

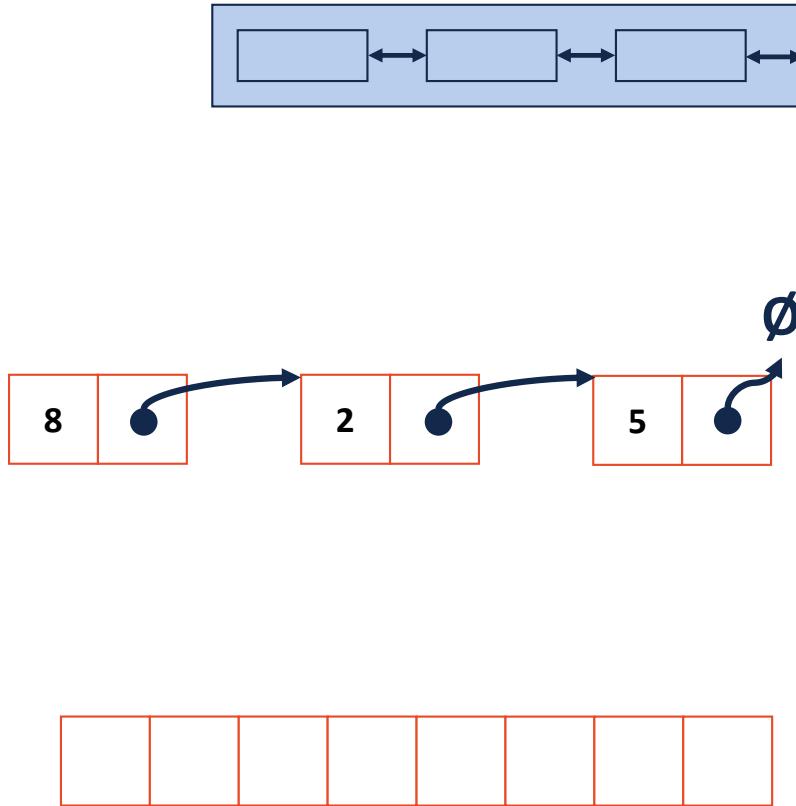


# CS 225

## Data Structures

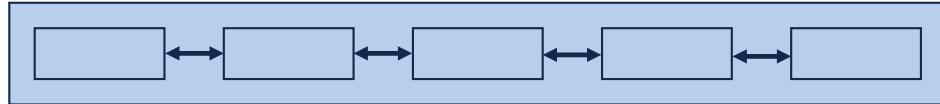
*February 6 – Iterators and Trees*  
*G Carl Evans*

Iterators encapsulated access to our data:



::begin	::end

Iterators encapsulated access to our data:



Get Data	Next Element	Compare
<code>*it</code>	<code>++it</code>	<code>it1 != it2</code>

## stlList.cpp

```
1 #include <list>
2 #include <string>
3 #include <iostream>
4
5 struct Animal {
6     std::string name, food;
7     bool big;
8     Animal(std::string name = "blob", std::string food = "you", bool big = true) :
9         name(name), food(food), big(big) { /* nothing */ }
10    };
11
12 int main() {
13     Animal g("giraffe", "leaves", true), p("penguin", "fish", false), b("bear");
14     std::vector<Animal> zoo;
15
16     zoo.push_back(g);
17     zoo.push_back(p); // std::vector's insertAtEnd
18     zoo.push_back(b);
19
20     for ( std::vector<Animal>::iterator it = zoo.begin(); it != zoo.end(); ++it ) {
21         std::cout << (*it).name << " " << (*it).food << std::endl;
22     }
23
24     return 0;
25 }
```

## stlList.cpp

```
1 #include <list>
2 #include <string>
3 #include <iostream>
4
5 struct Animal {
6     std::string name, food;
7     bool big;
8     Animal(std::string name = "blob", std::string food = "you", bool big = true) :
9         name(name), food(food), big(big) { /* nothing */ }
10    };
11
12 int main() {
13     Animal g("giraffe", "leaves", true), p("penguin", "fish", false), b("bear");
14     std::vector<Animal> zoo;
15
16     zoo.push_back(g);
17     zoo.push_back(p); // std::vector's insertAtEnd
18     zoo.push_back(b);
19
20     for ( auto it = zoo.begin(); it != zoo.end(); ++it ) {
21         std::cout << (*it).name << " " << (*it).food << std::endl;
22     }
23
24     return 0;
25 }
```

## stlList.cpp

```
1 #include <list>
2 #include <string>
3 #include <iostream>
4
5 struct Animal {
6     std::string name, food;
7     bool big;
8     Animal(std::string name = "blob", std::string food = "you", bool big = true) :
9         name(name), food(food), big(big) { /* none */ }
10    };
11
12 int main() {
13     Animal g("giraffe", "leaves", true), p("penguin", "fish", false), b("bear");
14     std::vector<Animal> zoo;
15
16     zoo.push_back(g);
17     zoo.push_back(p); // std::vector's insertAtEnd
18     zoo.push_back(b);
19
20     for ( const Animal & animal : zoo ) {
21         std::cout << animal.name << " " << animal.food << std::endl;
22     }
23
24     return 0;
25 }
```

# For Each and Iterators

```
for ( const TYPE & variable : collection ) {  
    // ...  
}
```

```
14 std::vector<Animal> zoo;  
...  
20 for ( const Animal & animal : zoo ) {  
21     std::cout << animal.name << " " << animal.food << std::endl;  
22 }
```

# Trees

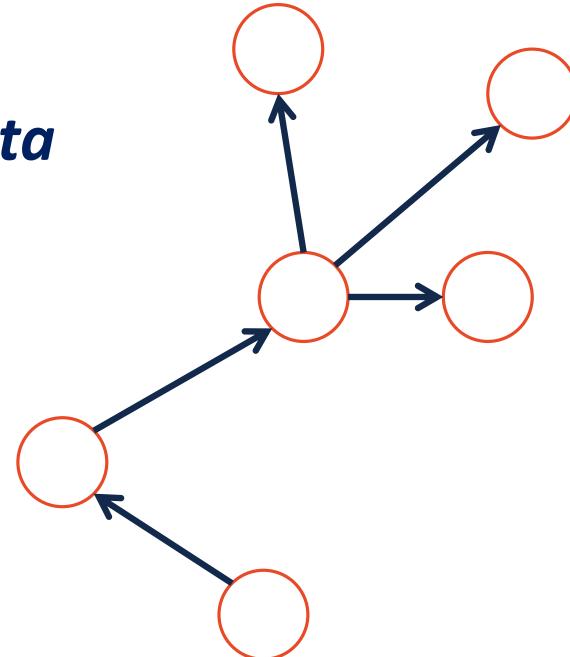
*“The most important non-linear data structure in computer science.”*

- David Knuth, *The Art of Programming, Vol. 1*

A tree is:

- 

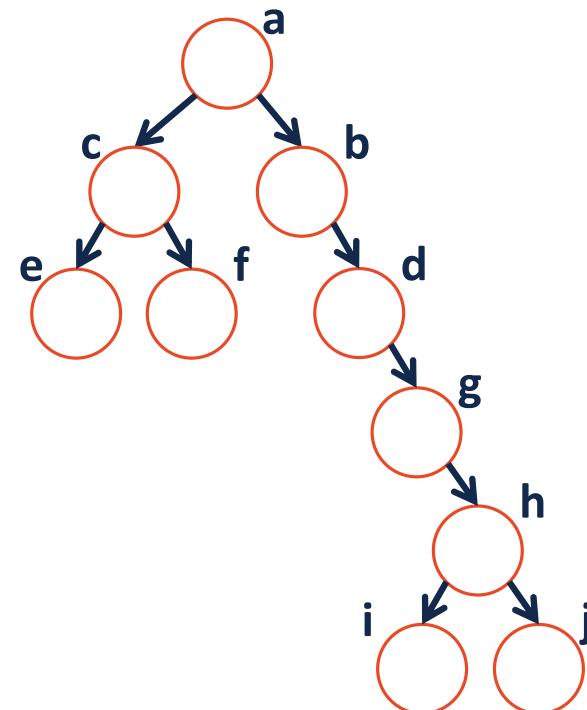
- 



# More Specific Trees

We'll focus on **binary trees**:

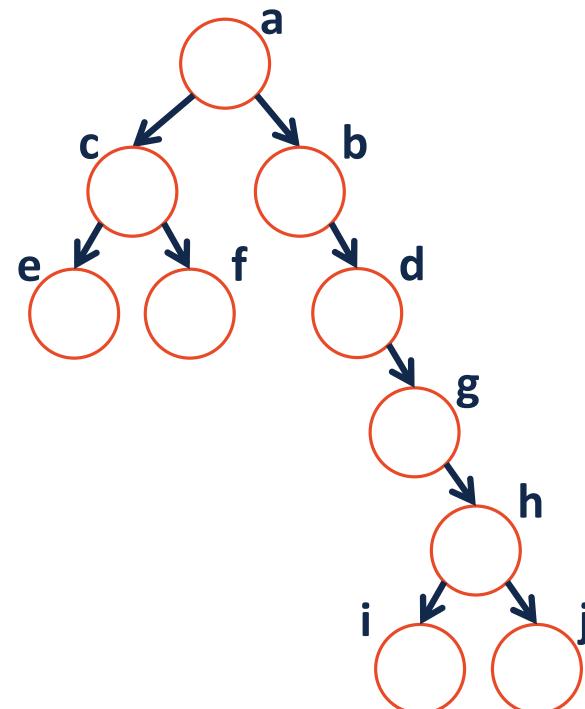
- A binary tree is **acyclic** – there are no cycles within the graph



# More Specific Trees

We'll focus on **binary trees**:

- A binary tree contains **two or fewer children** – where one is the “left child” and one is the “right child”:



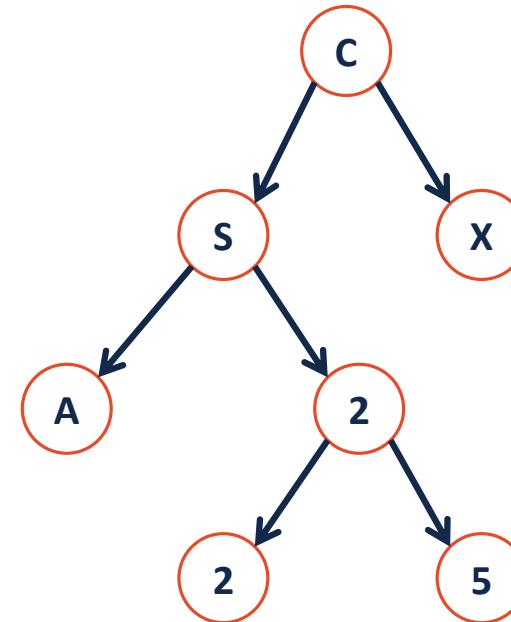
# Binary Tree – Defined

A *binary tree T* is either:

- 

OR

- 

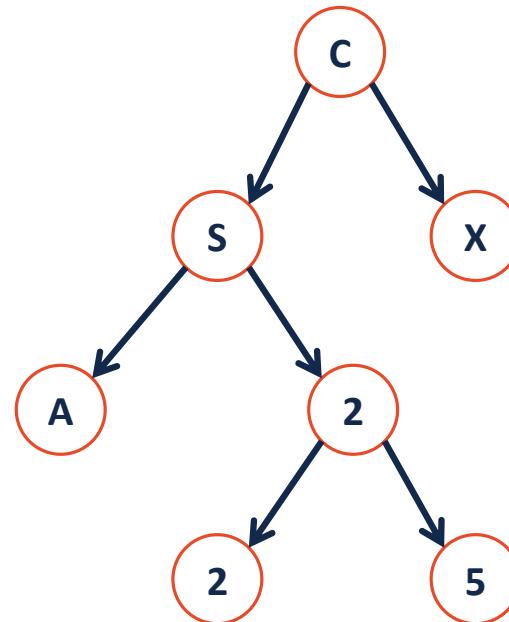


## Tree Property: height

**$height(T)$ :** length of the longest path from the root to a leaf

Given a binary tree T:

**$height(T) =$**

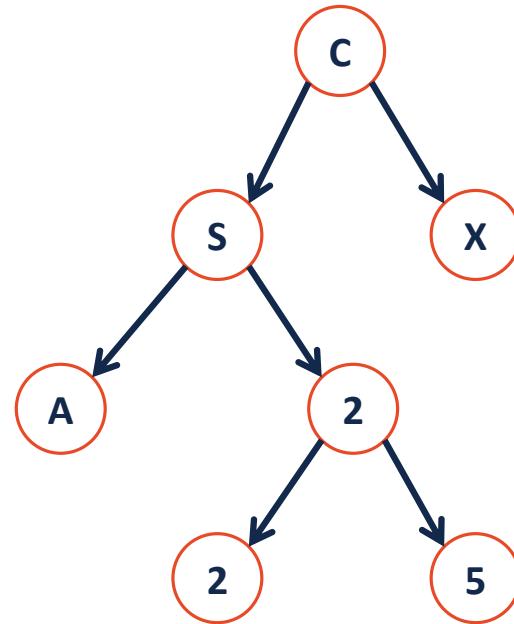


# Tree Property: full

A tree  $F$  is **full** if and only if:

1.

2.



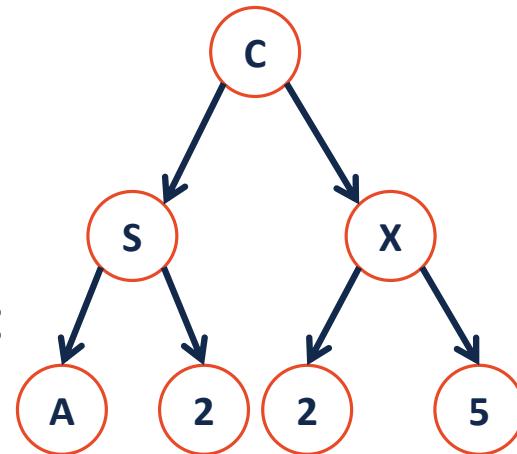
## Tree Property: perfect

A **perfect** tree  $P$  is defined in terms of the tree's height.

Let  $P_h$  be a perfect tree of height  $h$ , and:

1.

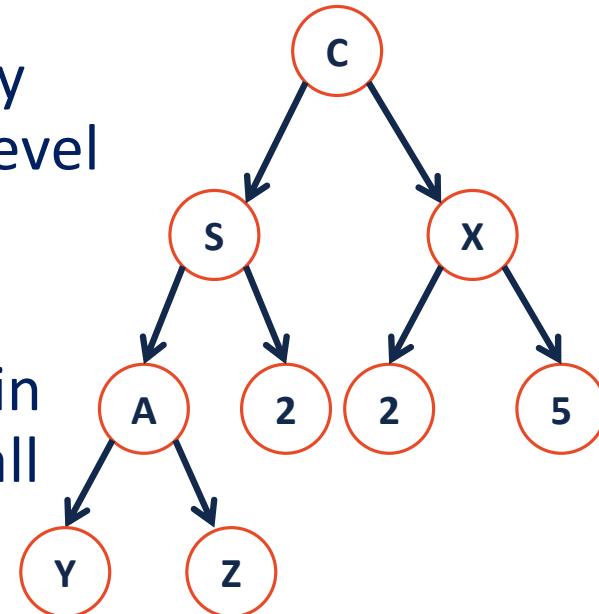
2.



## Tree Property: complete

**Conceptually:** A perfect tree for every level except the last, where the last level is “pushed to the left”.

**Slightly more formal:** For all levels  $k$  in  $[0, h-1]$ ,  $k$  has  $2^k$  nodes. For level  $h$ , all nodes are “pushed to the left”.



# Tree Property: complete

A **complete** tree  $C$  of height  $h$ ,  $C_h$ :

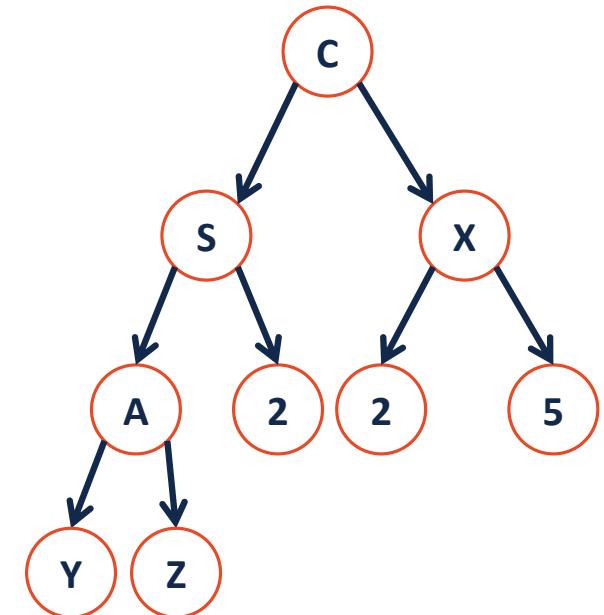
1.  $C_{-1} = \{\}$

2.  $C_h$  (*where  $h > 0$* ) =  $\{r, T_L, T_R\}$  and either:

$T_L$  is \_\_\_\_\_ and  $T_R$  is \_\_\_\_\_

OR

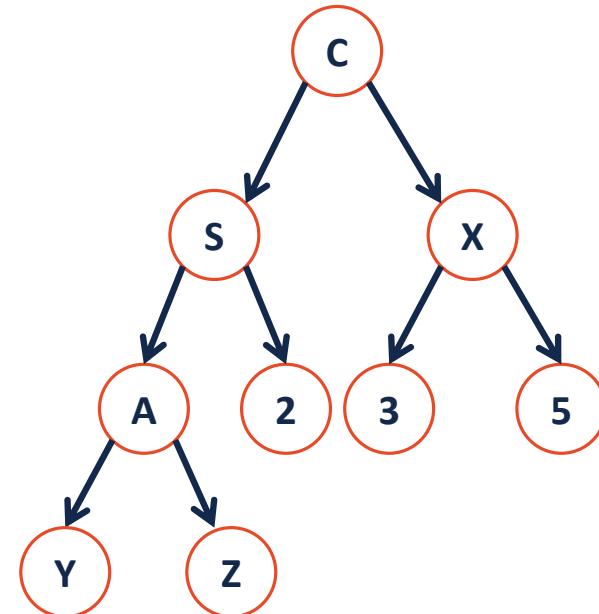
$T_L$  is \_\_\_\_\_ and  $T_R$  is \_\_\_\_\_



# Tree Property: complete

Is every **full** tree **complete**?

If every **complete** tree **full**?





# Tree ADT

# Tree ADT

**insert**, inserts an element to the tree.

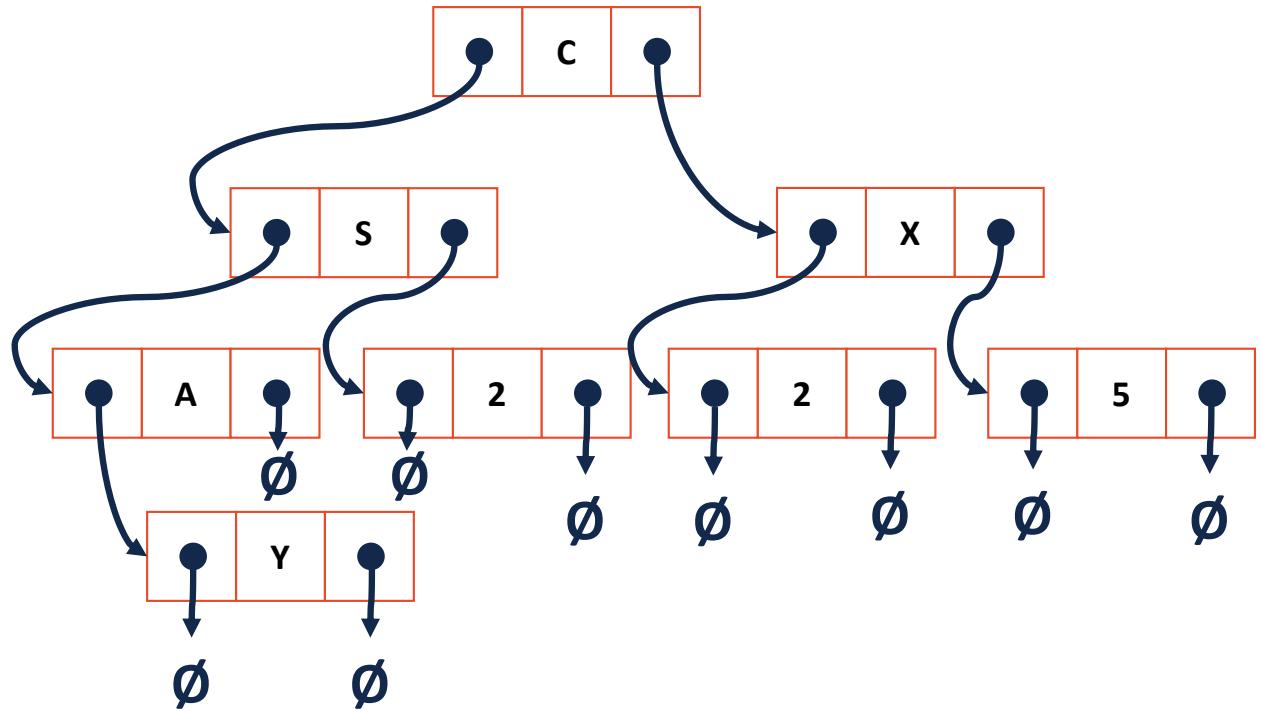
**remove**, removes an element from the tree.

**traverse**,

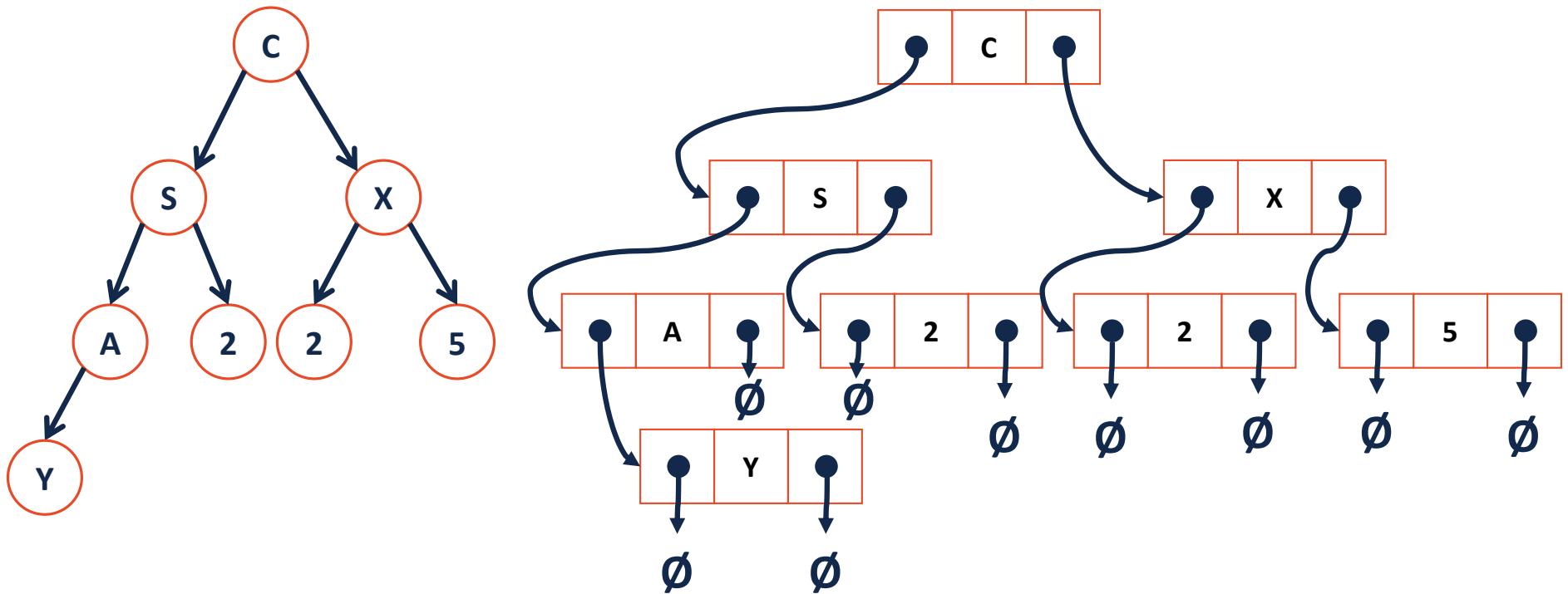
## BinaryTree.h

```
1 #pragma once
2
3 template <class T>
4 class BinaryTree {
5     public:
6         /* ... */
7
8     private:
9
10
11
12
13
14
15
16
17
18
19 };
```

Trees aren't new:



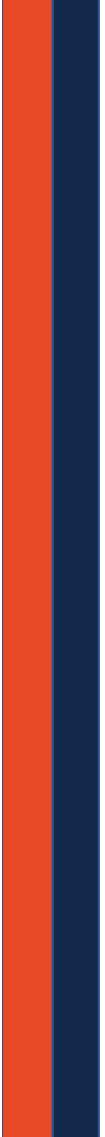
Trees aren't new:





## How many NULLs?

**Theorem:** If there are  $n$  data items in our representation of a binary tree, then there are \_\_\_\_\_ NULL pointers.



# How many NULLs?

**Base Cases:**

**n = 0:**

**n = 1:**

**n = 2:**



# How many NULLs?

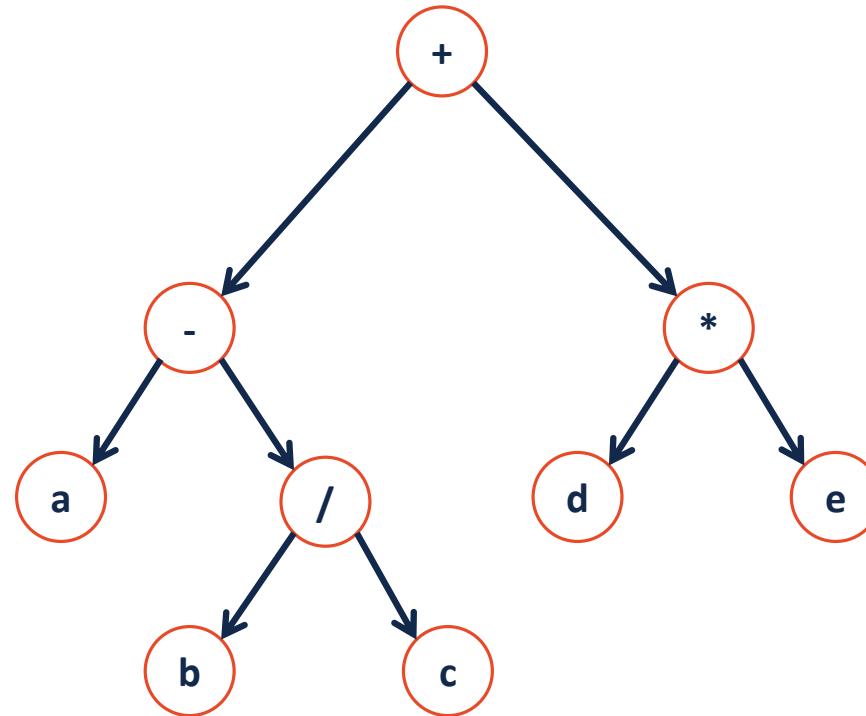
**Induction Hypothesis:**



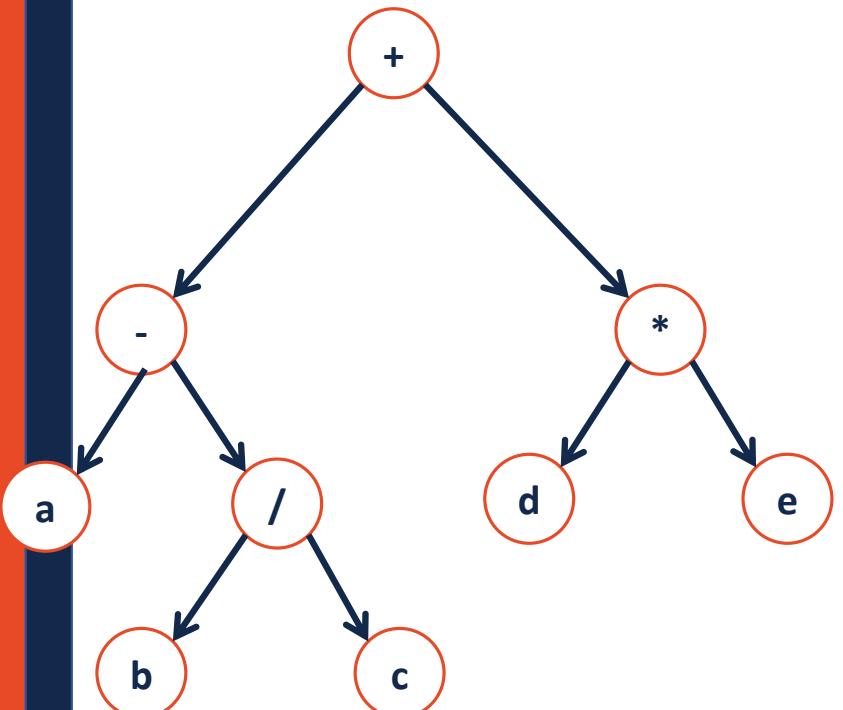
## How many NULLs?

Consider an arbitrary tree  $T$  containing  $n$  data elements:

# Traversals

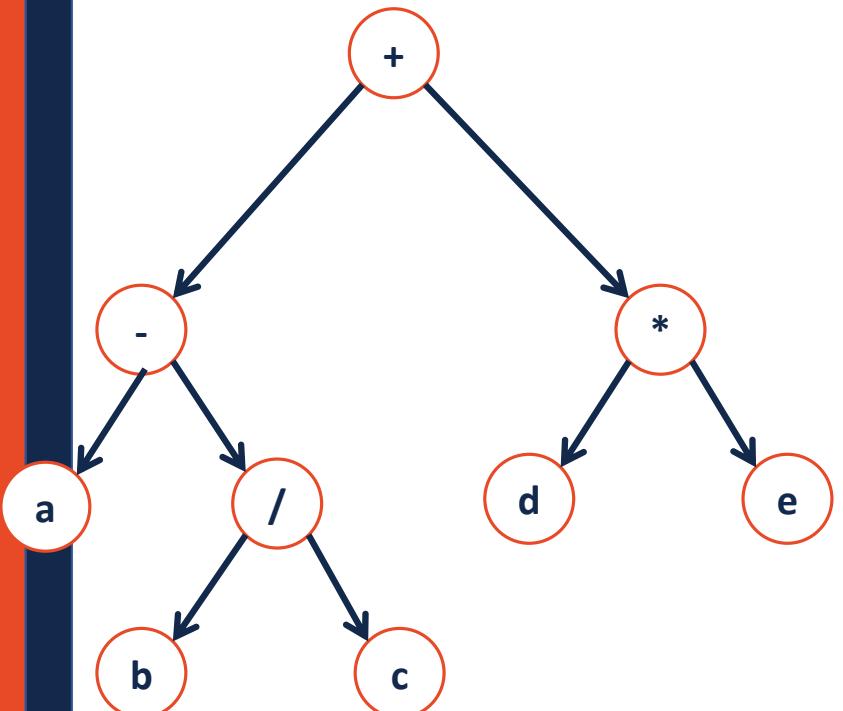


# Traversals



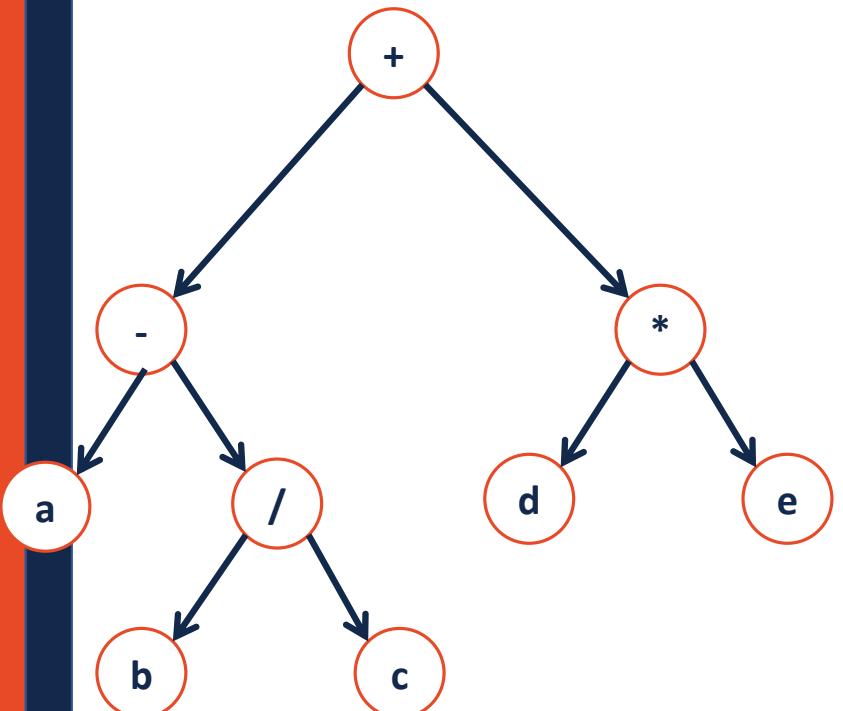
```
1 template<class T>
2 void BinaryTree<T>::__Order(TreeNode * root)
3 {
4     if (root != NULL) {
5
6         _____;
7
8         __Order(root->left);
9
10        _____;
11
12        __Order(root->right);
13
14        _____;
15
16    }
17 }
```

# Traversals



```
1 template<class T>
2 void BinaryTree<T>::__Order(TreeNode * root)
3 {
4     if (root != NULL) {
5
6         _____;
7
8         __Order(root->left);
9
10        _____;
11
12        __Order(root->right);
13
14        _____;
15
16    }
17 }
```

# Traversals



```
1 template<class T>
2 void BinaryTree<T>::__Order(TreeNode * root)
3 {
4     if (root != NULL) {
5
6         _____;
7
8         __Order(root->left);
9
10        _____;
11
12        __Order(root->right);
13
14        _____;
15
16    }
17 }
```