This assignment is best completed in groups. Choose the following roles for yourselves. If you only have three people, combine Reporter and Process Analyst.

<table>
<thead>
<tr>
<th>Team Roles</th>
<th>Team Member</th>
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<tbody>
<tr>
<td><strong>Manager</strong></td>
<td>Reads the questions out loud, keeps track of time, and makes sure everyone contributes appropriately.</td>
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<tr>
<td><strong>Reporter</strong></td>
<td>Talks to the instructor and other teams.</td>
</tr>
<tr>
<td><strong>Quality Control</strong></td>
<td>Records all answers and questions and provides team reflection to team and instructor.</td>
</tr>
<tr>
<td><strong>Process Analyst</strong></td>
<td>Considers how the team could work and learn more effectively.</td>
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Objectives

We are going to learn an algorithm for determining the network flow capacity of a graph. In the following graphs, the edge weights represent capacity. There are four sections: spend 10–15 minutes on each one.

Graph G1

We are given as input graph G1.

We create two new graphs: a flow graph F and a residual graph R.

Problem 1.

The algorithm works by selecting paths from the residual graph R. The first path selected is $A \rightarrow B \rightarrow C \rightarrow F$ in graph $R$. This path’s flow capacity is 3. What do you think determines the flow capacity?
The algorithm uses the path to modify graphs $F$ and $R$. Here is the result.

**Graph $F$**

```
A  B  C  D  E  F
0  3  3  0  0  0
```

**Graph $R$**

```
A  B  C  D  E  F
4  5  0  4  7  8
```

**Problem 2.**

Examine the new versions of $F$ and $R$ above. What is being done with the path selected from $R$ to modify these graphs?

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**Problem 3.**

The next path selected was $A \rightarrow D \rightarrow E \rightarrow F$ in graph $R$. What is the flow capacity of that path?

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The resulting working graphs are these:

**Graph $F$**

```
A  B  C  D  E  F
4  3  3  4  4  4
```

**Graph $R$**

```
A  B  C  D  E  F
0  5  0  0  3  4
```

**Problem 4.**

We select path $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$. What is the flow capacity of that path?

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**Problem 5.**

The paths selected always start from node $A$ and end with node $F$. What is different about these nodes compared to the others?
Here are the final working graphs $F$ and $R$.

Problem 6.
At this point, the algorithm is finished. How can we know the algorithm is done by examining graph $R$?

Problem 7.
For nodes $B$, $C$, $D$, and $E$, what is the relationship between the in-flows and the out-flows? Why does that relationship have to exist?

Problem 8.
Using the final flow graph $F$ above, determine the maximum flow of graph $G_1$.

Problem 9.
In graph $F$, the outflow of $A$ is equal to the inflow of $F$. Should that always be the case?

Problem 10.
Node $A$ is called a source node and node $F$ is called a sink node. Would this technique work if there were multiple source and sink nodes? Why or why not?
Graph $G_2$

Now we are going to look at a case that messes up the algorithm.

![Graph $G_2$ Flow Graph Residual Graph](image_url)

**Problem 11.**
The algorithm picks path $A \rightarrow B \rightarrow C \rightarrow D$. What is the capacity of that path?

**Problem 12.**
Update the flow and residual graphs as a result of selecting this path.

![Flow Graph Residual Graph](image_url)

**Problem 13.**
Select path $A \rightarrow B \rightarrow D$ from the above residual graph. What is the capacity of that path?

**Problem 14.**
Update the flow and residual graphs as a result of selecting this path.
Problem 15.
Select path $A \rightarrow C \rightarrow D$. What is the capacity of that path?

Problem 16.
Update the flow and residual graphs as a result of selecting this path.

At this point, the algorithm is finished.

Problem 17.
What is the maximum network flow of $G_2$, according to the algorithm?

Problem 18.
Is this number correct? Why or why not? Examine $G_2$ to verify your answer.

Problem 19.
Suppose a maximum flow is running through $G_2$. What would be the flow on edge $B \rightarrow C$ in this situation? Would it change the total flow of $G_2$ if we deleted $B \rightarrow C$?
Graph $G_3$

We are going to modify the algorithm. Starting again with the previous graph, we make a new kind of residual graph. The dotted edges are added, and are legal edges to be traversed in the residual graph.

![Graph $G_3$](image1)

Flow Graph

Residual Graph

Problem 20.

Select path $A \rightarrow B \rightarrow C \rightarrow D$. What is the capacity of that path?

Here are the updated flow and residual graphs:

![Flow Graph](image2)

![Residual Graph](image3)

Now we select path $A \rightarrow C \rightarrow B \rightarrow D$.

Here are the updated flow and residual graphs:

![Flow Graph](image4)

![Residual Graph](image5)
**Problem 21.**
Our algorithm adds capacity to the reverse edges when we update the residual graph. In your own words, can you explain what the reverse edges represent?

**Problem 22.**
Select path $A \rightarrow B \rightarrow C \rightarrow D$. (Yes, we are repeating this path.) What are the resulting flow and residual graphs?

![Flow Graph](image1)

![Residual Graph](image2)

**Problem 23.**
Now we select path $A \rightarrow C \rightarrow B \rightarrow D$.
What are the updated flow and residual graphs?

![Flow Graph](image3)

![Residual Graph](image4)

**Problem 24.**
At this point, the algorithm should be done. Is the final network flow accurate now?
Reflection

Congratulations! You have discovered the Ford-Fulkerson method of determining the maximum flow of a network!

**Problem 25.**
What was the strongest aspect of your performance as a team?

**Problem 26.**
How could your team perform even more effectively on the next activity?

**Problem 27.**
What insights did you have working on this activity?