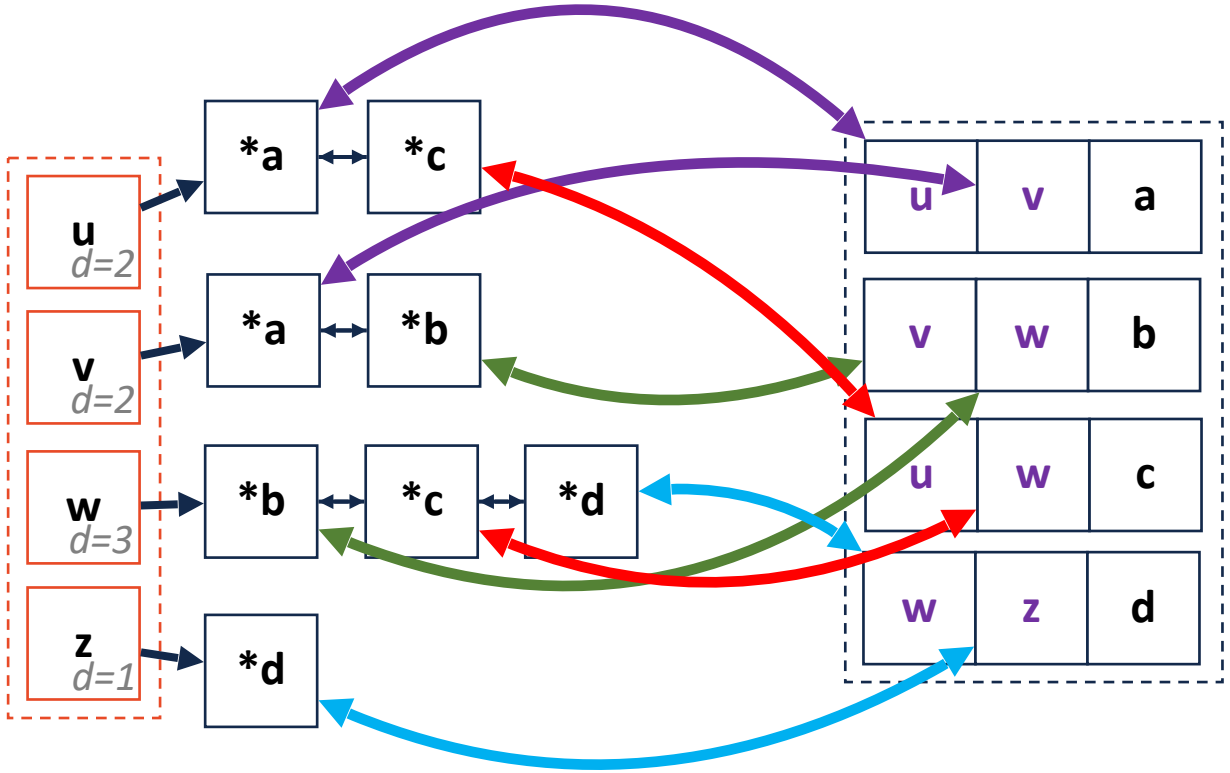


insertVertex(K key):

Add new empty list to vertex list.

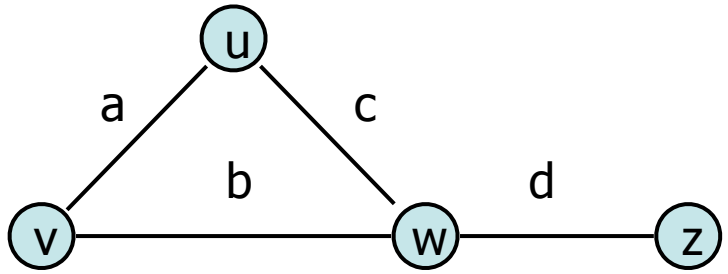
Adjacency List

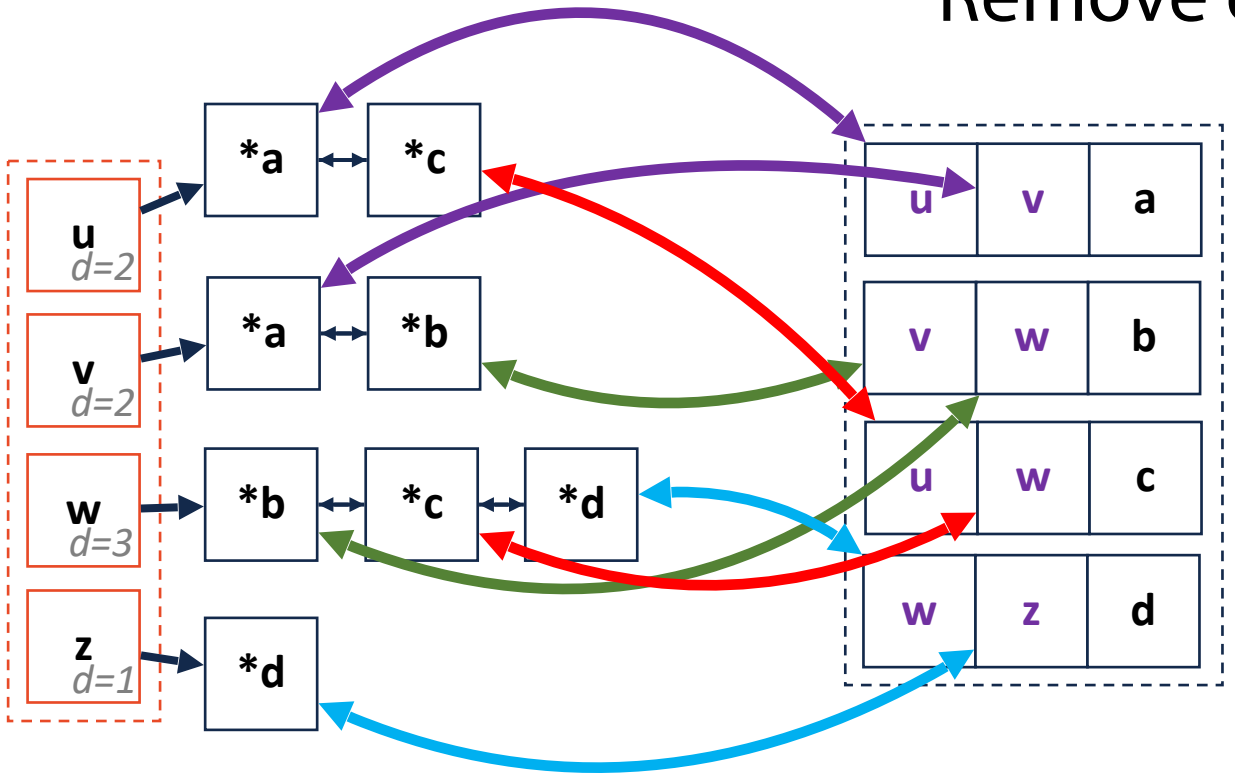




removeVertex(Vertex v):

Adjacency List





removeVertex(Vertex v):

Look up vertex in vertex list

Walk across list

Remove mirrored entry using pointers

Remove edge from edge list

Remove entry from vertex list

Join Code: 225

What are my Big Os?

|V|= n, |E|= m

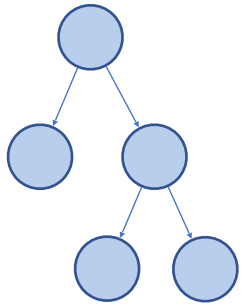
Expressed as O(f)	Edge List	Adjacency Matrix	Adjacency List
Space	n+m	n^2	n+m
insertVertex(v)	1*	n*	1*
removeVertex(v)	n+m	n	deg(v)
insertEdge(u, v)	1	1	1*
removeEdge(u, v)	m	1	min(deg(u), deg(v))
incidentEdges(v)	m	n	deg(v)
areAdjacent(u, v)	m	1	min(deg(u), deg(v))

Graph Traversals

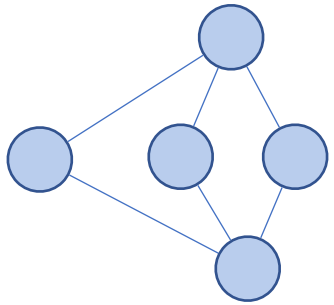
There is no clear order in a graph (even less than a tree!)

How can we systematically go through a complex graph in the fewest steps?

Tree traversals won't work — lets compare:



- Rooted
- Acyclic
-



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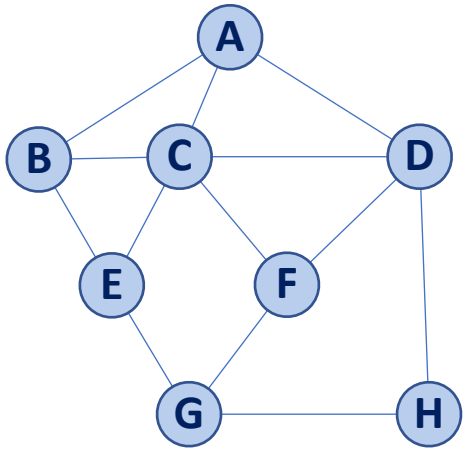
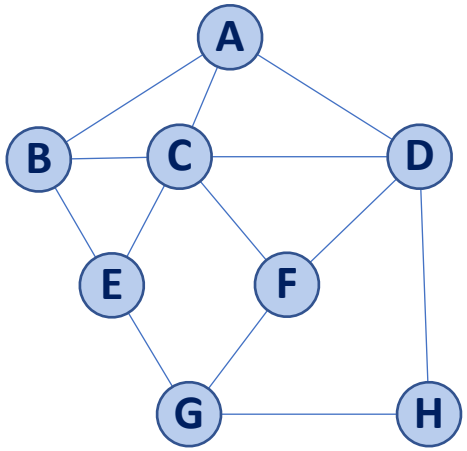
Traversal: BFS



Traversal: BFS



v	d	P	Adjacent Edges
A			B C D
B			A C E
C			A B D E F
D			A C F H
E			B C G
F			C D G
G			E F H
H			D G