CS 225

Data Structures

April 7 – Dijkstra's Algorithm

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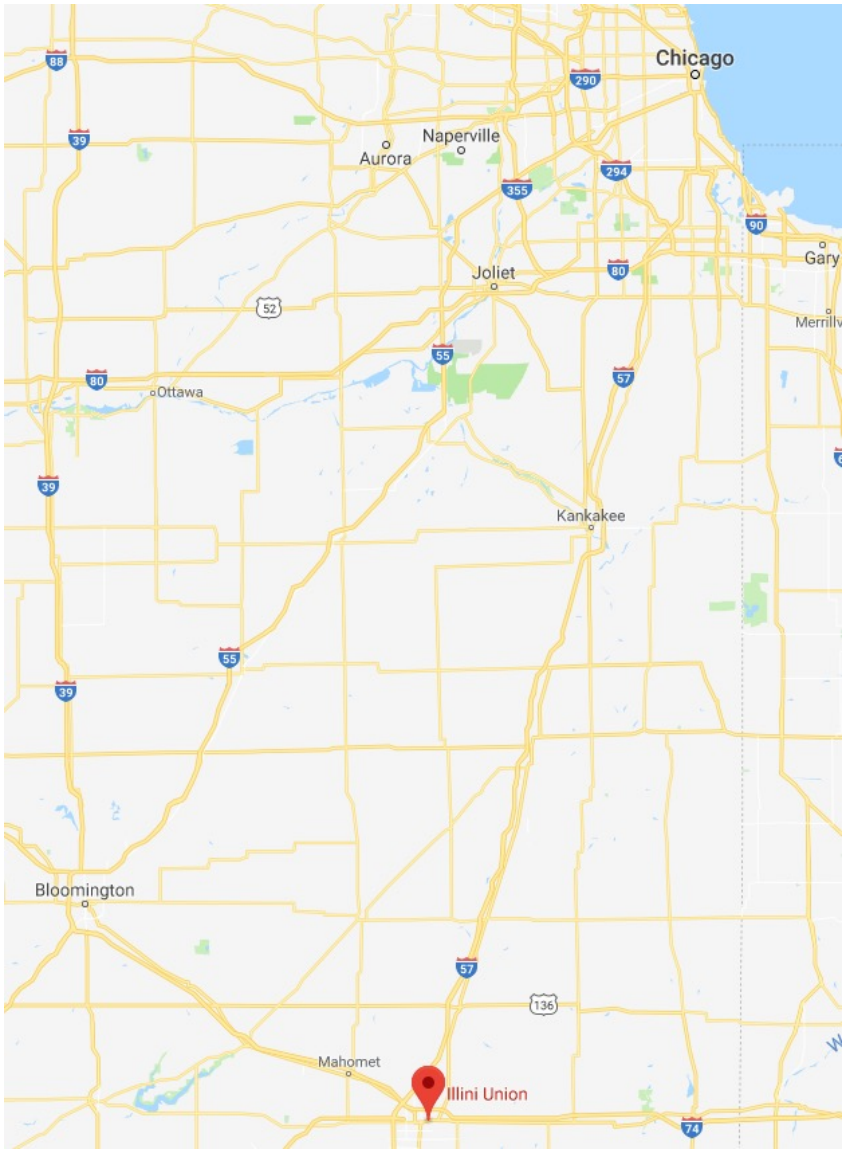
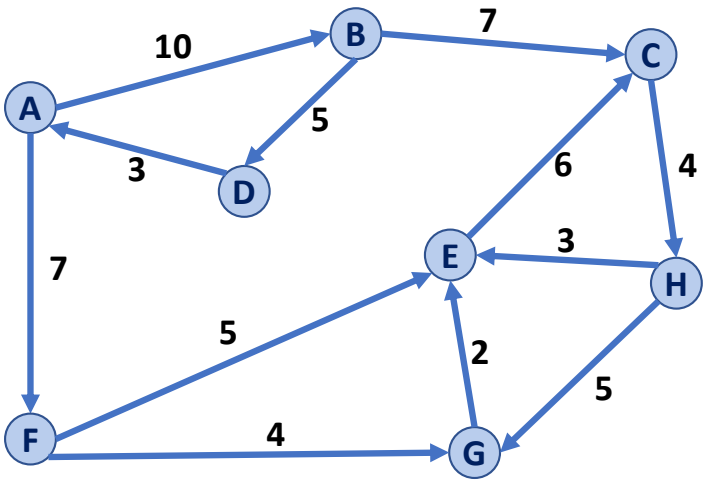
Shortest Path

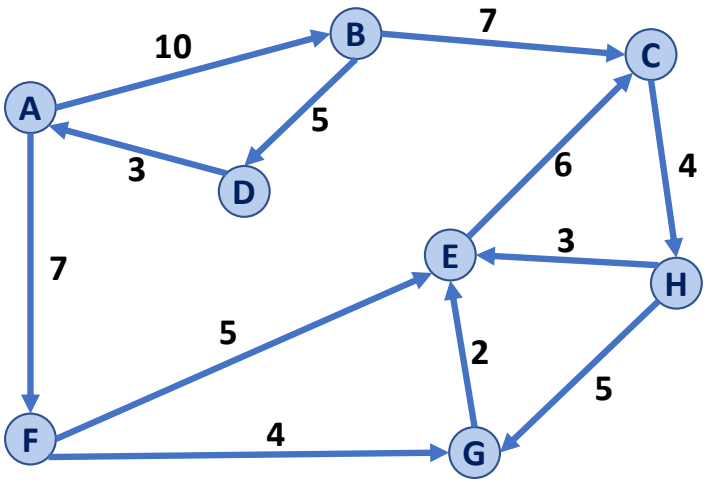


Dijkstra's Algorithm (SSSP)



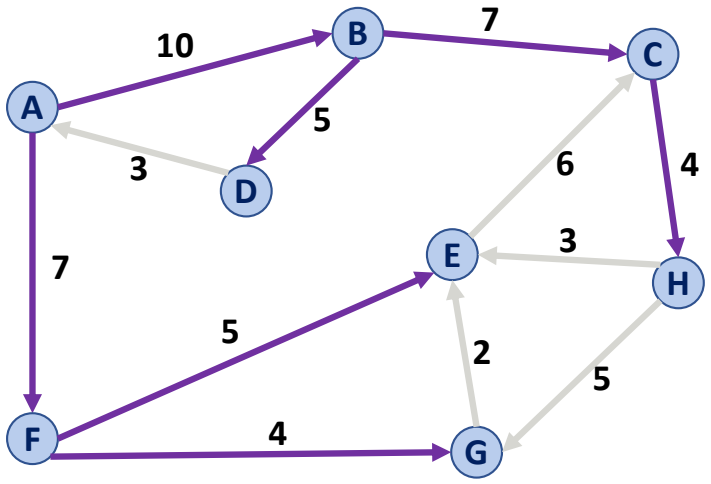

```
    PrimMST(G, s):
 6    foreach (Vertex v : G):
 7      d[v] = +inf
 8      p[v] = NULL
 9    d[s] = 0
10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13
14    repeat n times:
15      Vertex u = Q.removeMin()
16      foreach (Vertex v : neighbors of u not in T):
17        if cost(u, v) < d[v]:
18          d[v] = cost(u, v)
19          p[v] = u
20
21
```

Dijkstra's Algorithm (SSSP)




```
    DijkstraSSSP(G, s):
 6    foreach (Vertex v : G):
 7      d[v] = +inf
 8      p[v] = NULL
 9    d[s] = 0
10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13    Graph T          // "labeled set"
14
15    repeat n times:
16      Vertex u = Q.removeMin()
17      T.add(u)
18      foreach (Vertex v : neighbors of u not in T):
19        if _________________ < d[v]:
20          d[v] = ____________________
21          p[v] = u
```

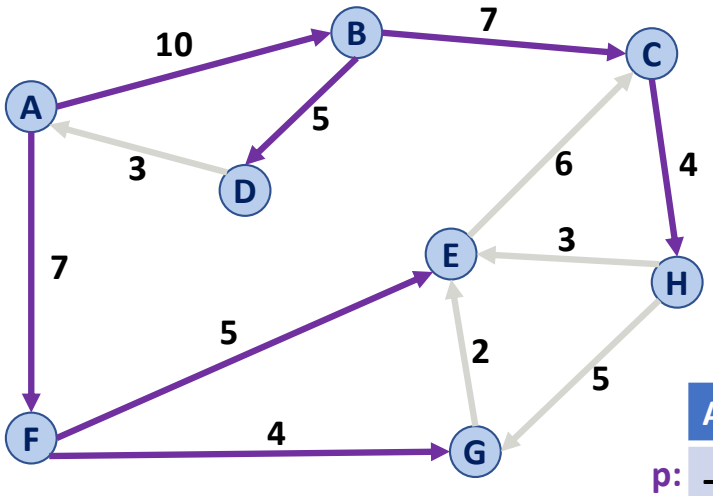
Dijkstra's Algorithm (SSSP)




```
    DijkstraSSSP(G, s):
 6    foreach (Vertex v : G):
 7      d[v] = +inf
 8      p[v] = NULL
 9    d[s] = 0
10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13
14    repeat n times:
15      Vertex u = Q.removeMin()
16      foreach (Vertex v : neighbors of u not in T):
17        if cost(u, v) + d[u] < d[v]:
18          d[v] = cost(u, v) + d[u]
19          p[v] = u
20
21
```

Dijkstra's Algorithm (SSSP)

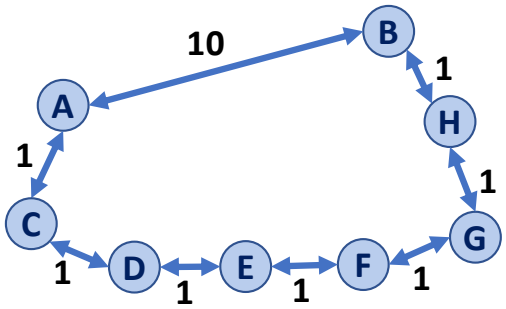
Dijkstra gives us the shortest path from our path (single source) to **every** connected vertex!



	A	B	C	D	E	F	G	H
p:	--	A	B	B	F	A	F	C
d:	0	10	17	15	12	7	11	21

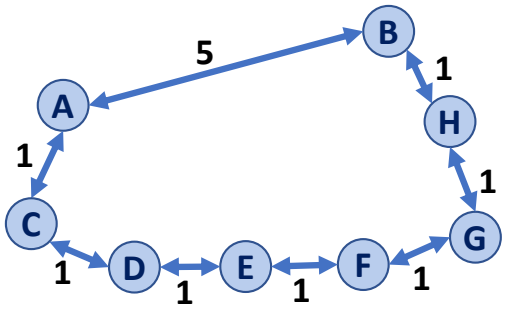
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



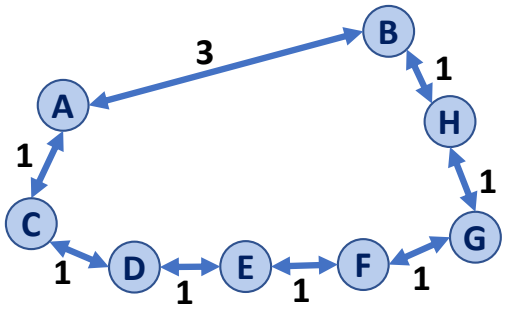
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle a single heavy-weight path vs. many light-weight paths?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle undirected graphs?



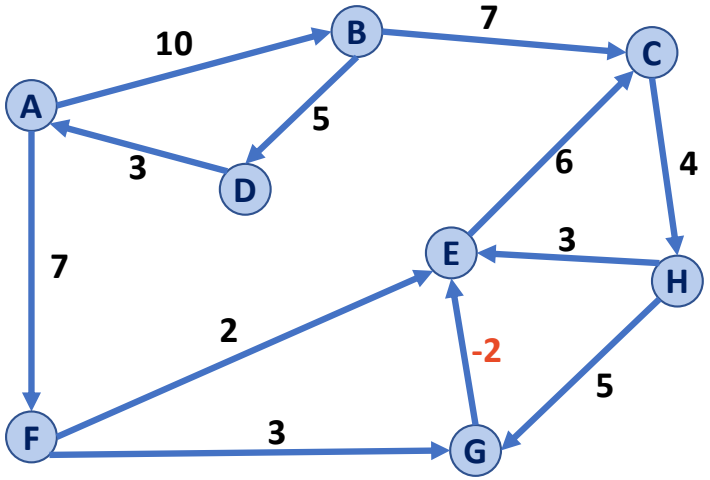
Dijkstra's Algorithm (SSSP)

What is Dijkstra's running time?

```
    DijkstraSSSP(G, s):
 6    foreach (Vertex v : G):
 7      d[v] = +inf
 8      p[v] = NULL
 9    d[s] = 0
10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13
14    repeat n times:
15      Vertex u = Q.removeMin()
16      foreach (Vertex v : neighbors of u not in T):
17        if cost(u, v) + d[u] < d[v]:
18          d[v] = cost(u, v) + d[u]
19          p[v] = u
20
21    return T
22
23
```

Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight edges?

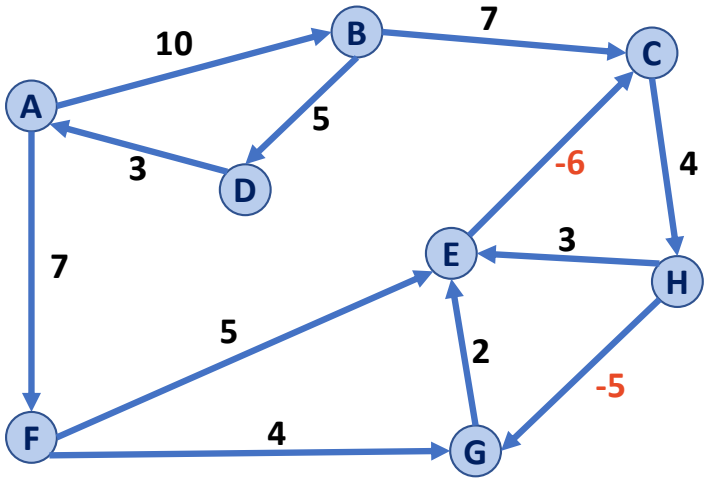


Modified Dijkstra's Algorithm (SSSP)

```
    DijkstraSSSP(G, s):
 6    foreach (Vertex v : G):
 7      d[v] = +inf
 8      p[v] = NULL
 9    d[s] = 0
10
11    PriorityQueue Q // min distance, defined by d[v]
12    Q.buildHeap(G.vertices())
13    Graph T          // "labeled set"
14
15    repeat until Q.empty() times:
16      Vertex u = Q.removeMin()
17      foreach (Vertex v : neighbors of u not in T):
18        if cost(u, v) + d[u] < d[v]:
19          d[v] = cost(u, v) + d[u]
20          p[v] = u
21          Q.push(v)
22
23    return T
```

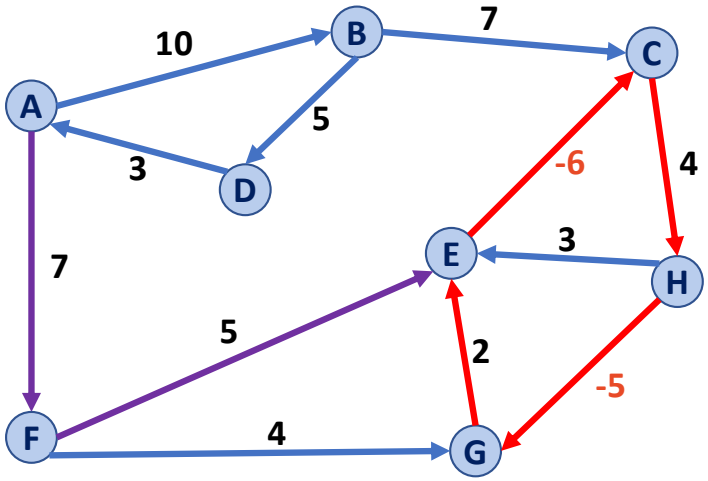
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



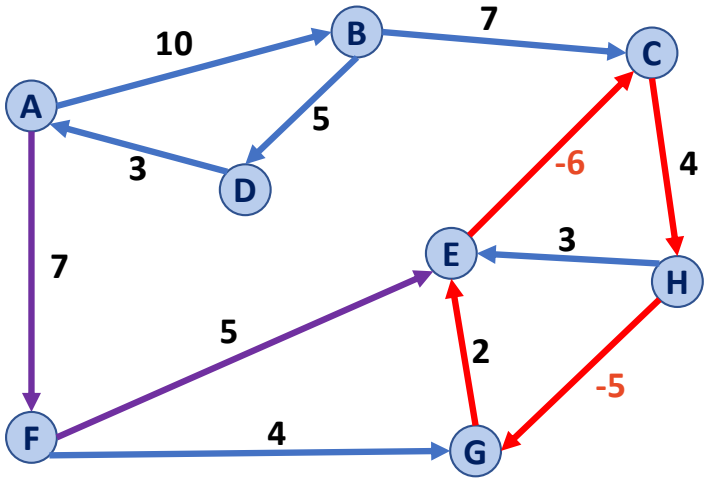
Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Dijkstra's Algorithm (SSSP)

Q: How does Dijkstra handle negative weight cycles?



Shortest Path (A ➔ E): A → F → E → (C → H → G → E)*

Length: 12 Length: -5 (repeatable)