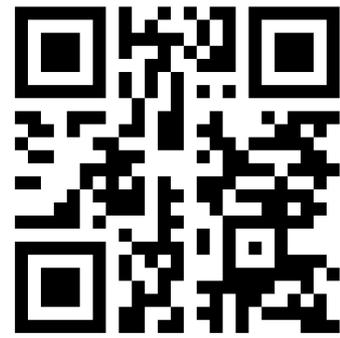


Announcements



Join Code: **225**

Thanks for being a great class!

EC Survey is closed
↳ Everyone gets 2pts EC!



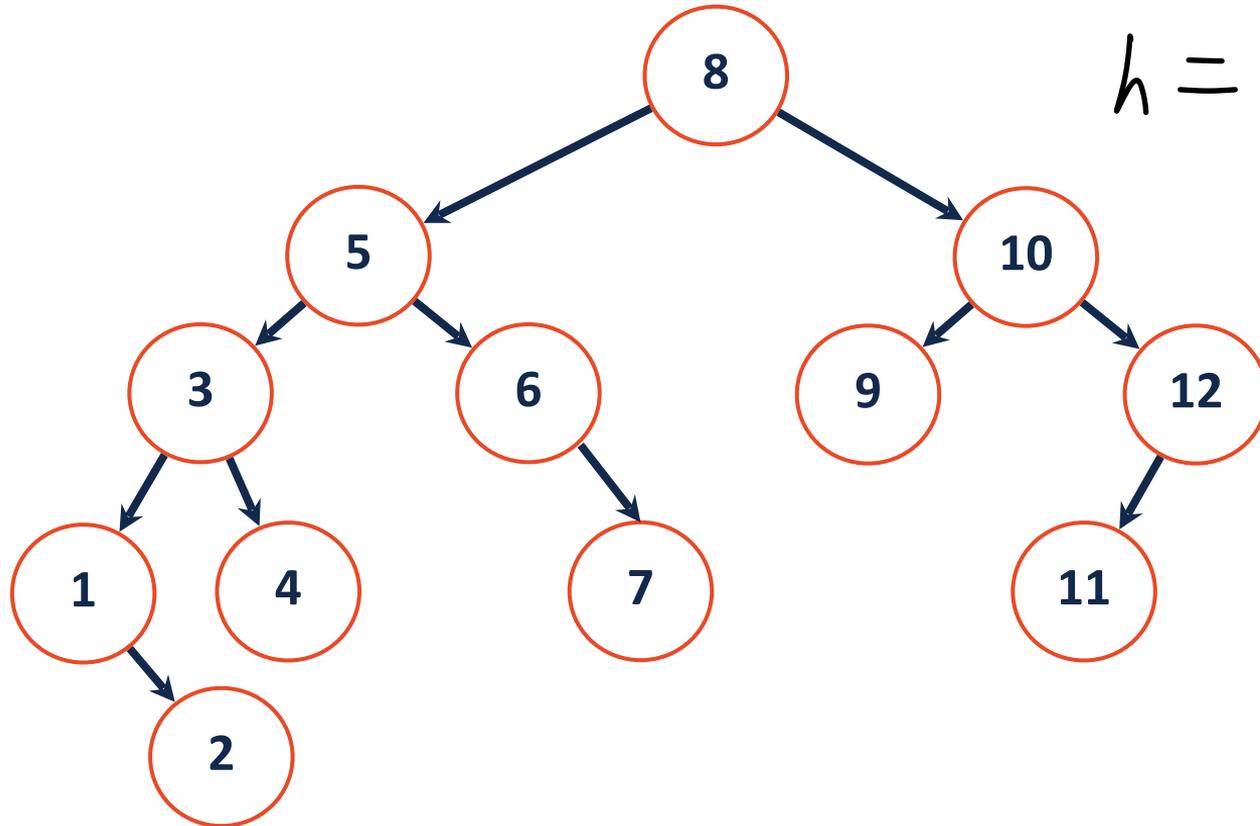
BTree Introduction

Learning Objectives

1. Understand the motivation behind BTrees
2. Know all the BTree Properties
3. Understand how to insert elements into a BTree
4. Implement Search in a BTree



AVL Trees



$$h = O(\log n)$$



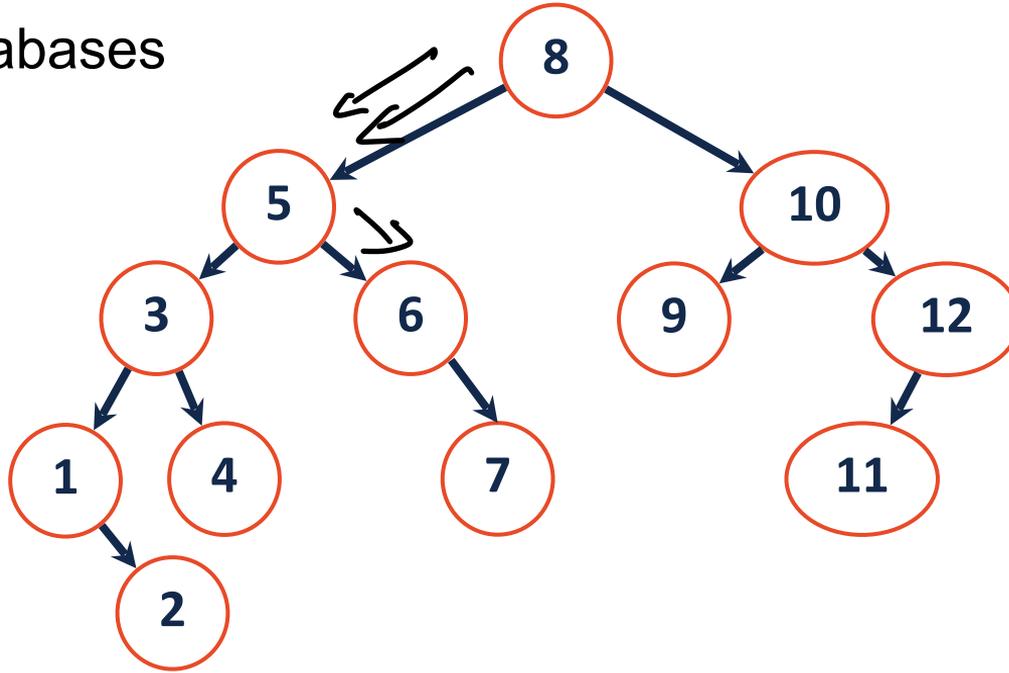
Rate Limiting Factor - Seeks

Distributed Databases

Hard Disks

Goal

- Shorter Tree
- More Data at each node



System Design

Goal: Minimize the number of reads! ^{seeks}

Build a tree that uses _____ / node

Ex. 1 billion integers

integer - 4 bytes

$$\frac{4096 \text{ bytes/node}}{4 \text{ bytes/integer}} = 1024 \frac{\text{int}}{\text{node}}$$

1. Distributed Database - [1 network packet]

a. 1500 Bytes (1 MTU) (Maximum Transmissible Unit) $\log_{1024}(10^9) \sim 3$

2. Hard Disk - [1 disk block]

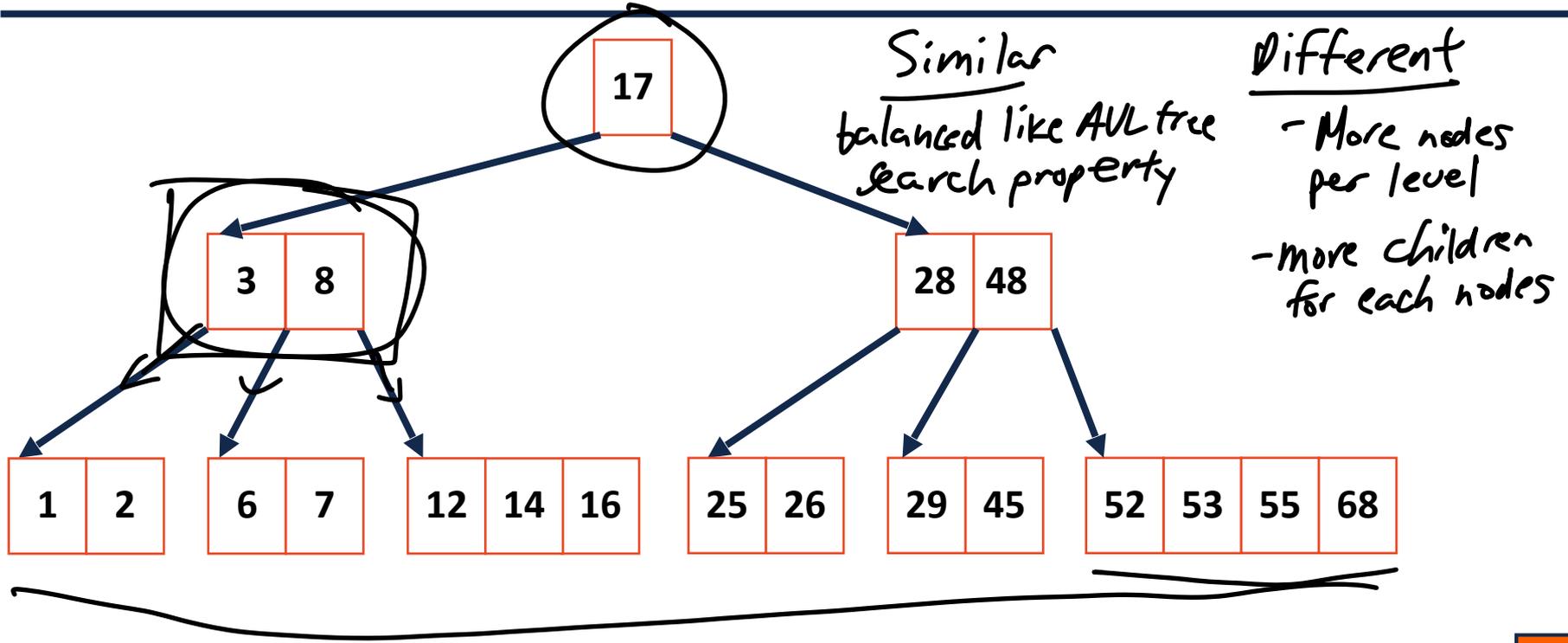
a. Old Systems 512 Bytes

b. New Systems 4096 Bytes

$$\log_2(10^9) \sim 30$$



What are some observations about B-Trees?



BTree Properties

A **BTree** of order m is an m -way tree:

Definition

- All keys within a node are ordered
- All nodes contain no more than $m-1$ keys
- All internal nodes have **one more child than keys**
- All leaves are on the same level



B-Tree Operations

Insert

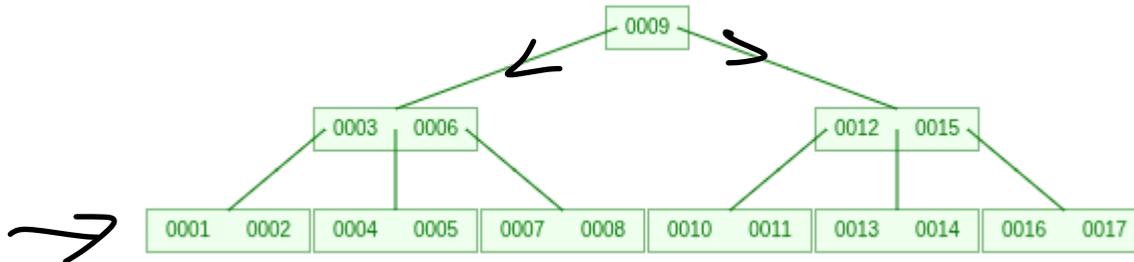
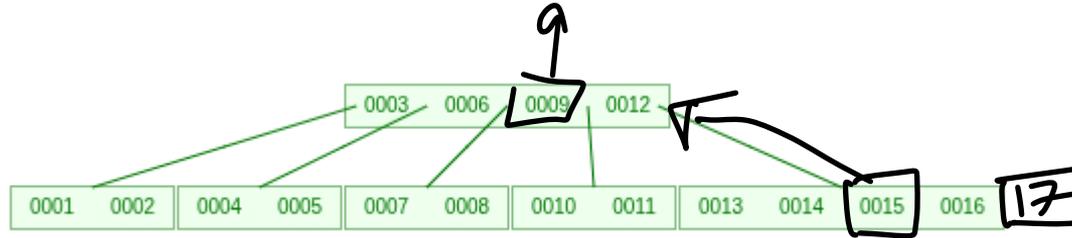
Find

Remove - More complicated, there is a POTD about this, not discussed in this class

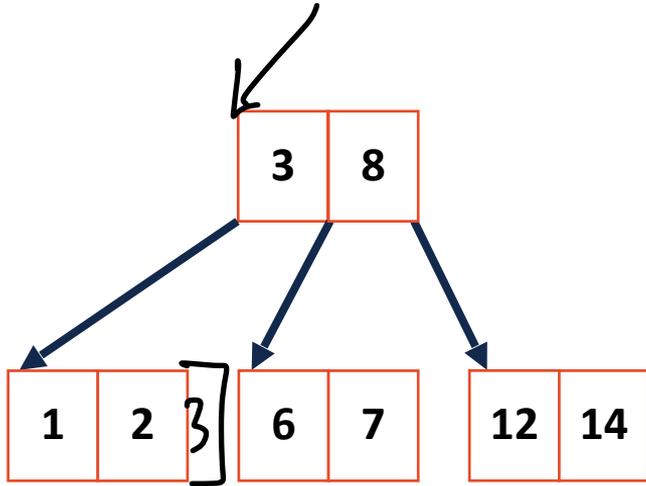


Insert - Recursive Split

Insert(17)

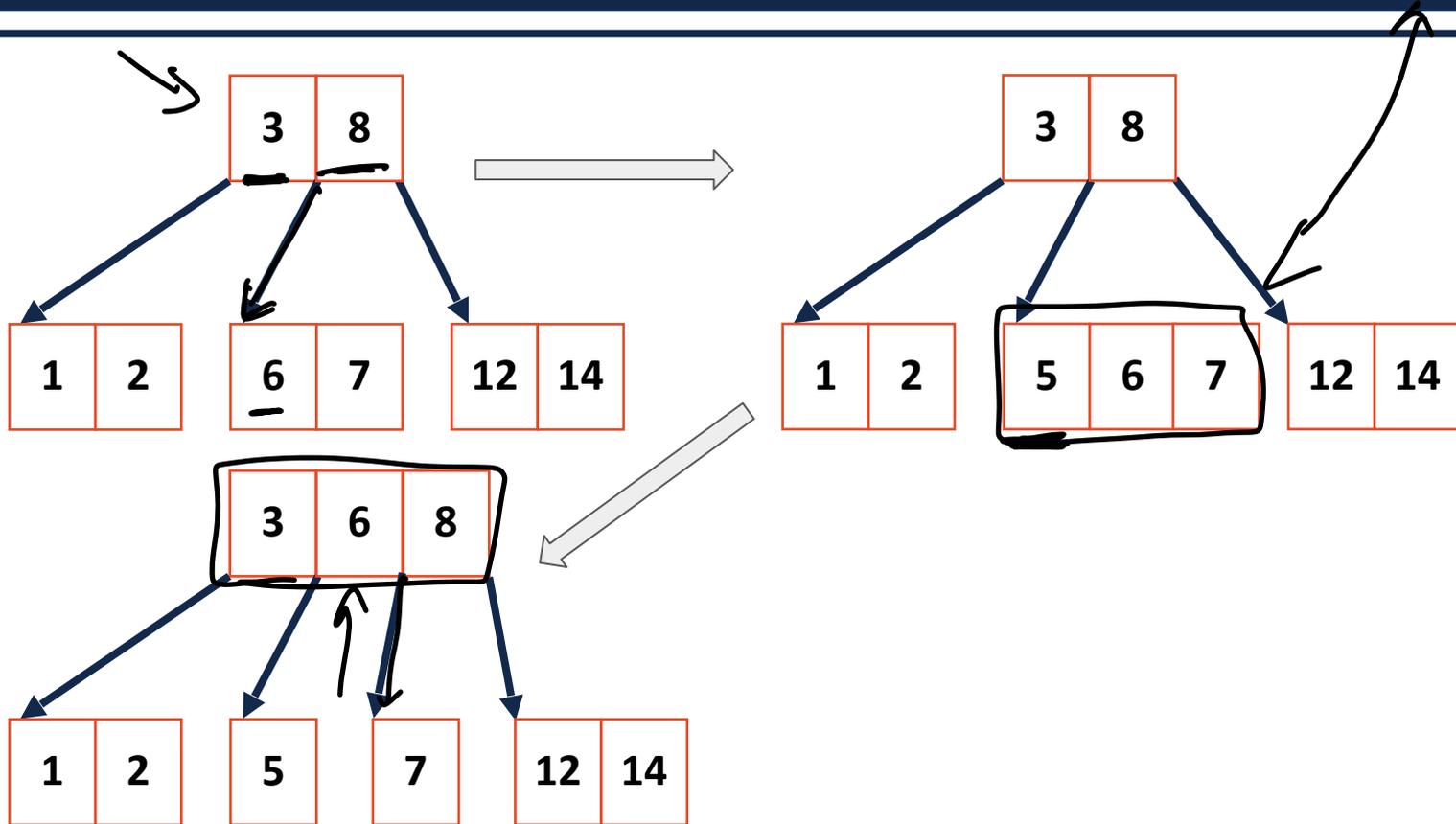


What will be the root after insert(5)? Assume $m = 3$

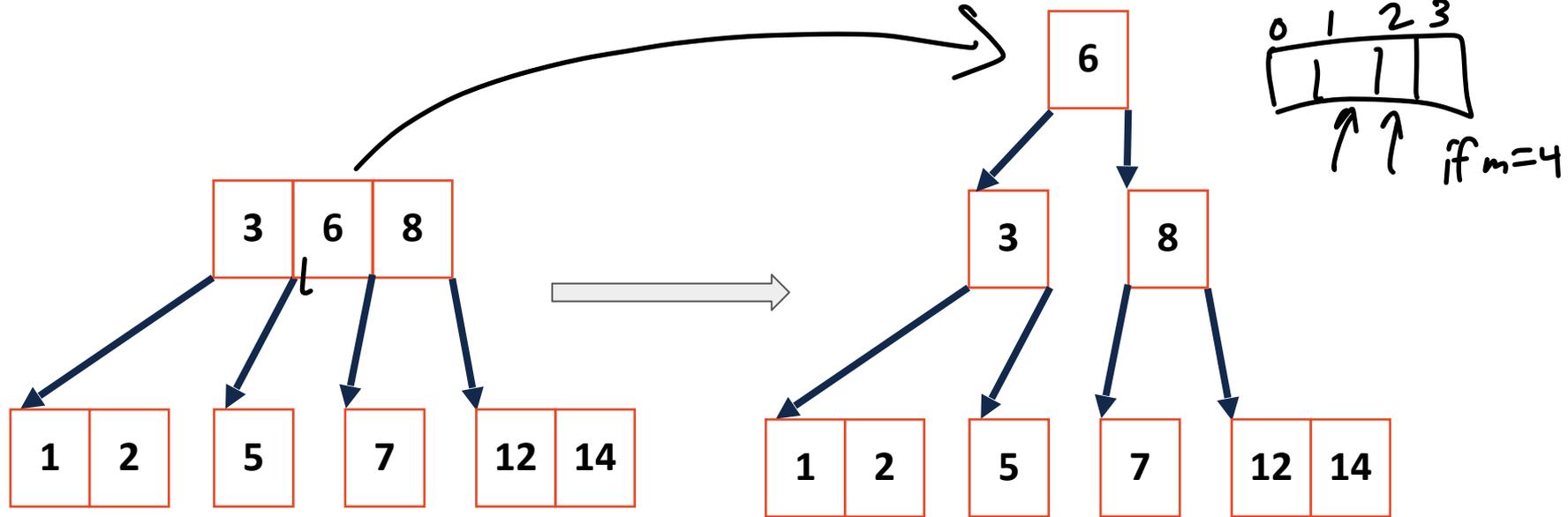


Join Code: 225

What will be the root after insert(5)? Assume $m = 3$



What will be the root after insert(5)? Assume $m = 3$



B-Tree Insert

- Always inserts at leaf level ←
- Split if ^{keys} ~~m nodes~~ are reached ←
in a node
- Split could propagate up the tree recursively

→ Split by "throwing up" the median



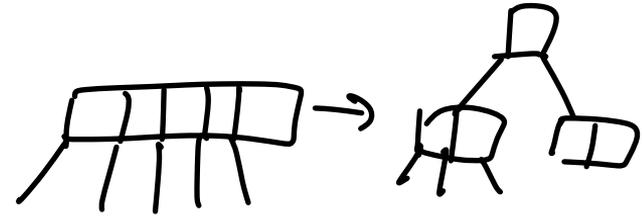
BTree Properties

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Definition

- All keys within a node are ordered
- All nodes contain no more than **m-1** keys
- All internal nodes have **one more child than keys**
- All leaves are on the same level
- Root node is a leaf or has **[2, m]** children.
- All non-root, internal nodes have **[ceil(m/2), m]** children

$$m=5$$



$$\text{ceil}\left(\frac{5}{2}\right) = 3$$

$$m=4$$



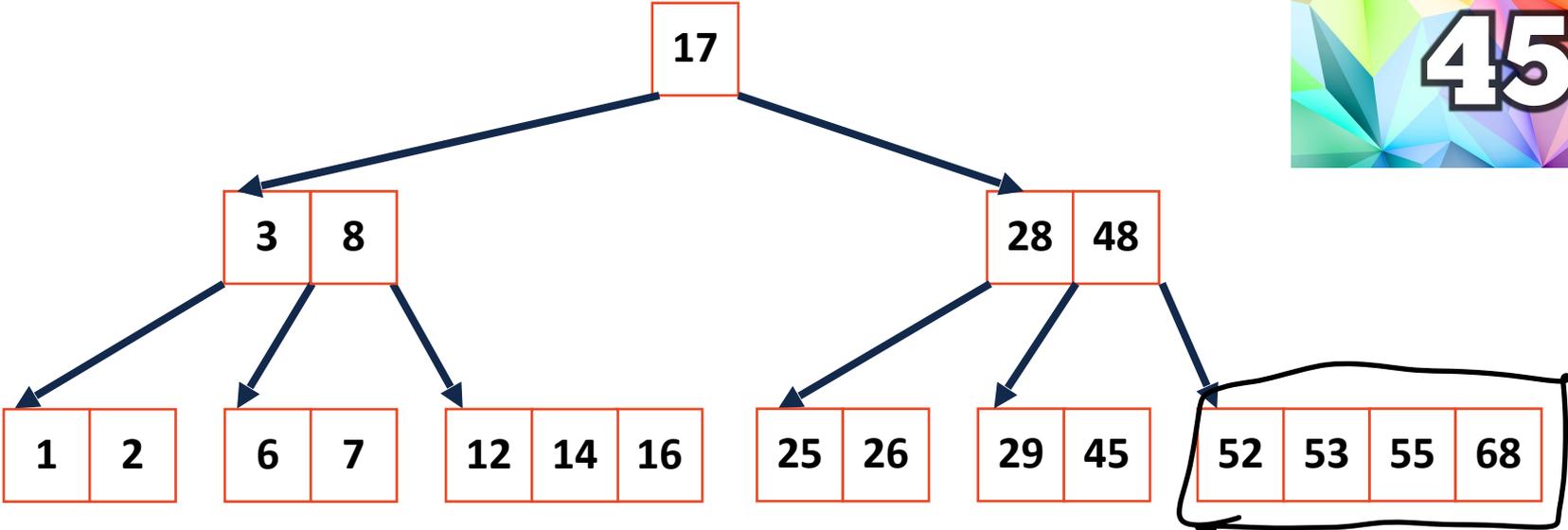
$$\text{ceil}\left(\frac{4}{2}\right) = 2$$



If this is a valid BTree, what is the lower bound for m?



Join Code: 225



m = 5

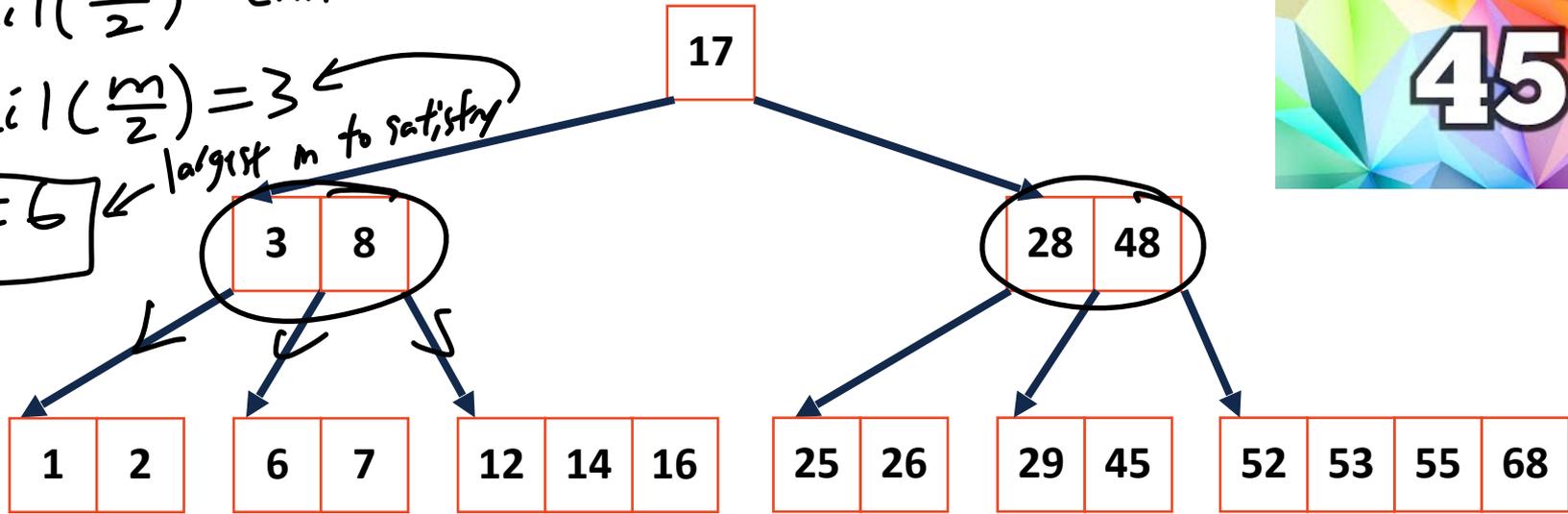


If this is a valid BTree, what is the upper bound for m?

$\text{Ceil}(\frac{m}{2})$ children at least

$\text{Ceil}(\frac{m}{2}) = 3$ ← largest m to satisfy

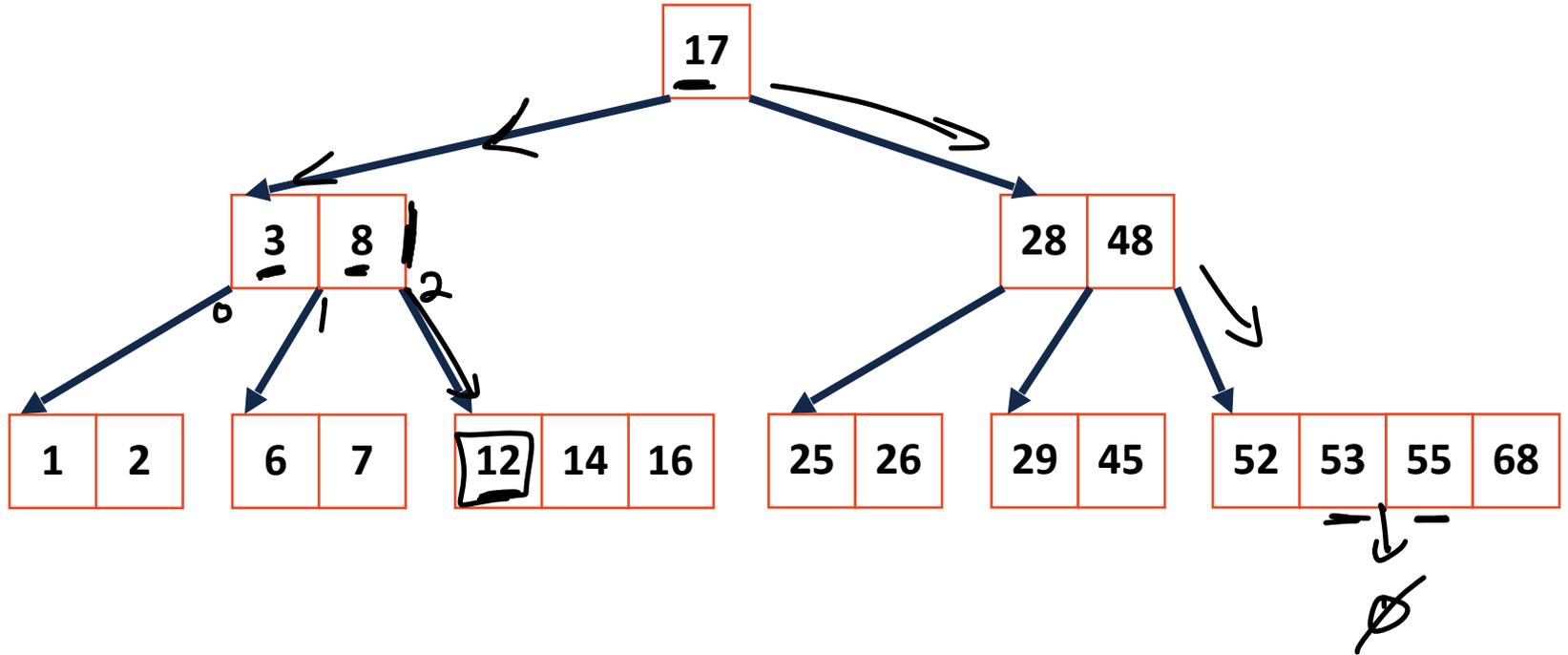
$m = 6$



If $n=7$, $\text{Ceil}(\frac{m}{2})=4$. non root internal nodes only have 3 children, so $m \neq 7$



B-Tree Find: How would find(12) work? find(54)?



BTree Search

```
1  template <typename K, typename V>
2  V BTree<K, V>::_find(BTreeNode * node, const K & key) const {
3  unsigned i;      stopping at end      node equal or larger
4  for (i = 0; i < node.keys_ct_ && key > node.keys_[i]; i++) { }
5
6  if (i < node.keys_ct_ && key == node.keys_[i]) { } } found key
7  return node.values_[i];
8  }
9
10 if (node.isLeaf()) { } } return not found
11 return V();
12 }
13
14 BTreeNode nextChild = node._fetchChild(i); } } recursively call
15 return _find(nextChild, key); on child
16 }
```

↑
seek

BTree Properties

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