Data Structures

Stacks and Queues

CS 225
Brad Solomon

September 10, 2025

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Department of Computer Science

Exam 1 (2/09 — 2/11)

Autograded MC and one coding question

Manually graded short answer prompt

Practice exam will be released on PL

Topics covered can be found on website

Register now

https://courses.engr.illinois.edu/cs225/exams/

Preparing for Exams

Make sure you understand the coding assignments

Review lecture slides — especially review slides!

Take a look at 'staff notes' — added to website for past lectures

Do the practice exam before watching practice exam solution video

Learning Objectives

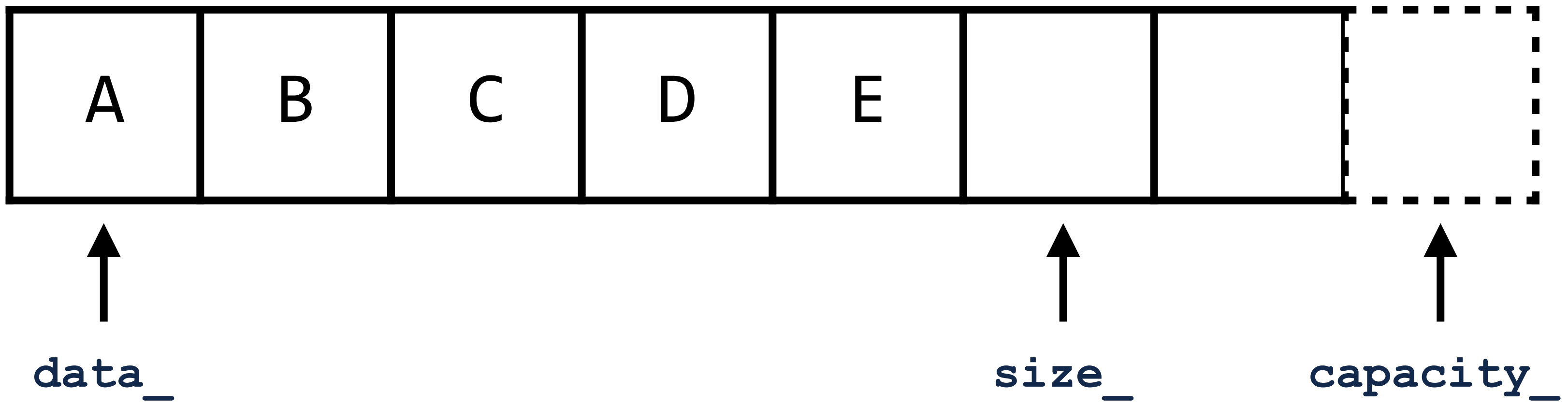
Discuss amortized analysis

Consider extensions to lists (data structure tradeoffs)

Introduce the stack and the queue data structure

Introduce and explore iterators

Array List



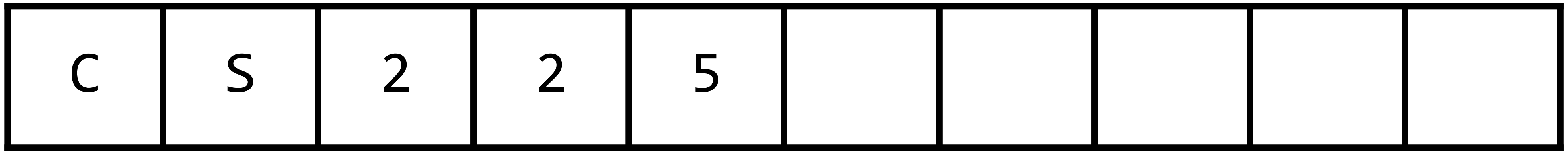
In C++, vector is implemented as:

1) **Data:** Stored as a pointer to array start

2) **Size:** Stored as a pointer to the next available space

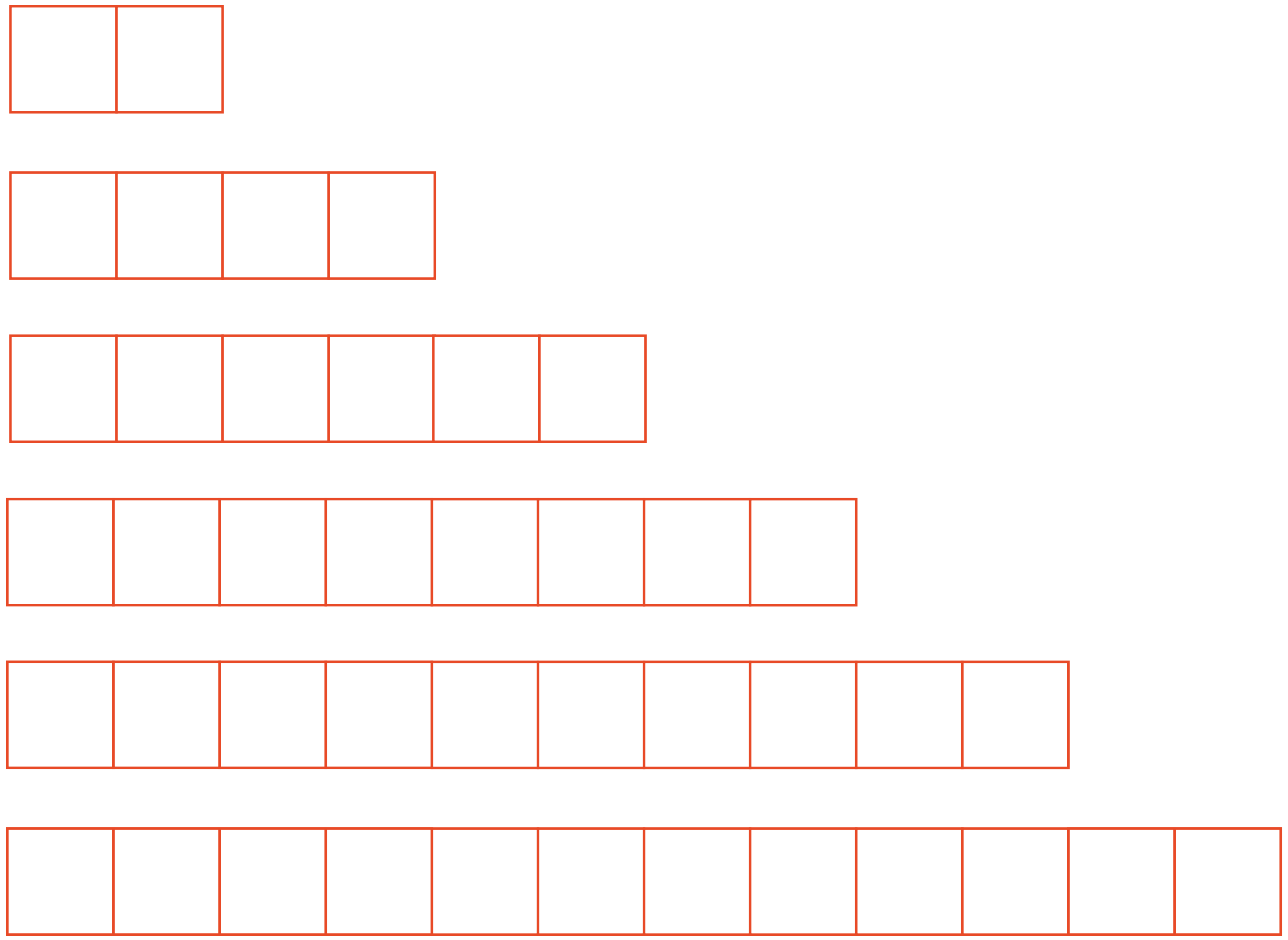
3) **Capacity:** Stored as a pointer past the end of the array

Array List: Not at capacity



	@Front	@Back	@Index
Insert	O(n)	O(1)	O(n)
Delete	O(n)	O(1)	O(n)

Resize Strategy: +2 elements every time



Resize Strategy: +2 elements every time

Total copies for N inserts: $\dfrac{N^2 + 2N}{4}$

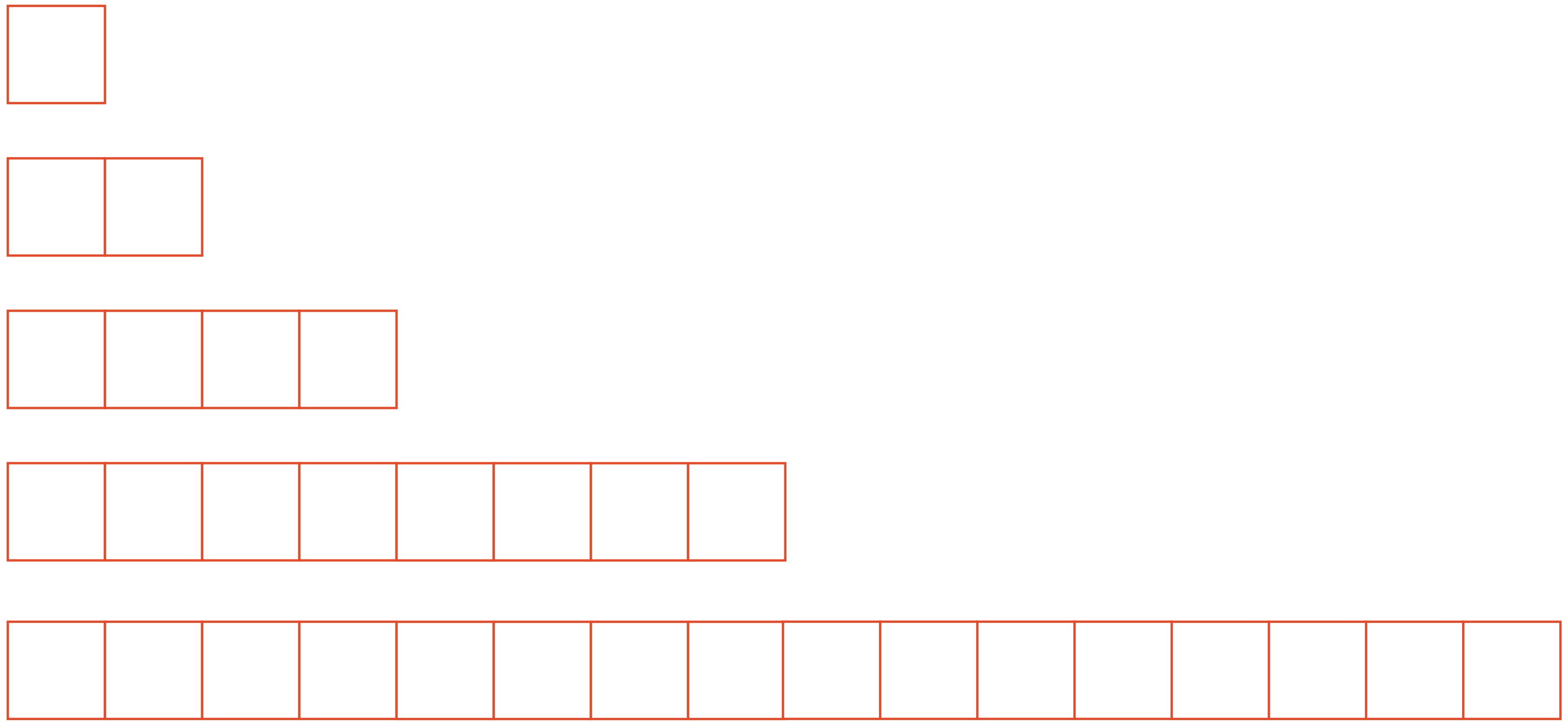
Amortized:

Precise total work over N calls

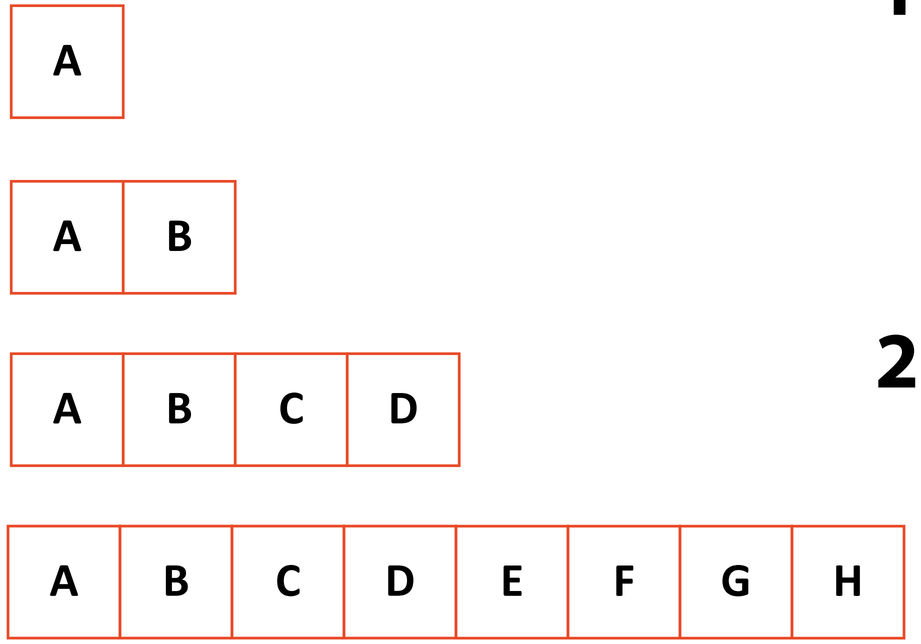
Big O:

Upperbound on worst case

Resize Strategy: x2 elements every time



Resize Strategy: x2 elements every time



1) How many copy calls per reallocation?

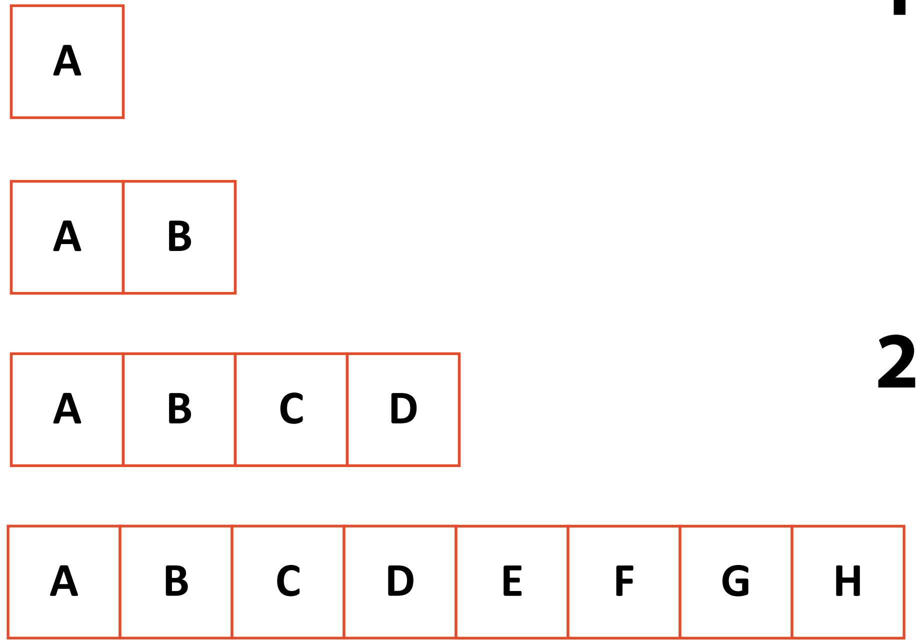
For reallocation i, 2^i copy calls are made

2) Total reallocations for N objects?

k = final realloc needed = $\lceil log_2 n \rceil$

Total number of copy calls:

Resize Strategy: x2 elements every time



1) How many copy calls per reallocation?

For reallocation i, 2^i copy calls are made

2) Total reallocations for N objects?

k = final realloc needed = $\lceil log_2 n \rceil$

Total number of copy calls: $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

… For N objects: $2n - 1$

Resize Strategy: x2 elements every time

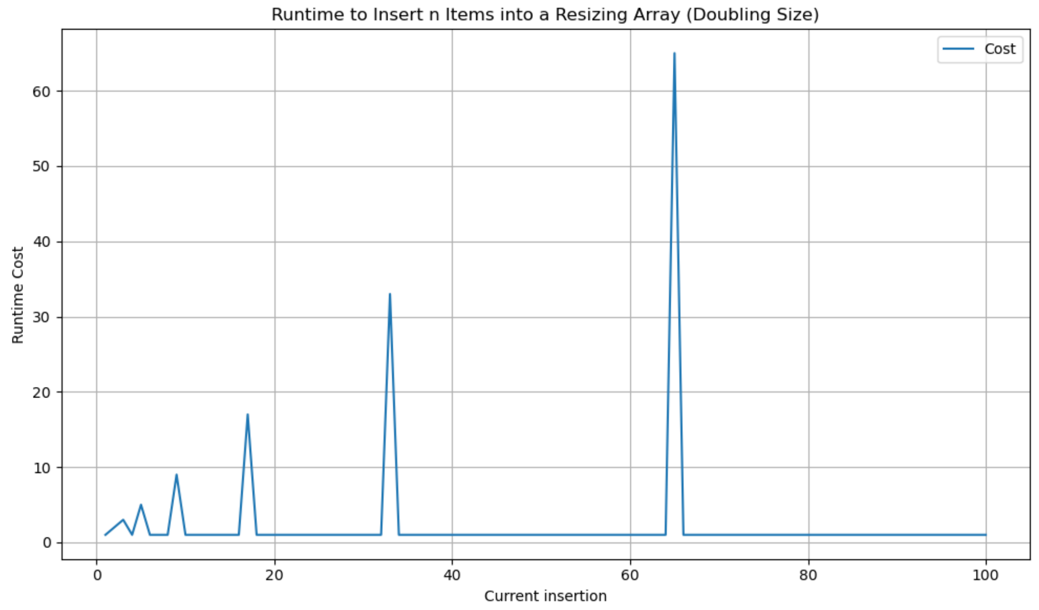
Total copies for n inserts: $2n - 1$

Amortized:

Precise total work over N calls

Big O:

Upperbound on worst case



List Implementation

	Singly Linked List	Array
Look up **arbitrary** location		
Insert after **given** element		
Remove after **given** element		
Insert at **arbitrary** location		
Remove at **arbitrary** location		
Search for an input **value**		

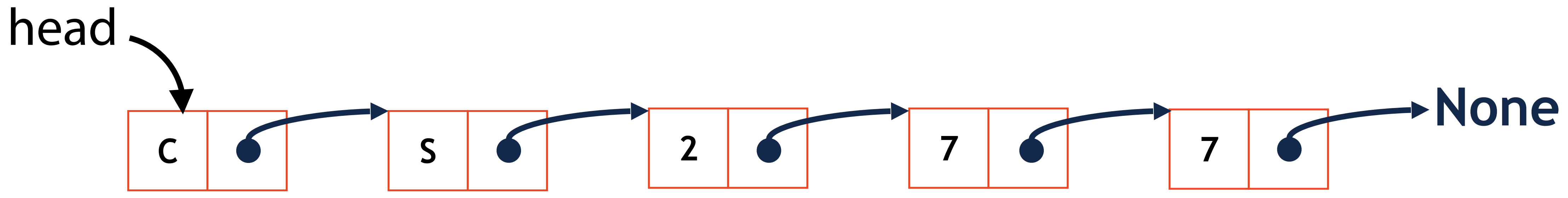
Thinking critically about lists: tradeoffs

The implementations shown are foundational (simple).

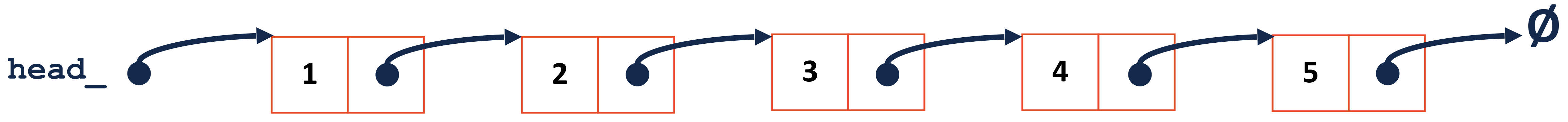
Can we make our lists better at some things? What is the cost?

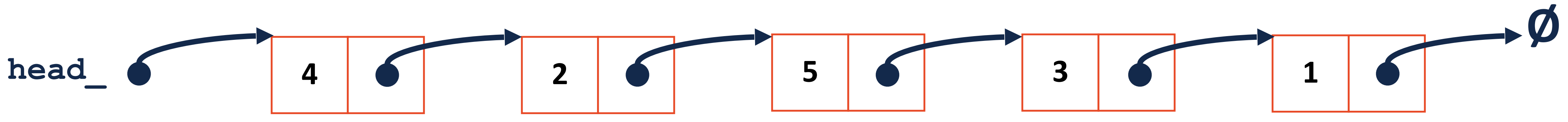
Thinking critically about lists: tradeoffs

Getting the size of a linked list has a Big O of:



Thinking critically about lists: tradeoffs



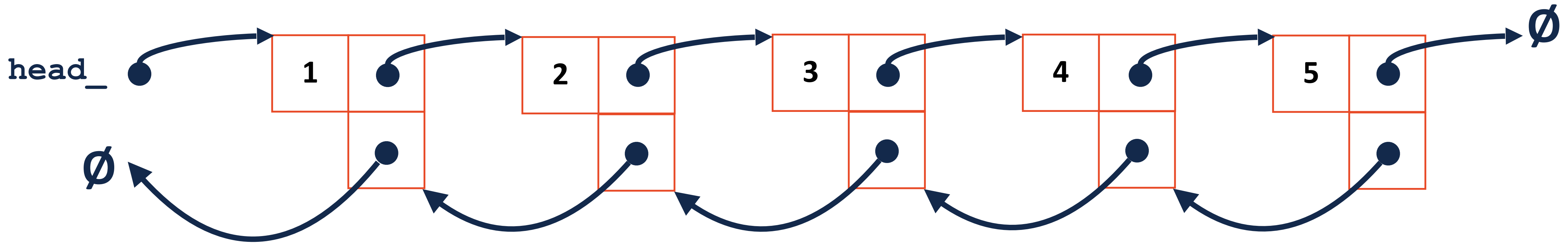


Thinking critically about lists: tradeoffs

2	7	5	9	7	14	1	0	8	3

0	1	2	3	5	7	7	8	9	14

Thinking critically about lists: tradeoffs



Thinking critically about lists: tradeoffs

As we progress in the class, we will see that $O(n)$ isn't very good.

Take searching for a specific list value:

2	7	5	9	7	14	1	0	8	3

0	1	2	3	5	7	7	8	9	14

Thinking critically about lists: tradeoffs

Can we make a ‘list’ that is O(1) to insert and remove?

Stack Data Structure

A **stack** stores an ordered collection of objects (like a list)

However you can only do three* operations:

Push: Put an item on top of the stack

Pop: Remove the top item of the stack

Top: Return the top item of the stack

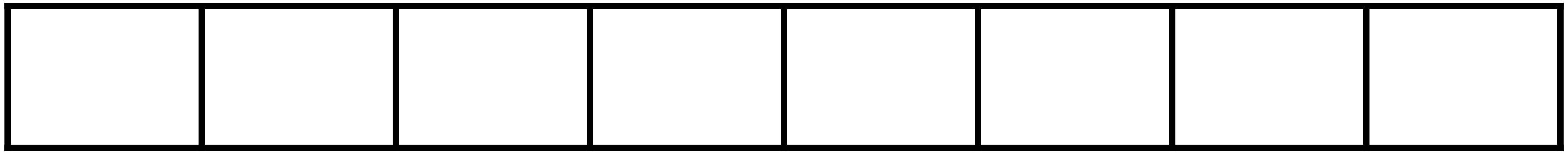
```
push(3); push(5); pop(); push(2)
```

Stack Data Structure

C++ has a built-in stack

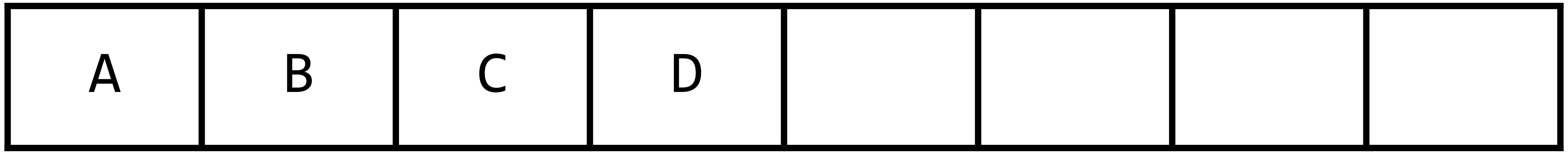
Underlying implementation is vector or deque

```
#include <stack>
int main() {
  stack<int> stack;
  stack.push(3);
  stack.push(8);
  stack.push(4);
  stack.pop();
  stack.push(7);
  stack.pop();
  stack.pop();
}
```



Stack Data Structure

`Push(X)` is equivalent to …

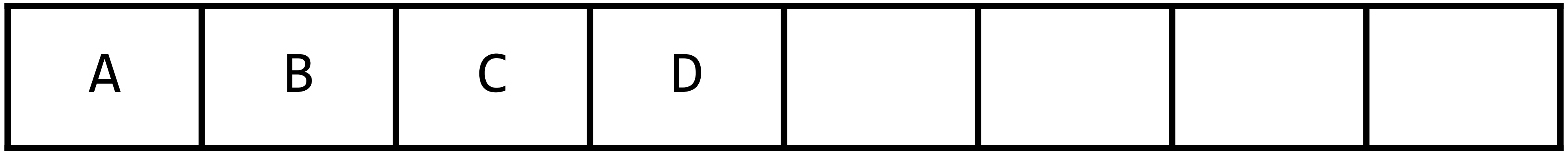


Stack Data Structure

`Push(X)` is equivalent to `insertBack(X)`

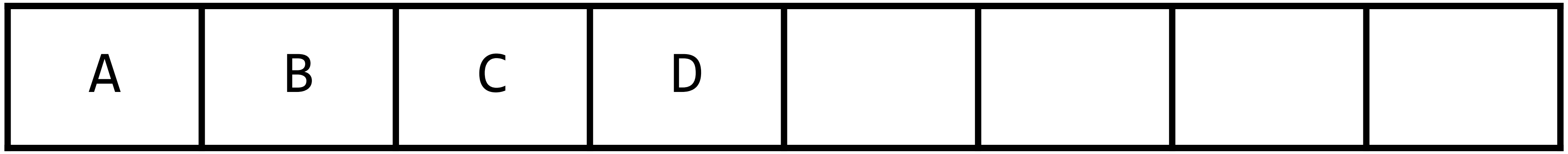
```
*size = X;

size++;
```



Stack Data Structure

Pop( ) is equivalent to…

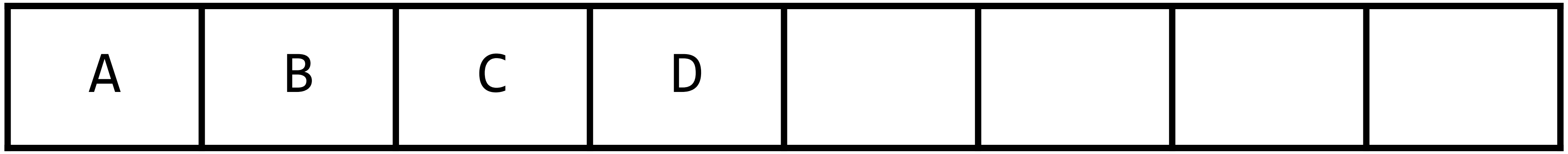


Stack Data Structure

`Pop()` is equivalent to `removeBack()`

```
size--;
```

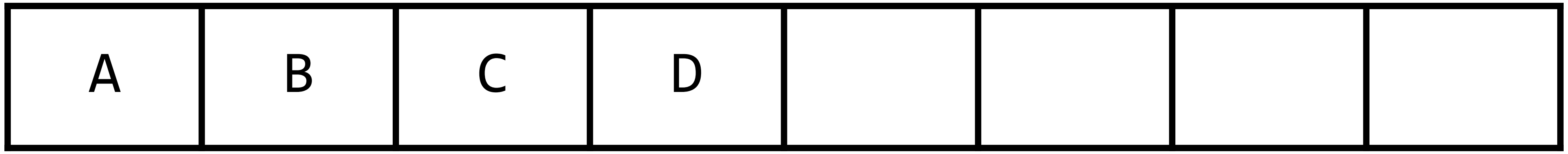
```
T tmp = *size;
```



Stack Data Structure

`Top()` is tricky — remember size points to next available space!

```
return *(size - 1);
```



Stack ADT

- [Order]:
- [Implementation]:
- [Runtime]:

Queue Data Structure

A **queue** stores an ordered collection of objects (like a list)

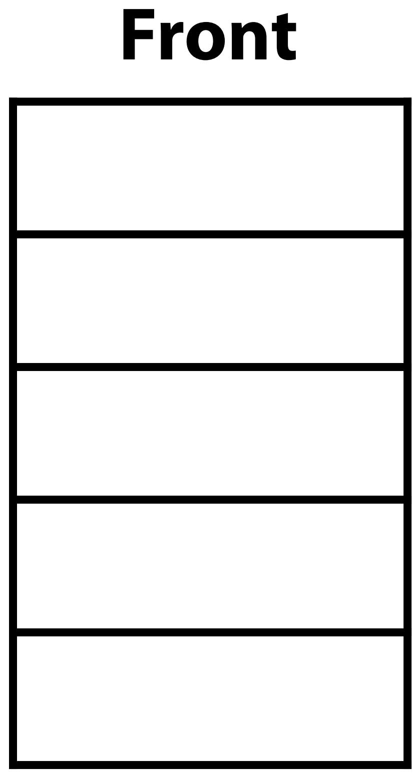
However you can only do three* operations:

Enqueue: Put an item at the back of the queue

Dequeue: Remove the front item of the queue

Front: Return the front item of the queue

```
enqueue(3); enqueue(5); dequeue(); enqueue(2)
```



Queue Data Structure

The queue is a **first in — first out** data structure (FIFO)

What data structure excels at removing from the front?

Can we make that same data structure good at inserting at the end?

Queue Data Structure

The C++ implementation of a queue is also a vector or deque — why?

Engineering vs Theory Efficiency

	Time x1 billion	Like
L1 cache reference	0.5 seconds	Heartbeat 💓
Branch mispredict	5 seconds	Yawn 😲
L2 cache reference	7 seconds	Long yawn 😲 😲 😲
Mutex lock/unlock	25 seconds	Make coffee ☕
Main memory reference	100 seconds	Brush teeth
Compress 1K bytes	50 minutes	TV show 📺
Send 2K bytes over 1 Gbps network	5.5 hours	(Brief) Night's sleep 🛌
SSD random read	1.7 days	Weekend
Read 1 MB sequentially from memory	2.9 days	Long weekend
Read 1 MB sequentially from SSD	11.6 days	2 weeks for delivery 📦
Disk seek	16.5 weeks	Semester
Read 1 MB sequentially from disk	7.8 months	Human gestation 🐣
Above two together	1 year	🌍 ☀️
Send packet CA->Netherlands->CA	4.8 years	Ph.D. 🎓

(Care of https://gist.github.com/hellerbarde/2843375)

Engineering vs Theory Efficiency

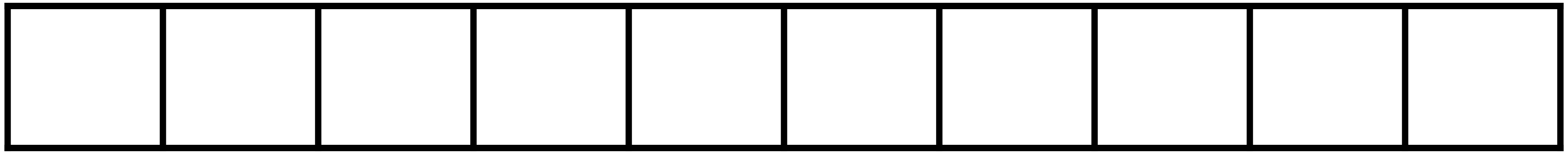
	Time x1 billion	Like
L1 cache reference	0.5 seconds	Heartbeat 💓
Main memory reference	100 seconds	Brush teeth
SSD random read	1.7 days	Weekend
Disk seek	16.5 weeks	Semester
Send packet CA->Netherlands->CA	4.8 years	Ph.D. 🎓

(Care of https://gist.github.com/hellerbarde/2843375)

Queue Data Structure

```
q.enqueue(8);
q.enqueue(4);
q.dequeue();
```

What do we need to track to maintain a queue with an array list?



Queue Data Structure

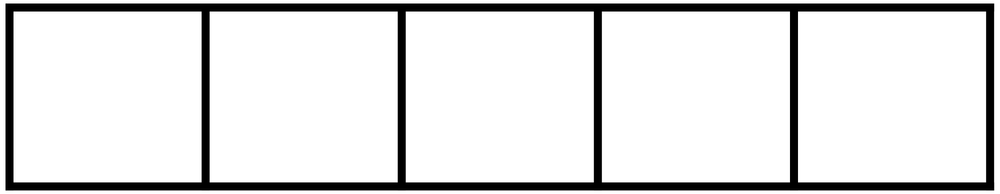
Unlike the array list, it is easier to implement a Queue using unsigned ints

Queue.h

```
#pragma once

template <typename T>
class Queue {
  public:
    void enqueue(T e);
    T dequeue();
    bool isEmpty();

  private:
    T *data_;
    unsigned size_;
    unsigned capacity_;
    unsigned front_;
};
```



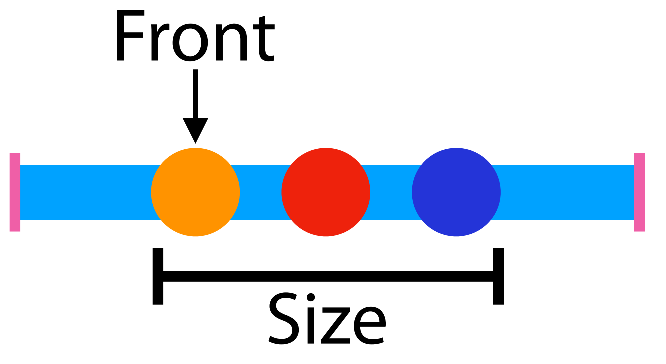
(Circular) Queue Data Structure

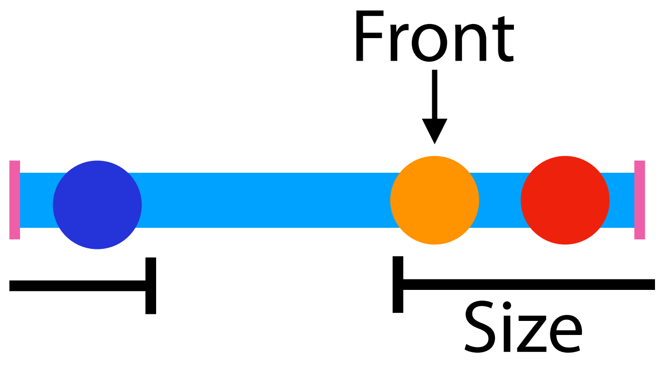
Queue.h

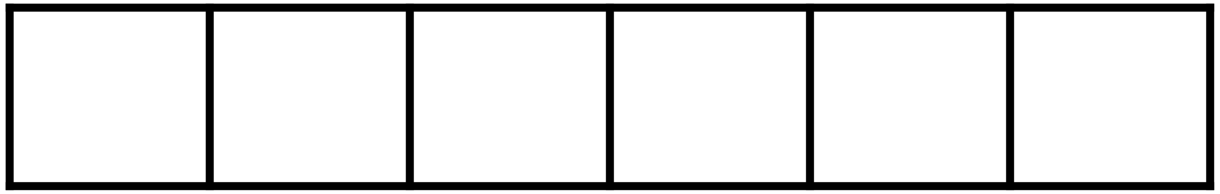
```cpp
#pragma once

template <typename T>
class Queue {
  public:
    void enqueue(T e);
    T dequeue();
    bool isEmpty();

  private:
    T *data_;
    unsigned capacity_;
    unsigned size_;
    unsigned front_;
};
```







Enqueue(D):

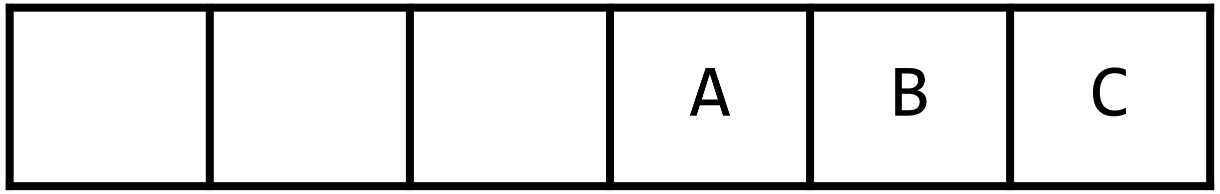
Dequeue():

Size:

Front:

```
Queue<int> q;
q.enqueue(3);
q.enqueue(8);
q.enqueue(4);
q.dequeue();
q.enqueue(7);
q.dequeue();
q.dequeue();
q.enqueue(2);
q.enqueue(1);
q.enqueue(3);
q.enqueue(5);
q.dequeue();
q.enqueue(9);
```

Capacity:



```
Queue<int> q;
…
q.enqueue(D);
q.dequeue();
q.dequeue();
q.dequeue();
q.dequeue();
q.enqueue(E);
```

Enqueue(D): Add data to ‘back’ of queue

Insert D at index **(size+front) % capacity**

size++ (as long as size != capacity)

Dequeue(): Remove data at index front

front = **(front+1) % capacity**

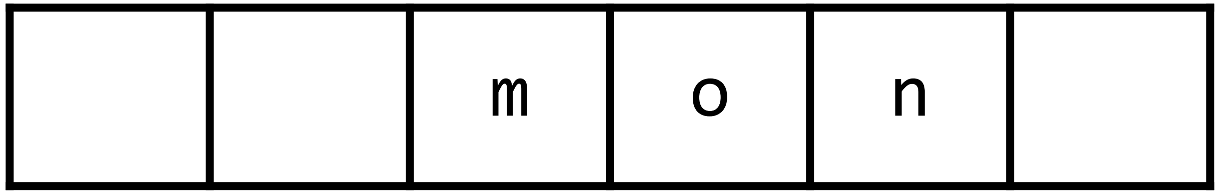
size-- (as long as size != 0)

Size: 3

Front: 3

Capacity: 6

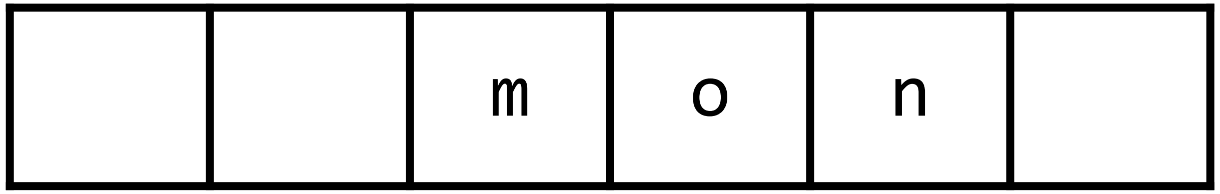
Queue Data Structure: Resizing

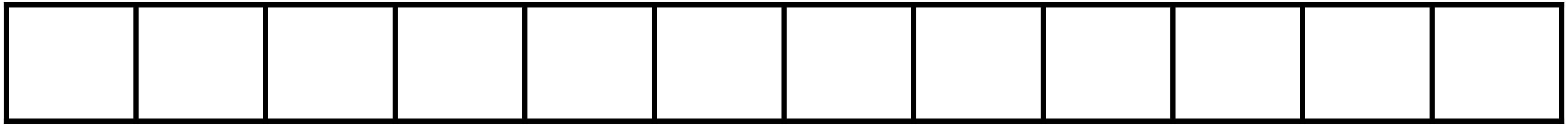


```
Queue<char> q;
...
q.enqueue(d);
q.enqueue(a);
q.enqueue(y);
q.enqueue(i);
q.enqueue(s);
```

Queue Data Structure: Resizing

```
Queue<char> q;
...
q.enqueue(d);
q.enqueue(a);
q.enqueue(y);
q.enqueue(i);
q.enqueue(s);
```



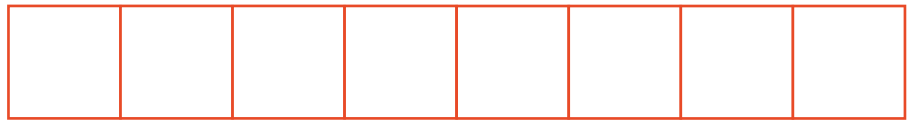


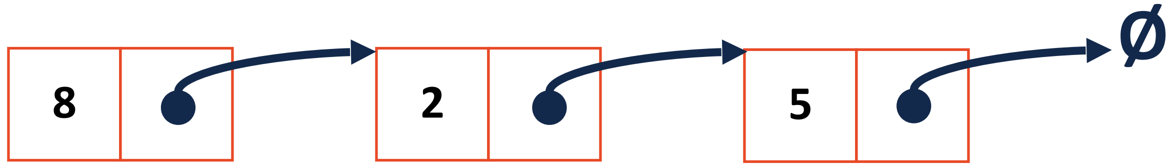
Queue ADT

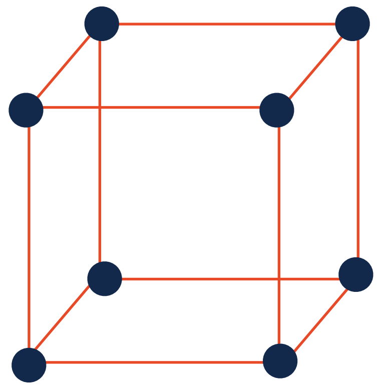
- [Order]:
- [Implementation]:
- [Runtime]:

Iterators

We want to be able to loop through all elements for any underlying implementation in a systematic way

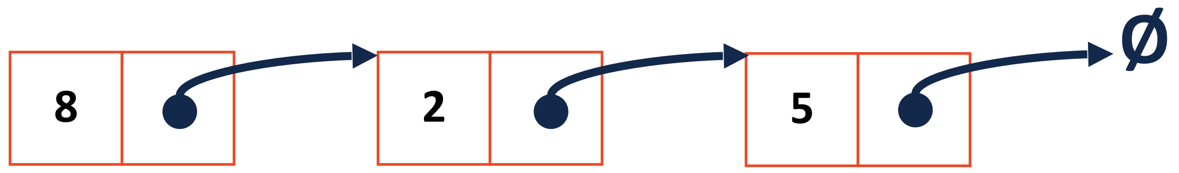


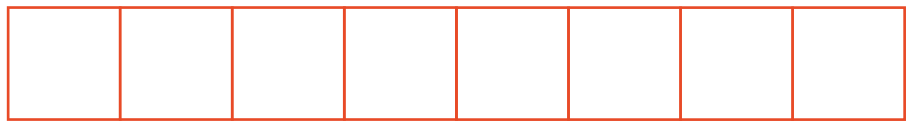


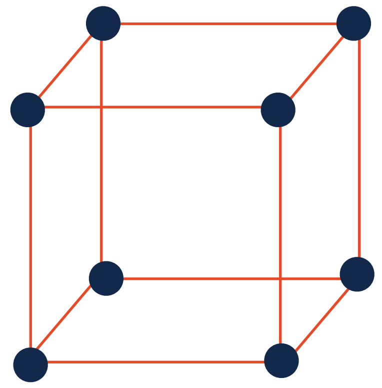


Iterators

We want to be able to loop through all elements for any underlying implementation in a systematic way



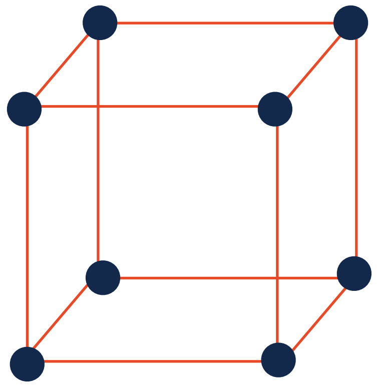




Cur. Location	Cur. Data	Next
`ListNode * curr`		
`unsigned index`		
`Some form of (x, y, z)`		

Iterators

Iterators provide a way to access items in a container without exposing the underlying structure of the container



```
Cube::Iterator start = myCube.begin();

while (it != myCube.end()) {
    std::cout << *it << " ";
    it++;
}

```

Iterators

For a class to implement an iterator, it needs two functions:

```
Iterator begin()
```

```
Iterator end()
```

Iterators

The actual iterator is defined as a class **inside** the outer class:

1. It must be of base class **`std::iterator`**

2. It must implement at least the following operations:

```
Iterator& operator ++()

const T & operator *()

bool operator !=(const Iterator &)
```

Iterators

Here is a (truncated) example of an iterator:

```
template <class T>
class List {

    class ListIterator : public
std::iterator<std::bidirectional_iterator_tag, T> {
      public:

        ListIterator& operator++();

        ListIterator& operator--()

        bool operator!=(const ListIterator& rhs);

        const T& operator*();
    };

    ListIterator begin() const;

    ListIterator end() const;
};
```

stlList.cpp

```
#include <list>
#include <string>
#include <iostream>

struct Animal {
  std::string name, food;
  bool big;
  Animal(std::string name = "blob", std::string food = "you", bool big = true) :
    name(name), food(food), big(big) { /* nothing */ }
};

int main() {
  Animal g("giraffe", "leaves", true), p("penguin", "fish", false), b("bear");
  std::vector<Animal> zoo;

  zoo.push_back(g);
  zoo.push_back(p);   // std::vector's insertAtEnd
  zoo.push_back(b);

  for ( std::vector<Animal>::iterator it = zoo.begin(); it != zoo.end(); ++it ) {
    std::cout << (*it).name << " " << (*it).food << std::endl;
  }

  return 0;
}
```

```

std::vector<Animal> zoo;


/* Full text snippet */

  for ( std::vector<Animal>::iterator it = zoo.begin(); it != zoo.end(); ++it ) {
    std::cout << (*it).name << " " << (*it).food << std::endl;
  }


/* Auto Snippet */

  for ( auto it = zoo.begin(); it != zoo.end; ++it ) {
    std::cout << animal.name << " " << animal.food << std::endl;
  }

/* For Each Snippet */

  for ( const Animal & animal : zoo ) {
    std::cout << animal.name << " " << animal.food << std::endl;
  }



```

Trees

"The most important non-linear data structure in computer science."

- David Knuth, The Art of Programming, Vol. 1

A tree is:

-
-