CS 225

Data Structures

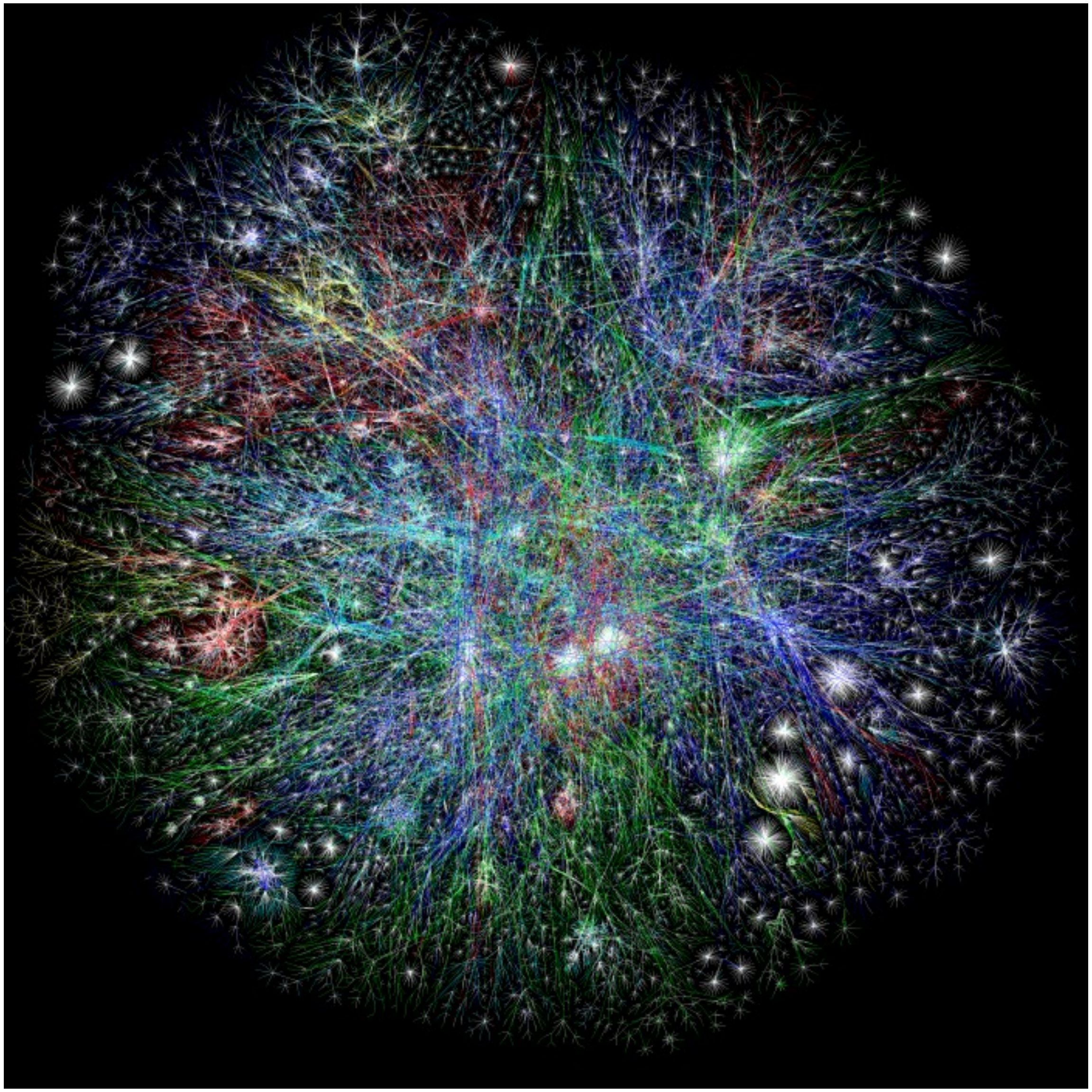
March 18– Graphs

G Carl Evans

In Review: Data Structures

Array
- Sorted Array
- Unsorted Array
  - Stacks
  - Queues
  - Priority Queues
    - Heaps
  - Disjoint Sets
    - UpTrees

Linked
- Doubly Linked List
- Trees
  - BTree
  - Binary Tree
    - Huffman Encoding
    - kd-Tree
    - AVL Tree

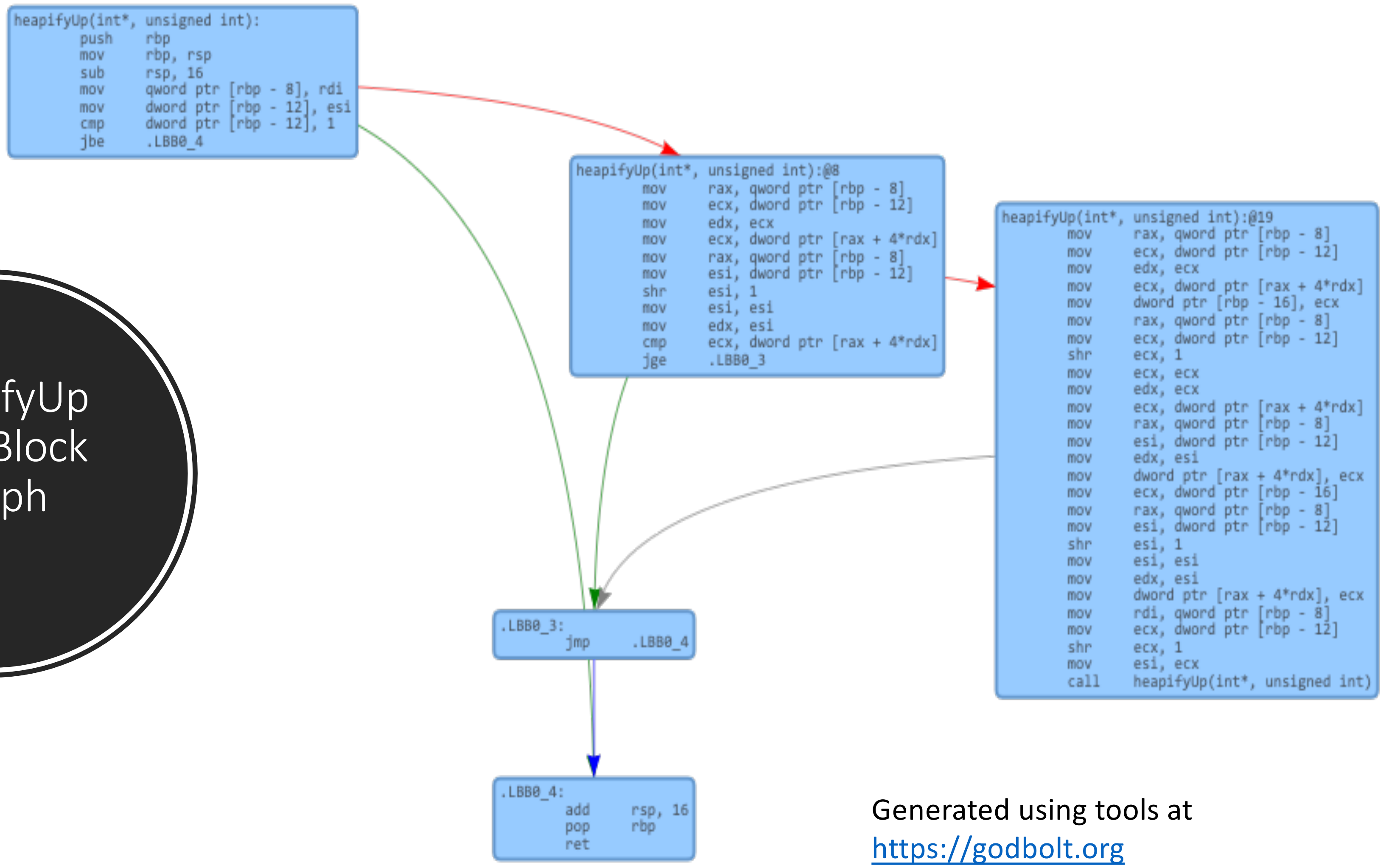


The Internet 2003

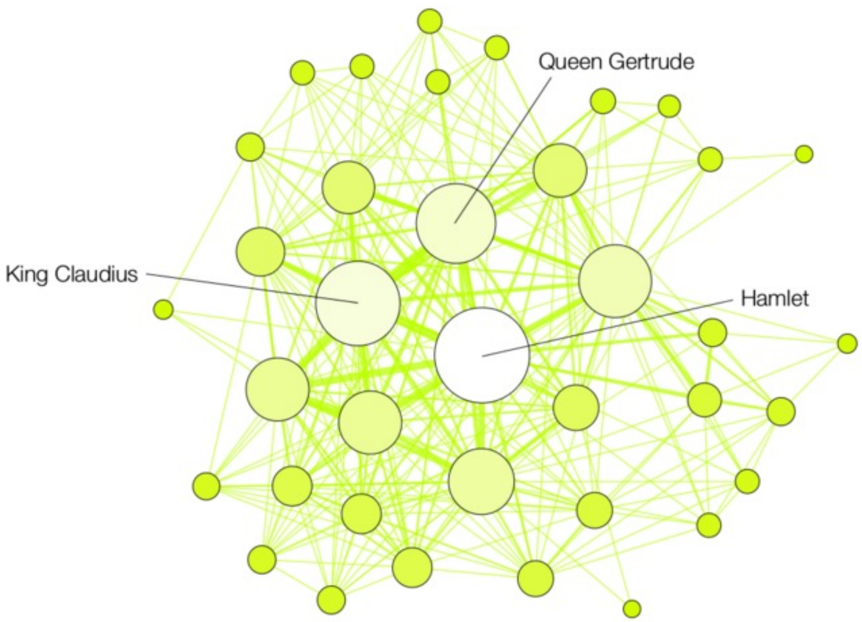
The OPTE Project (2003)

Map of the entire internet; nodes are routers; edges are connections.

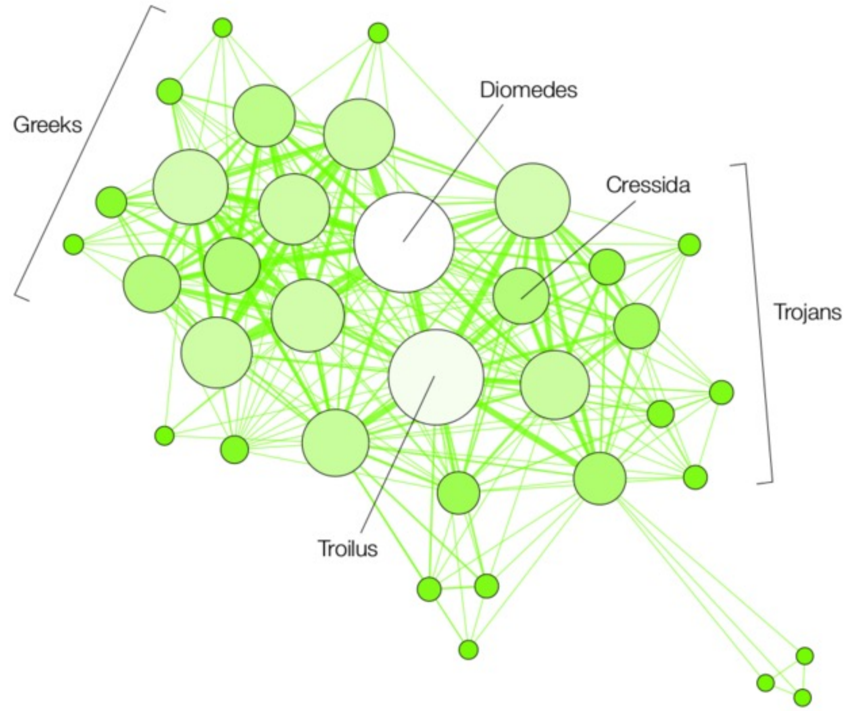
HeapifyUp BasicBlock Graph



Generated using tools at https://godbolt.org



HAMLET

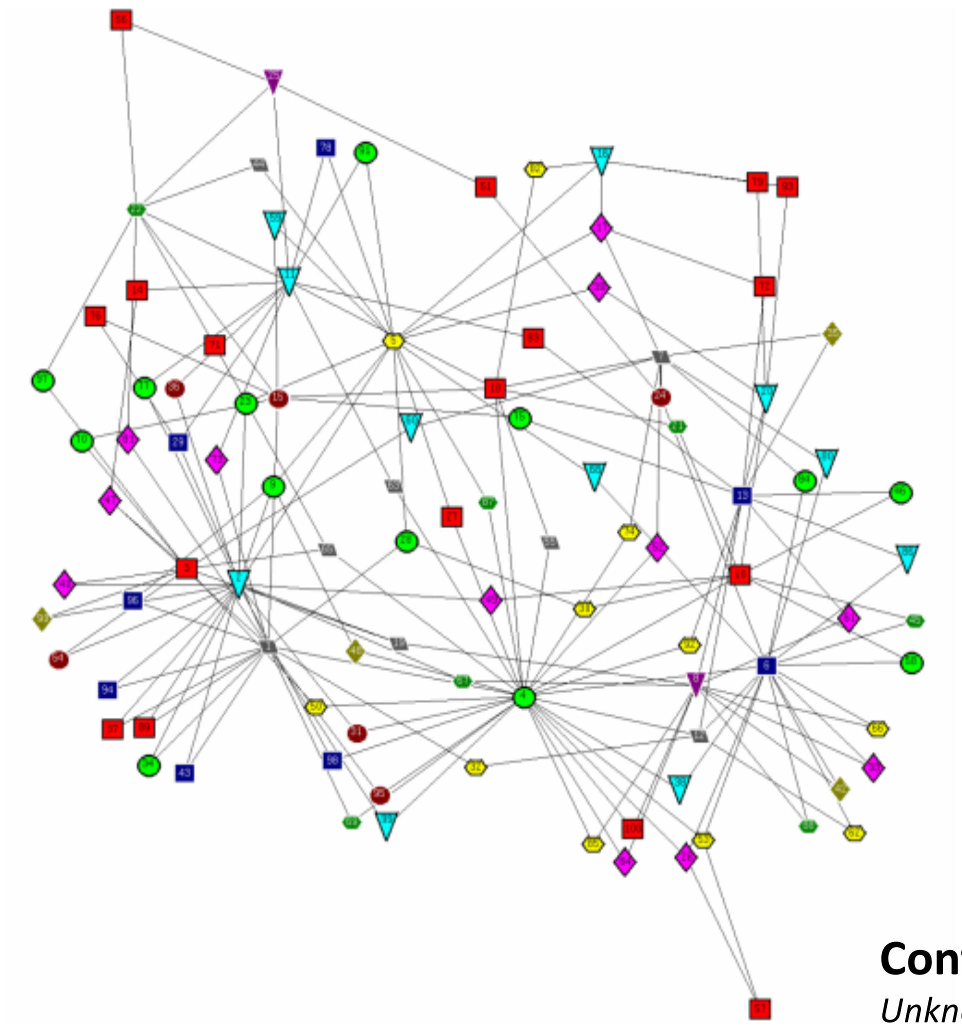


TROILUS AND CRESSIDA

Who's the real main character in Shakespearean tragedies?

Martin Grandjean (2016)

[*https://www.pbs.org/newshour/arts/whos-the-real-main-character-in-shakespearen-tragedies-heres-what-the-data-say*](https://www.pbs.org/newshour/arts/whos-the-real-main-character-in-shakespearen-tragedies-heres-what-the-data-say)



Conflict-Free Final Exam Scheduling Graph

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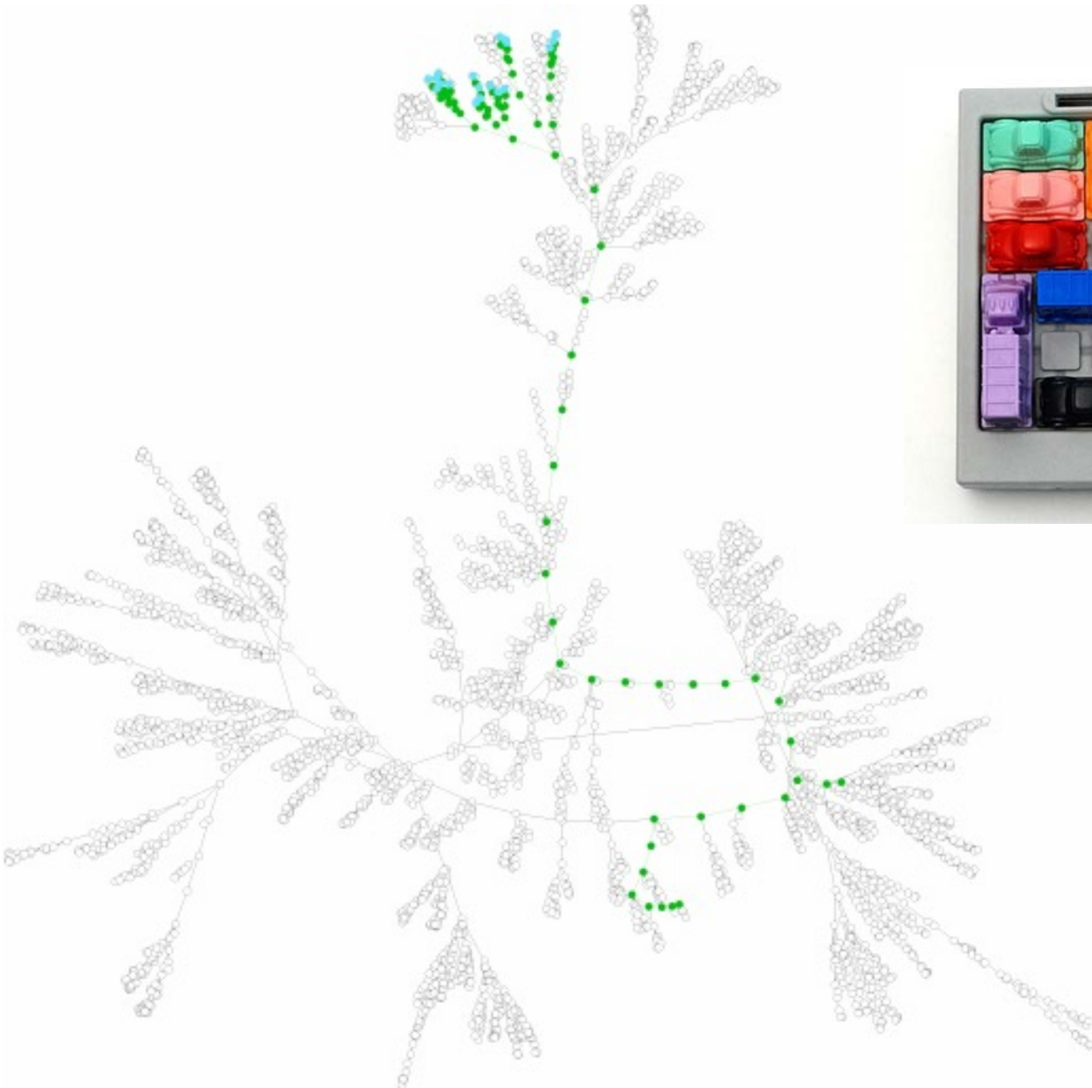
Presented by Cinda Heeren, 2016



“Rush Hour” Solution

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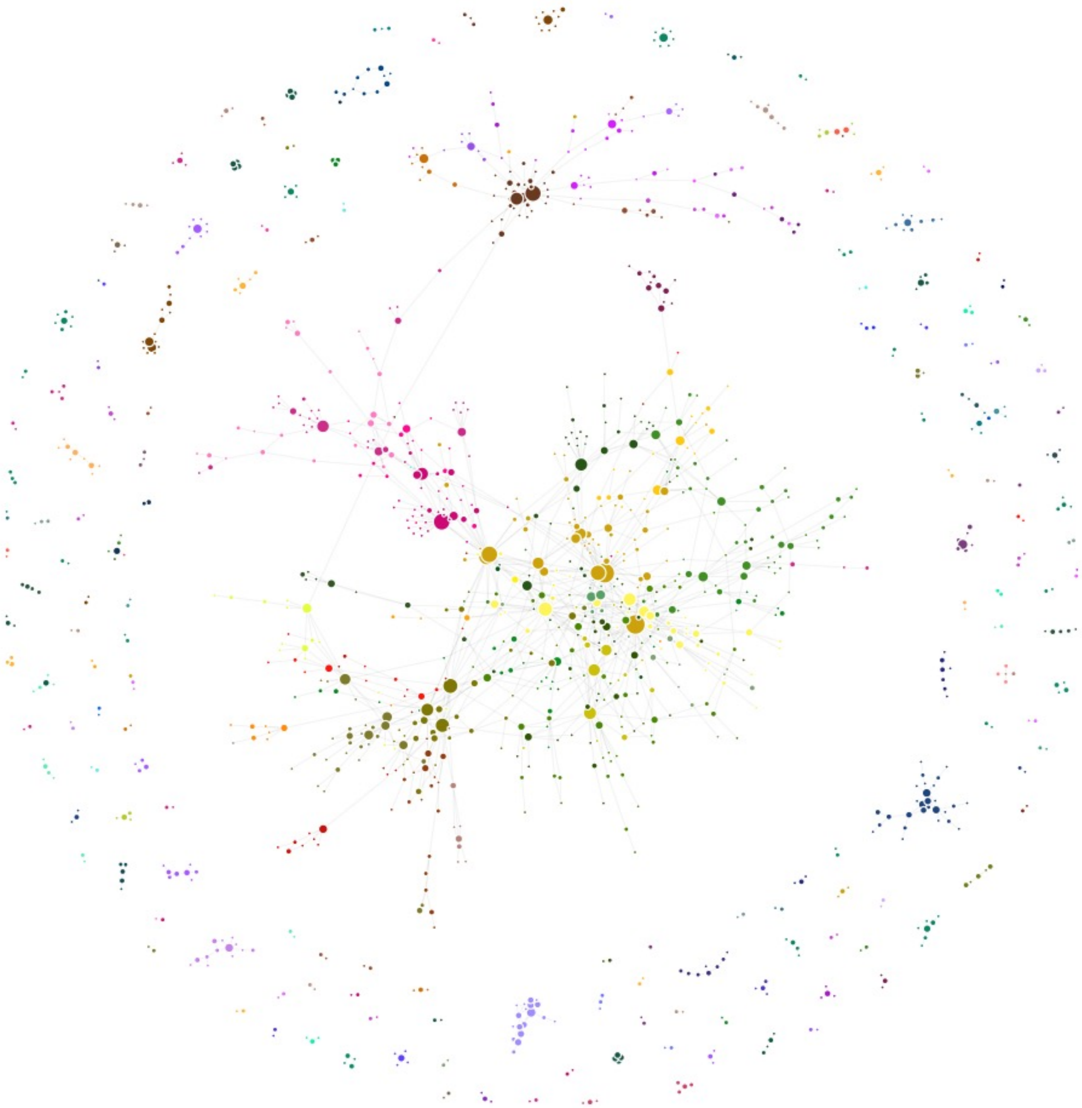
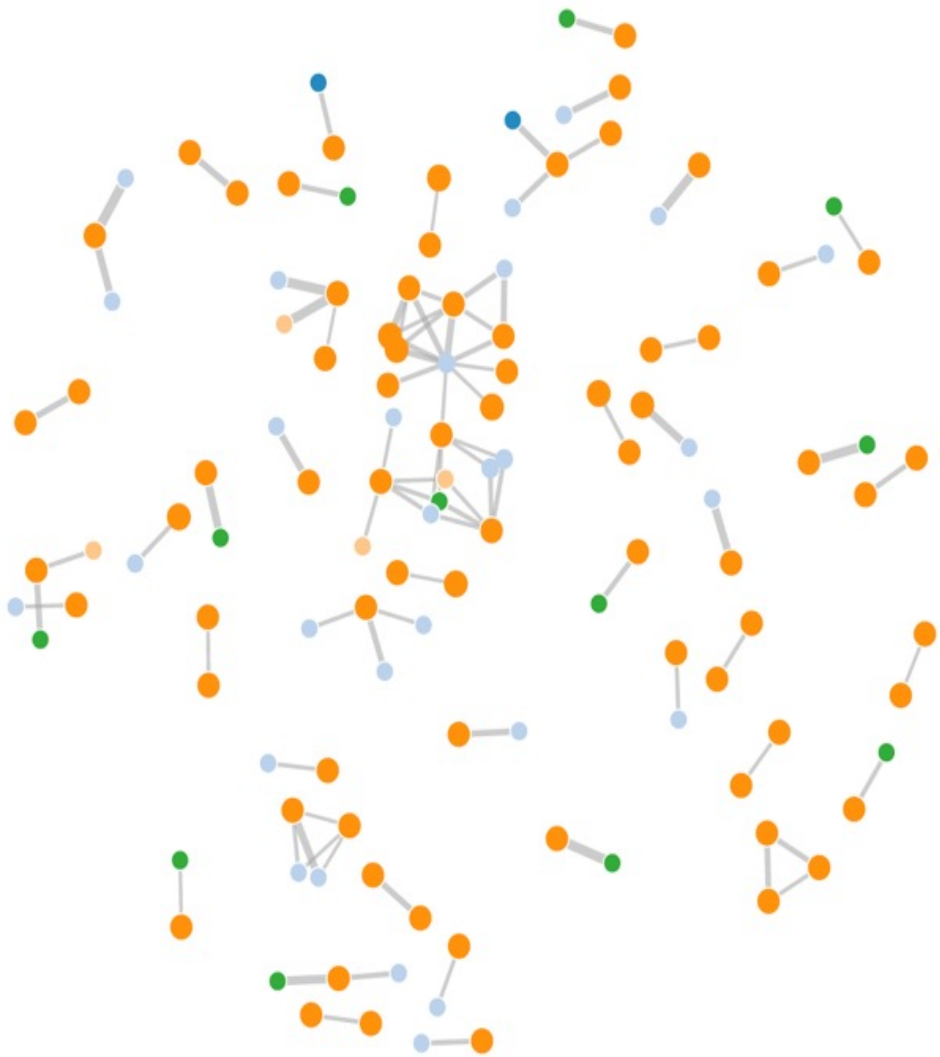


Class Hierarchy At University of Illinois Urbana-Champaign

A. Mori, W. Fagen-Ulmschneider, C. Heeren

Graph of every course at UIUC; nodes are courses, edges are prerequisites

http://waf.cs.illinois.edu/discovery/class_hierarchy_at_illinois/



MP Collaborations in CS 225

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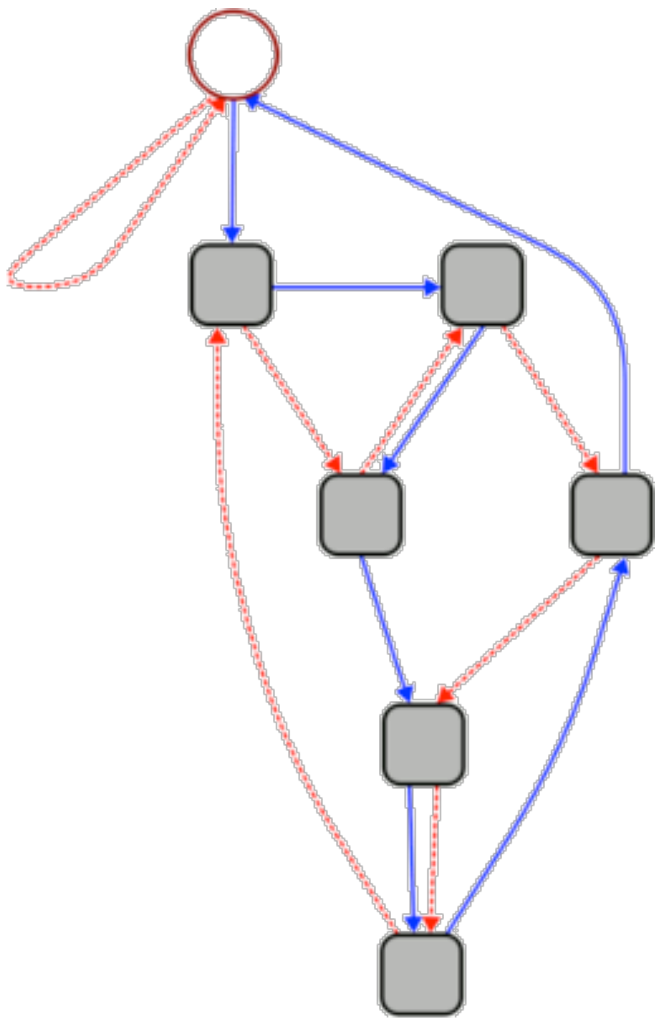

This graph can be used to quickly calculate whether a given number is divisible by 7.

1. Start at the circle node at the top.
2. For each digit **d** in the given number, follow **d** blue (solid) edges in succession. As you move from one digit to the next, follow **1** red (dashed) edge.
3. If you end up back at the circle node, your number is divisible by 7.

3703

"Rule of 7"

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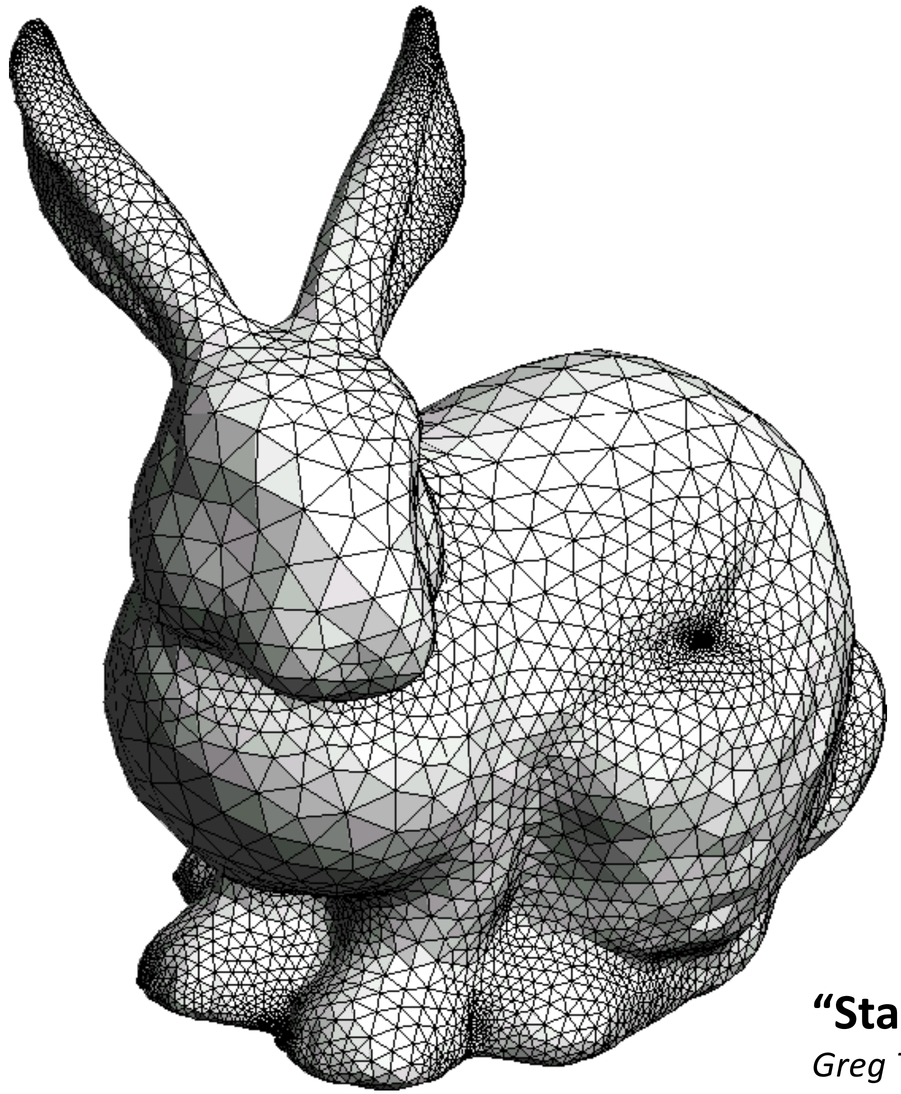
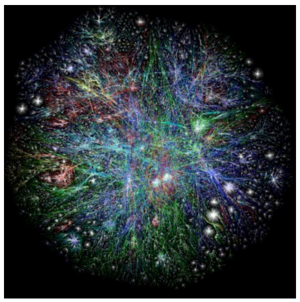
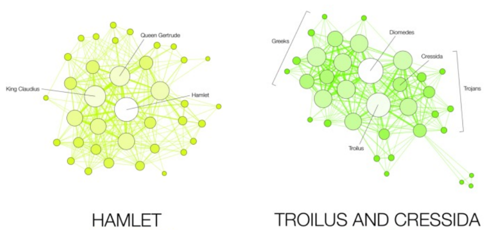
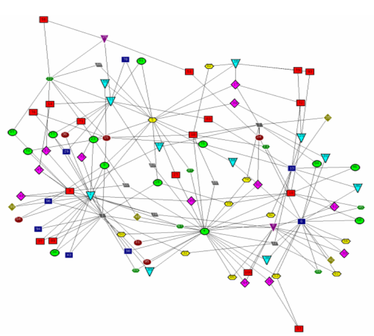


“Stanford Bunny”
Greg Turk and Mark Levoy (1994)

Graphs

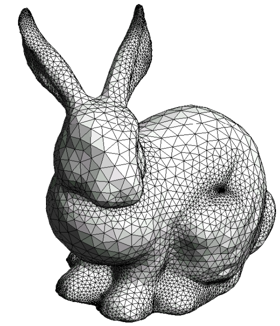


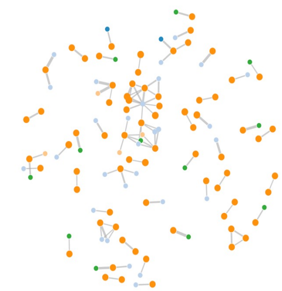


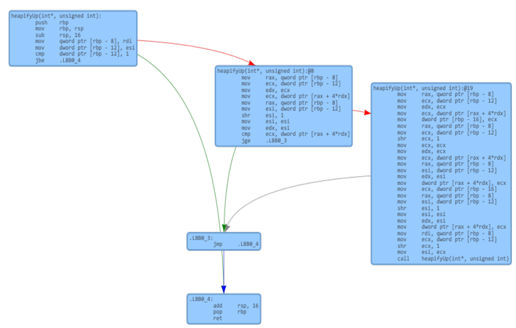


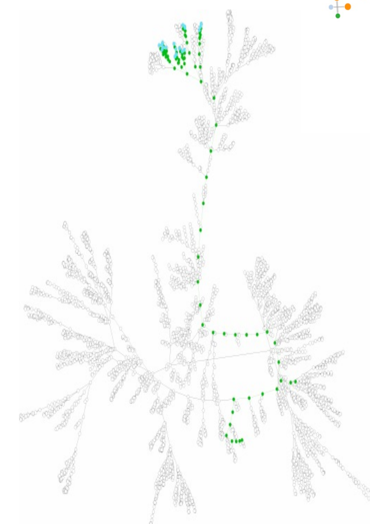
To study all of these structures:

1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms



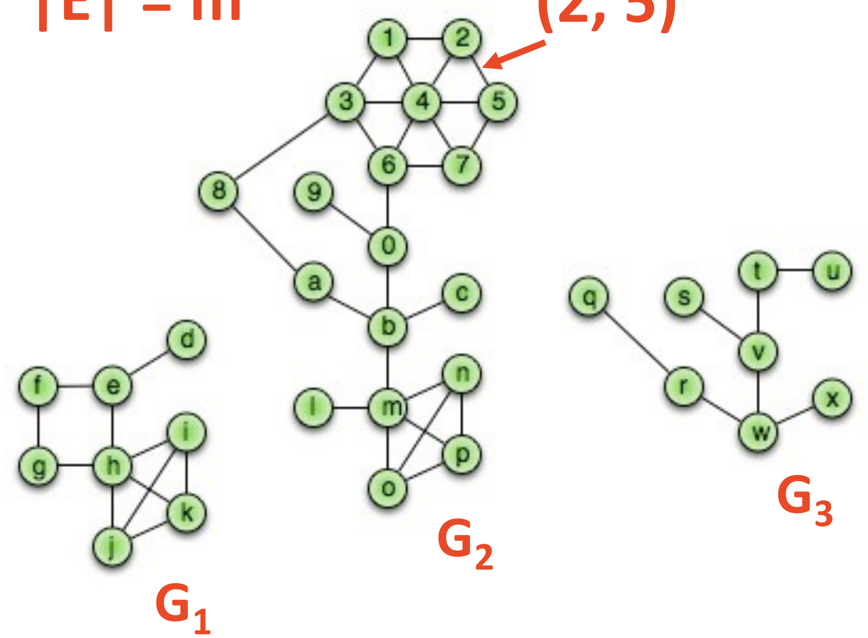






Graph Vocabulary

G = (V, E)
|V| = n
|E| = m



Incident Edges:
I(v) = { {x, v} in E }

Degree(v): |I|

Adjacent Vertices:
A(v) = { x : {x, v} in E }

Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex with at least 3 vertices.

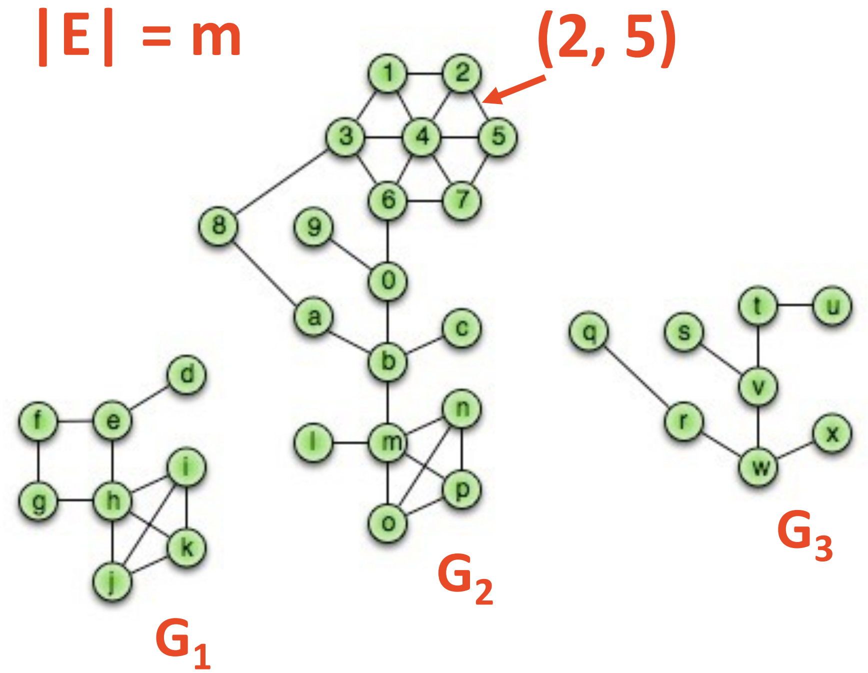
Simple Graph(G): A graph with no self loops or multi-edges.

Graph Vocabulary

$G = (V, E)$

$|V| = n$

$|E| = m$



Subgraph(G):

$G' = (V', E')$:

$V' \in V$, $E' \in E$, and

$(u, v) \in E' \rightarrow u \in V', v \in V'$

Complete subgraph(G)

Connected subgraph(G)

Connected component(G)

Acyclic subgraph(G)

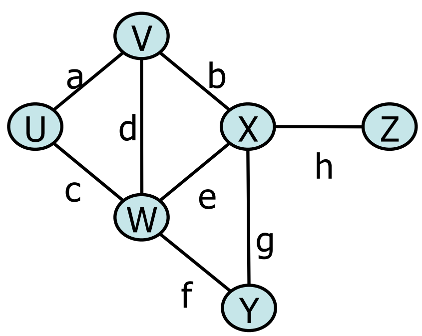
Spanning tree(G)

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

How many edges? **Minimum edges:**

Not Connected:

Connected*:



Maximum edges:

Simple:

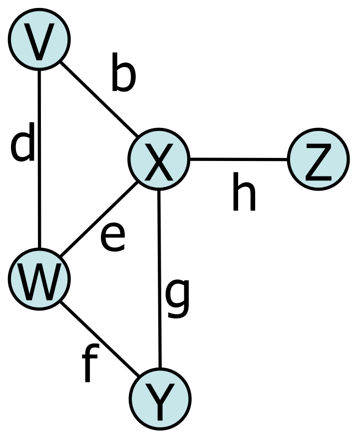
Not simple:

$$\sum_{v \in V} \deg(v) =$$

Graph ADT

Data:

- Vertices
- Edges
- Some data structure maintaining the structure between vertices and edges.



Functions:

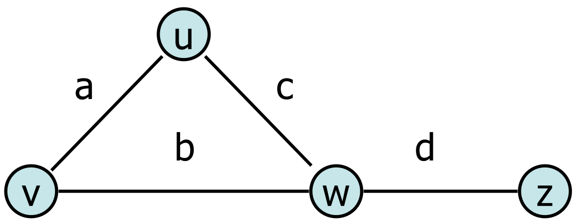
- insertVertex(K key);
- insertEdge(Vertex v1, Vertex v2, K key);

- removeVertex(Vertex v);
- removeEdge(Vertex v1, Vertex v2);

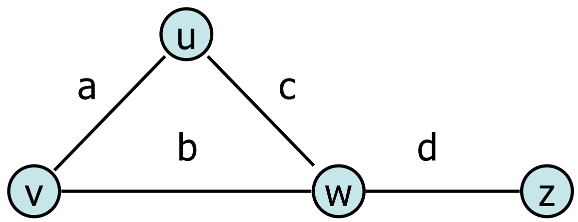
- incidentEdges(Vertex v);
- areAdjacent(Vertex v1, Vertex v2);

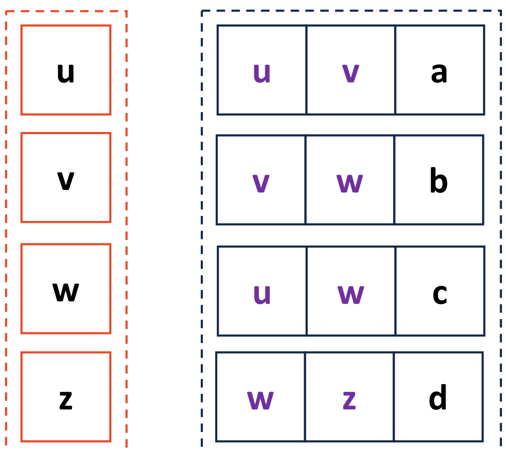
- origin(Edge e);
- destination(Edge e);

Graph Implementation Idea



Graph Implementation: Edge List

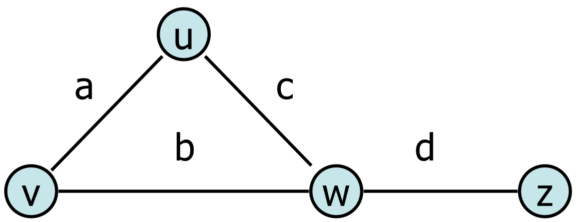


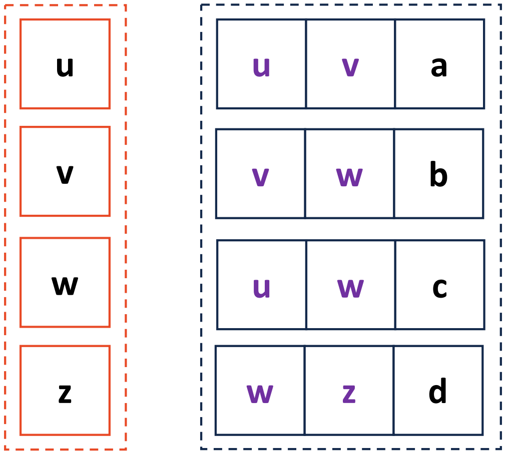


Vertex Collection:

Edge Collection:

Graph Implementation: Edge List

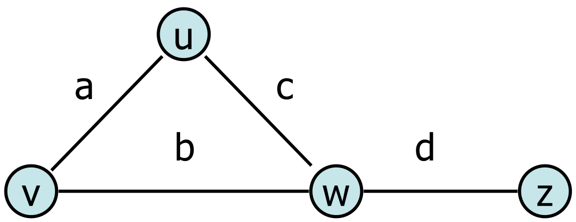




insertVertex(K key):

removeVertex(Vertex v):

Graph Implementation: Edge List



u
v
w
z

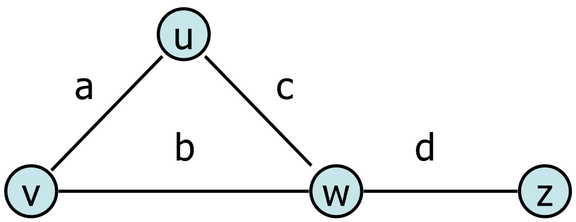
u	v	a
v	w	b
u	w	c
w	z	d

incidentEdges(Vertex v):

areAdjacent(Vertex v1, Vertex v2):

`G.incidentEdges(v1).contains(v2)`

Graph Implementation: Edge List

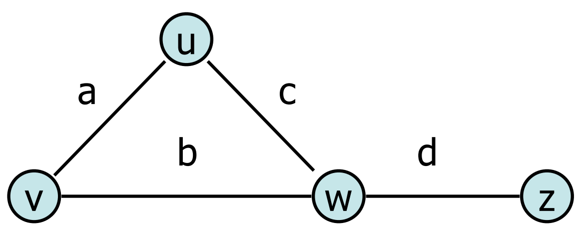


u
v
w
z

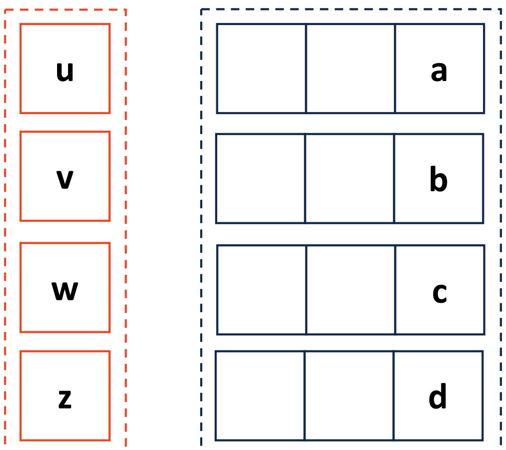
u	**v**	**a**
v	**w**	**b**
u	**w**	**c**
w	**z**	**d**

insertEdge(Vertex v1, Vertex v2, K key):

Graph Implementation: Adjacency Matrix



insertVertex(K key);
removeVertex(Vertex v);
areAdjacent(Vertex v1, Vertex v2);
incidentEdges(Vertex v);



	u	v	w	z
u				
v				
w				
z				