



# CS 225

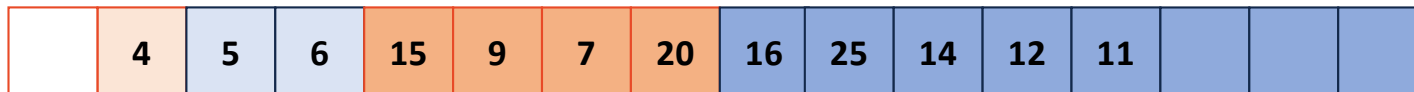
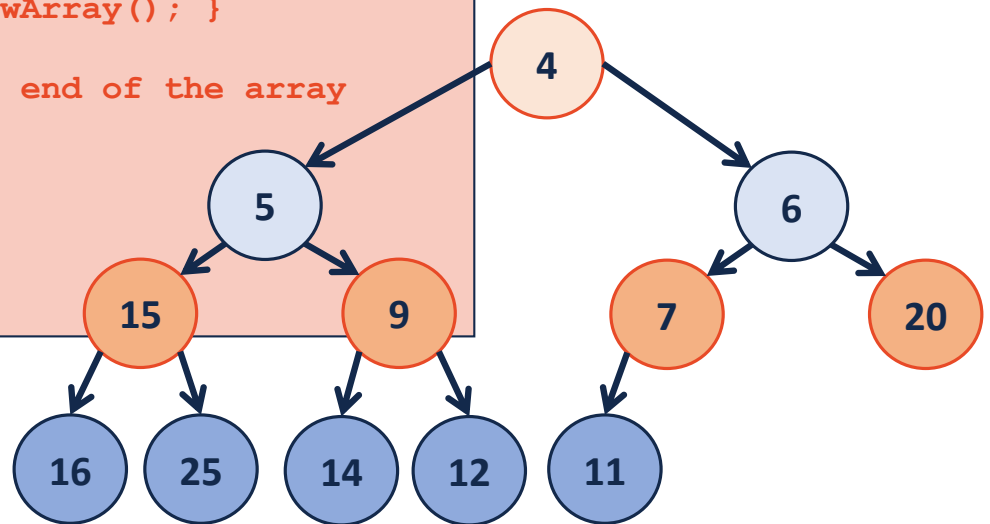
## Data Structures

*March 11– Heaps and Priority Queues*

*G Carl Evans*

# insert

```
1 template <class T>
2 void Heap<T>::_insert(const T & key) {
3     // Check to ensure there's space to insert an element
4     // ...if not, grow the array
5     if ( size_ == capacity_ ) { _growArray(); }
6
7     // Insert the new element at the end of the array
8     item_[++size] = key;
9
10    // Restore the heap property
11    _heapifyUp(size);
12 }
```





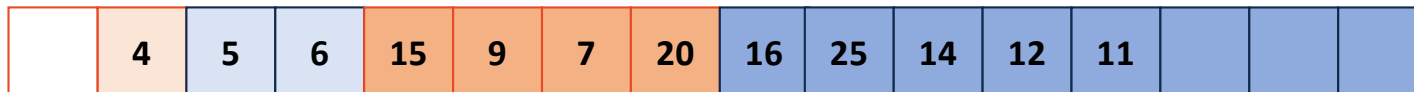
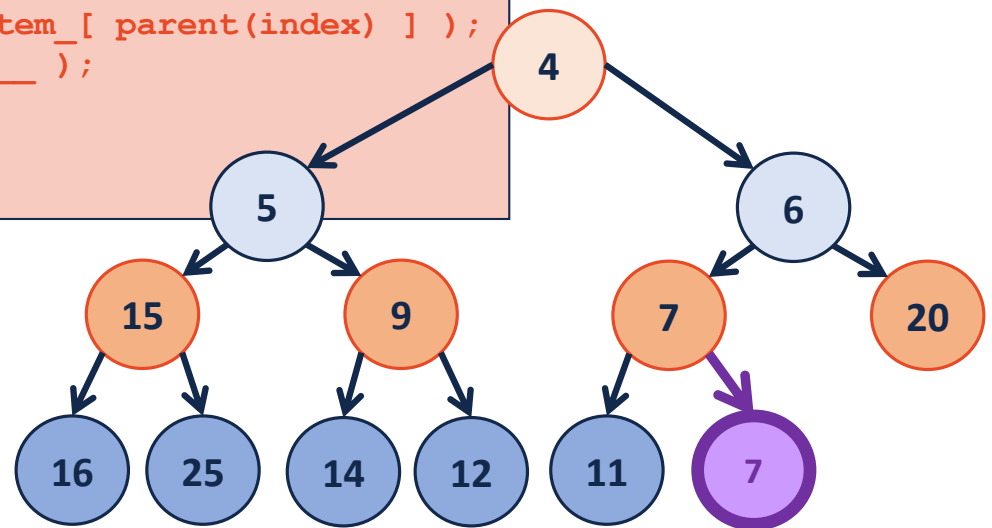
# insert - heapifyUp

```
1 template <class T>
2 void Heap<T>::_insert(const T & key) {
3     // Check to ensure there's space to insert an element
4     // ...if not, grow the array
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11    _heapifyUp(size);
12 }
```

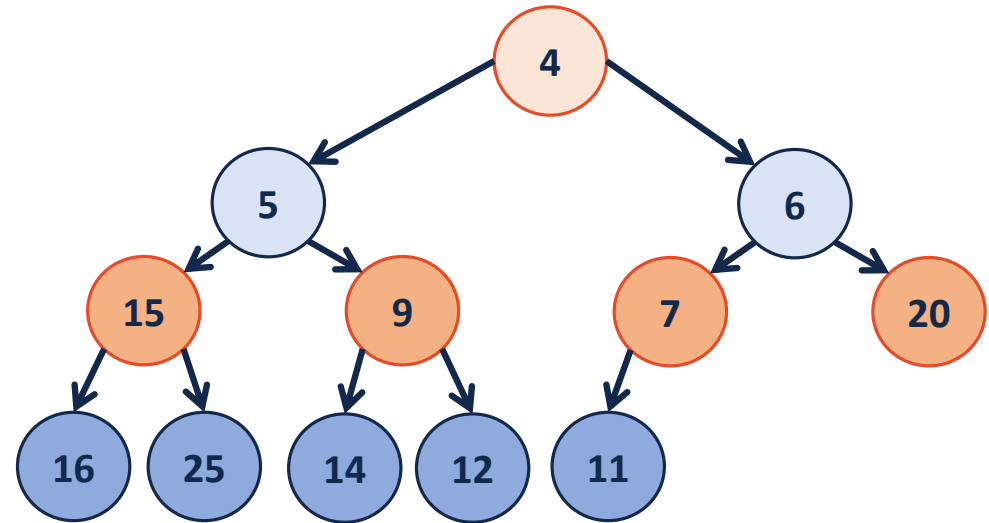
```
1 template <class T>
2 void Heap<T>::_heapifyUp( _____ ) {
3     if ( index > _____ ) {
4         if ( item_[index] < item_[ parent(index) ] ) {
5             std::swap( item_[index], item_[ parent(index) ] );
6             _heapifyUp( _____ );
7         }
8     }
9 }
```

# heapifyUp

```
1 template <class T>
2 void Heap<T>::_heapifyUp( _____ ) {
3     if ( index > _____ ) {
4         if ( item_[index] < item_[ parent(index) ] ) {
5             std::swap( item_[index], item_[ parent(index) ] );
6             _heapifyUp( _____ );
7         }
8     }
9 }
```



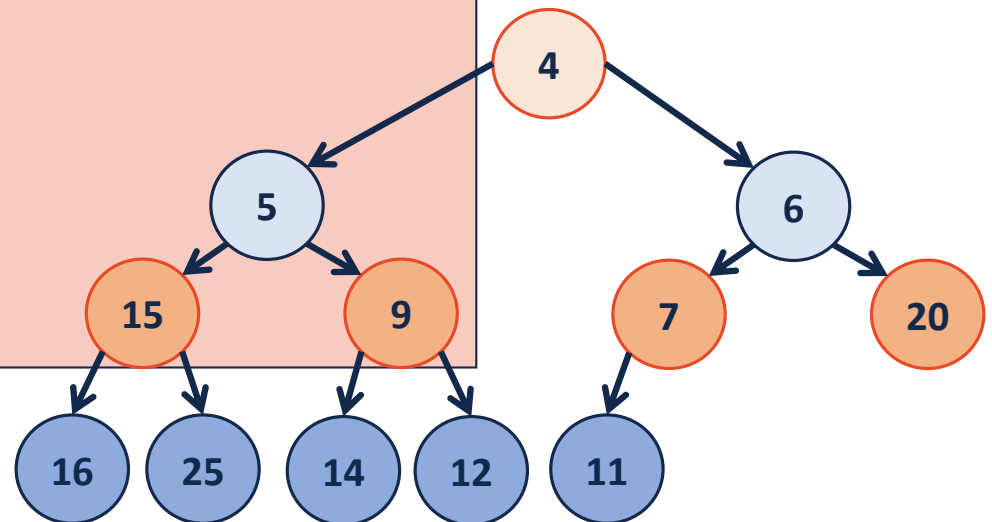
# removeMin



	4	5	6	15	9	7	20	16	25	14	12	11			
--	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--

# removeMin

```
1 template <class T>
2 void Heap<T>::_removeMin() {
3     // Swap with the last value
4     T minValue = item_[1];
5     item_[1] = item_[size_];
6     size--;
7
8     // Restore the heap property
9     heapifyDown();
10
11    // Return the minimum value
12    return minValue;
13 }
```



# removeMin - heapifyDown

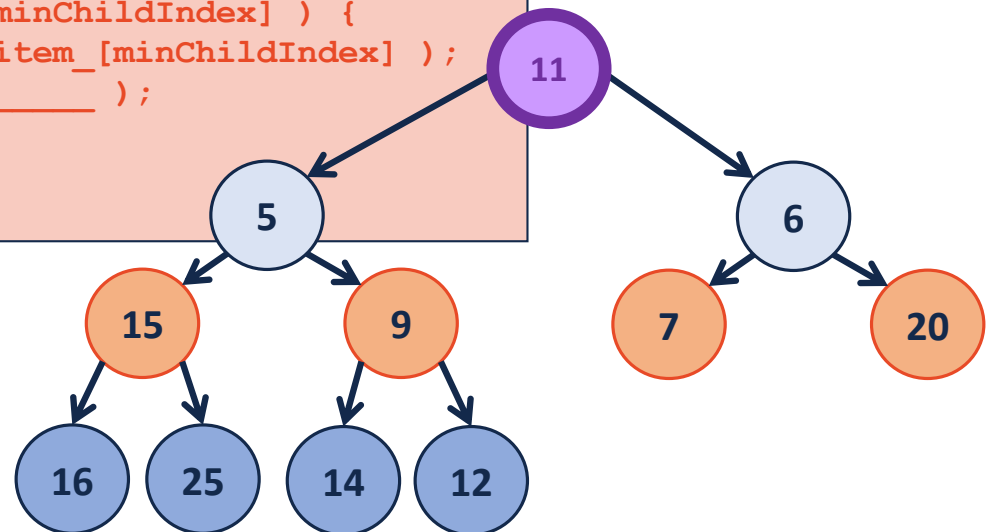
```
1  template <class T>
2  void Heap<T>::_removeMin() {
3      // Swap with the last value
4      T minValue = item_[1];
5      item_[1] = item_[size_];
6      size--;
7
8      // Restore the heap property
9      _heapifyDown();
10
11     // Return the minimum value
12     return minValue;
13 }
```

```
1  template <class T>
2  void Heap<T>::_heapifyDown(int index) {
3      if ( !_isLeaf(index) ) {
4          T minChildIndex = _minChild(index);
5          if ( item_[index] > item_[minChildIndex] ) {
6              std::swap( item_[index], item_[minChildIndex] );
7              _heapifyDown( minChildIndex );
8          }
9      }
10 }
```



# removeMin

```
1 template <class T>
2 void Heap<T>::_heapifyDown(int index) {
3     if ( !_isLeaf(index) ) {
4         T minChildIndex = _minChild(index);
5         if ( item_[index] > item_[minChildIndex] ) {
6             std::swap( item_[index], item_[minChildIndex] );
7             _heapifyDown( minChildIndex );
8         }
9     }
10 }
```

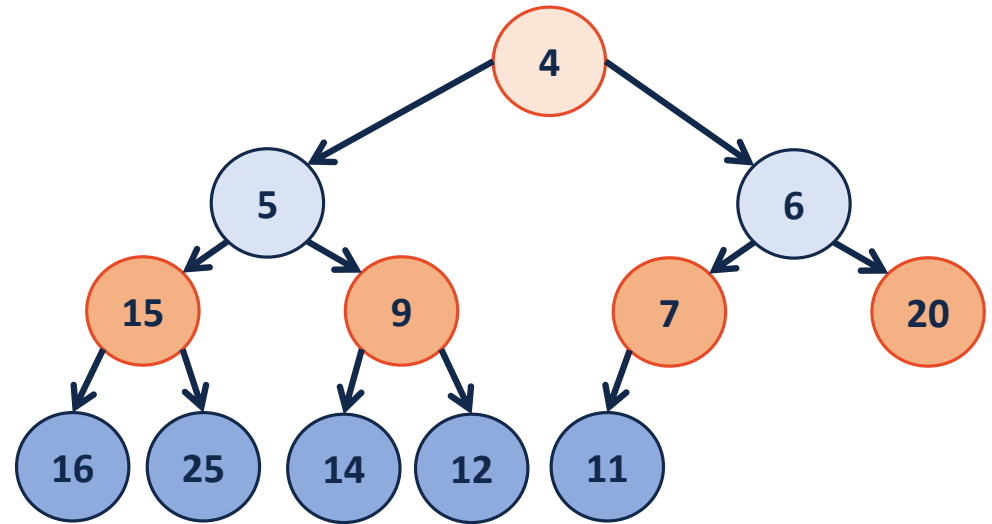


	4	5	6	15	9	7	20	16	25	14	12	11			
--	---	---	---	----	---	---	----	----	----	----	----	----	--	--	--

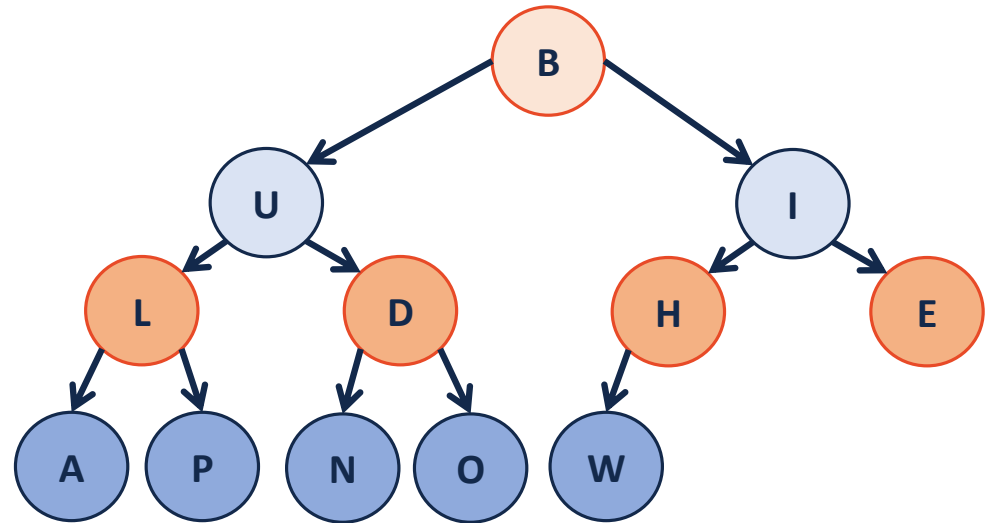
# (min)Heap

A complete binary tree  $T$  is a min-heap if:

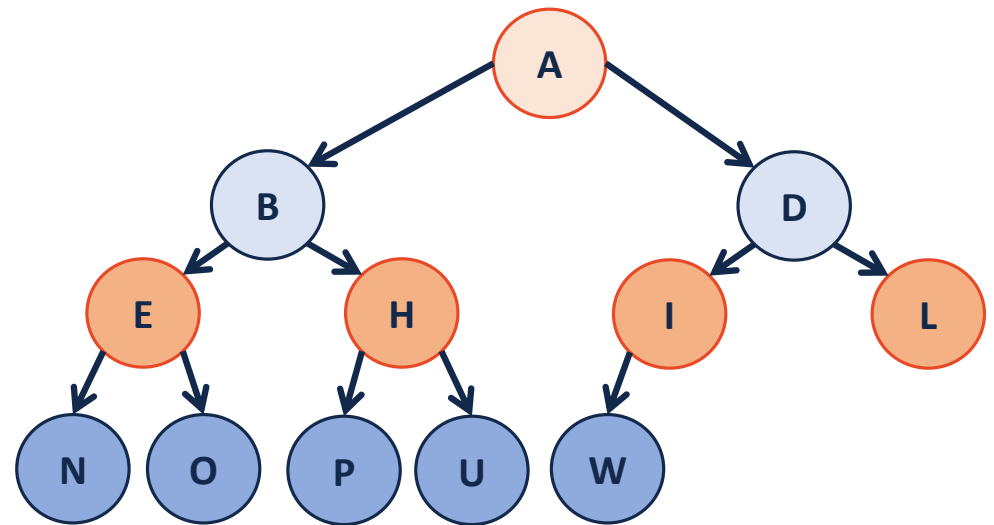
- $T = \{\}$  or
- $T = \{r, T_L, T_R\}$ , where  $r$  is less than the roots of  $\{T_L, T_R\}$  and  $\{T_L, T_R\}$  are min-heaps.



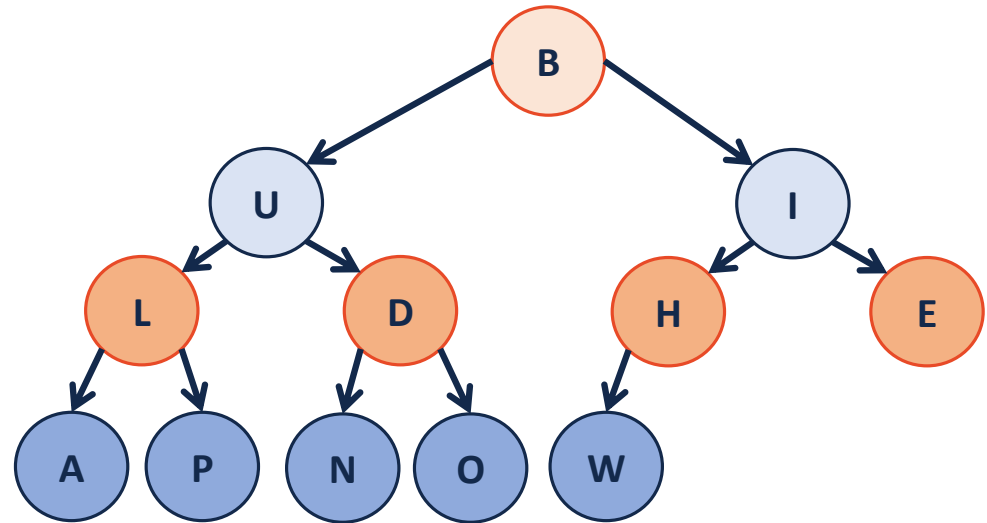
# buildHeap



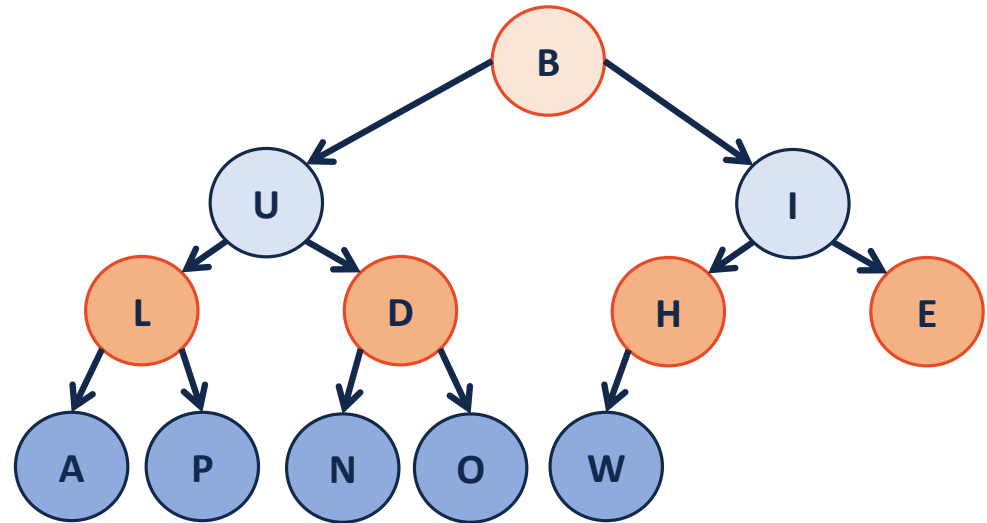
# buildHeap – sorted array



# buildHeap - heapifyUp



# buildHeap - heapifyDown



# buildHeap

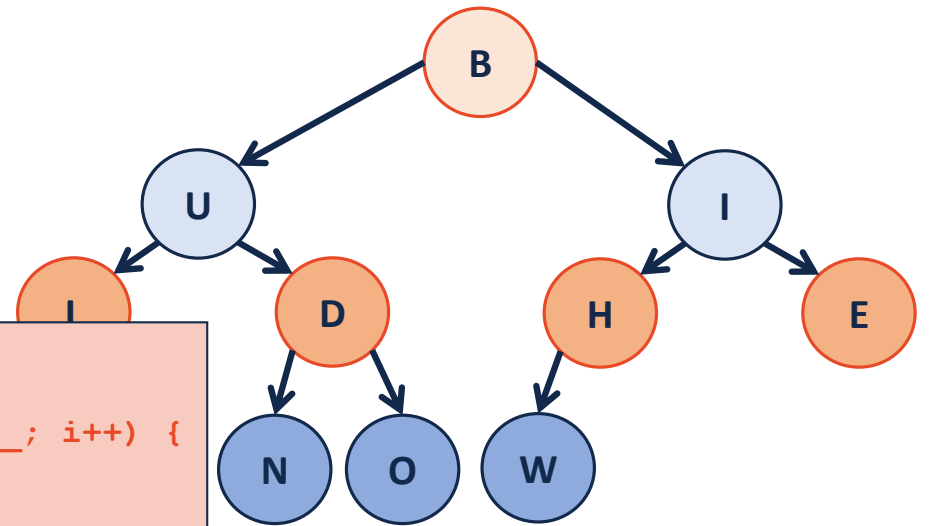
1. Sort the array – it's a heap!

2.

```
1 template <class T>
2 void Heap<T>::buildHeap() {
3     for (unsigned i = 2; i <= size_; i++) {
4         heapifyUp(i);
5     }
6 }
```

3.

```
1 template <class T>
2 void Heap<T>::buildHeap() {
3     for (unsigned i = parent(size); i > 0; i--) {
4         heapifyDown(i);
5     }
6 }
```



# buildHeap

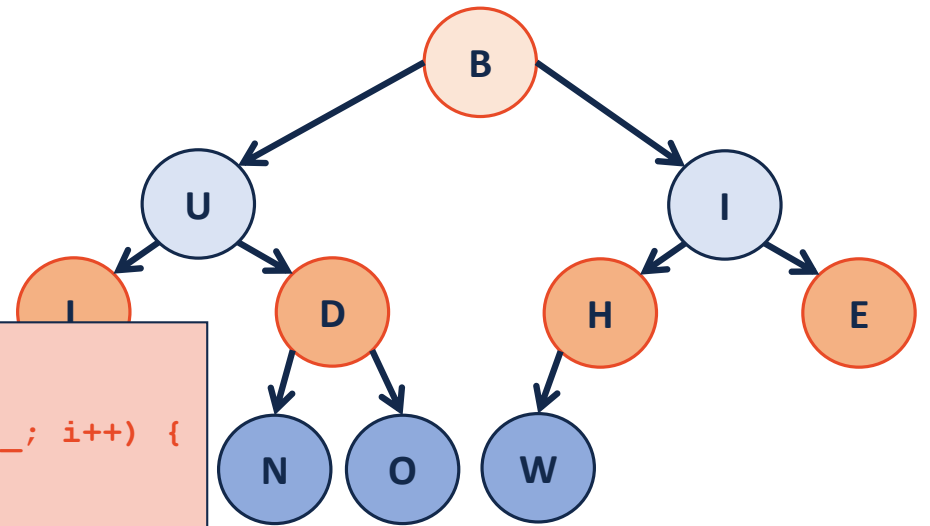
1. Sort the array – it's a heap!

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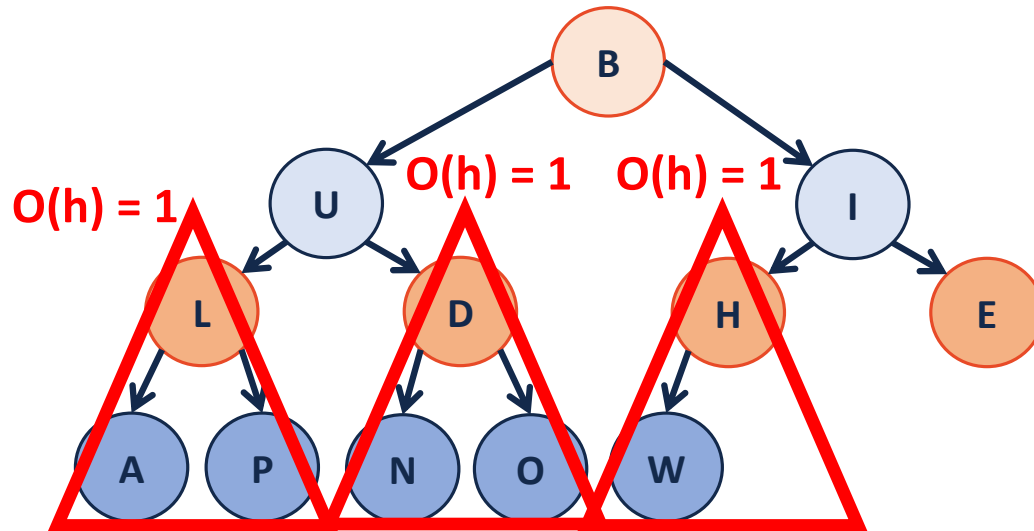
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2 void Heap<T>::buildHeap() {
3     for (unsigned i = 2; i <= size_; i++) {
4         heapifyUp(i);
5     }
6 }
```

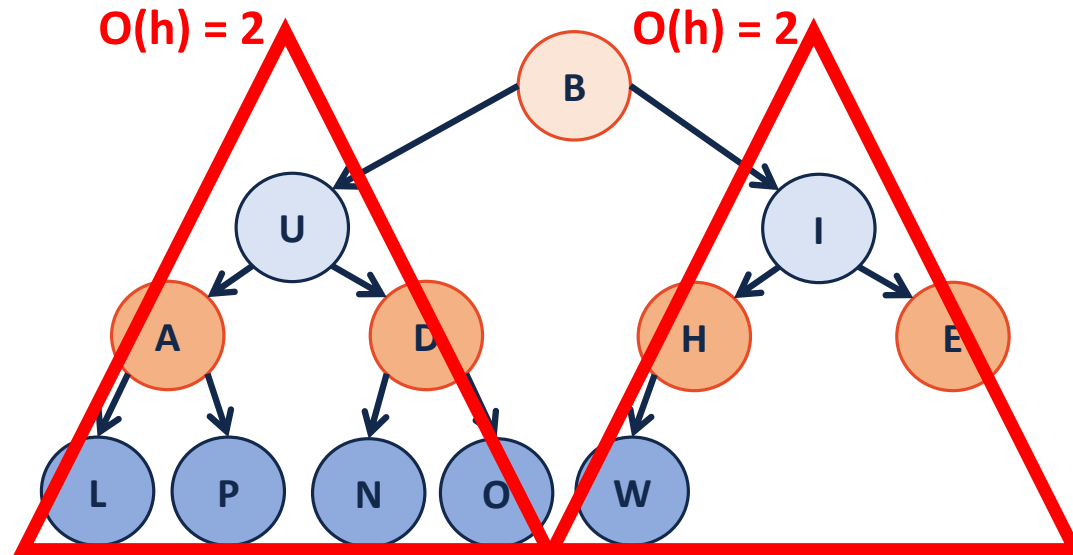
3.

```
1 template <class T>
2 void Heap<T>::buildHeap() {
3     for (unsigned i = parent(size); i > 0; i--) {
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6 }
```



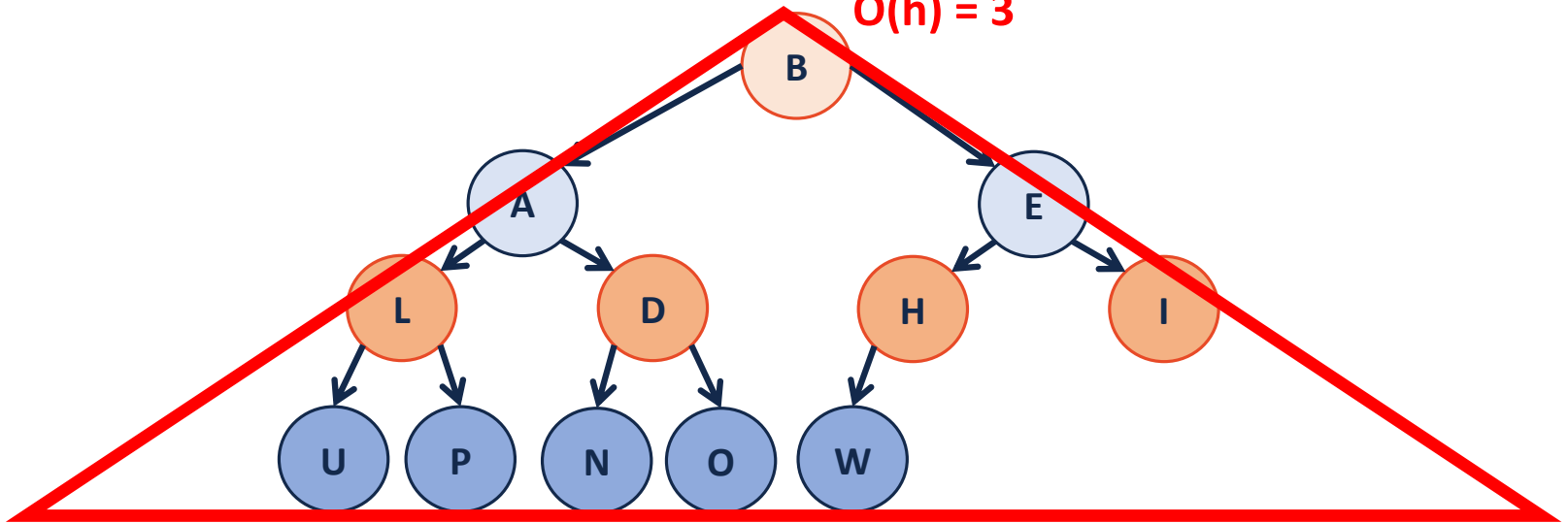








$O(h) = 3$





# Proving buildHeap Running Time

**Theorem:** The running time of buildHeap on array of size  $n$  is: \_\_\_\_\_.

**Strategy:**

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# Proving buildHeap Running Time

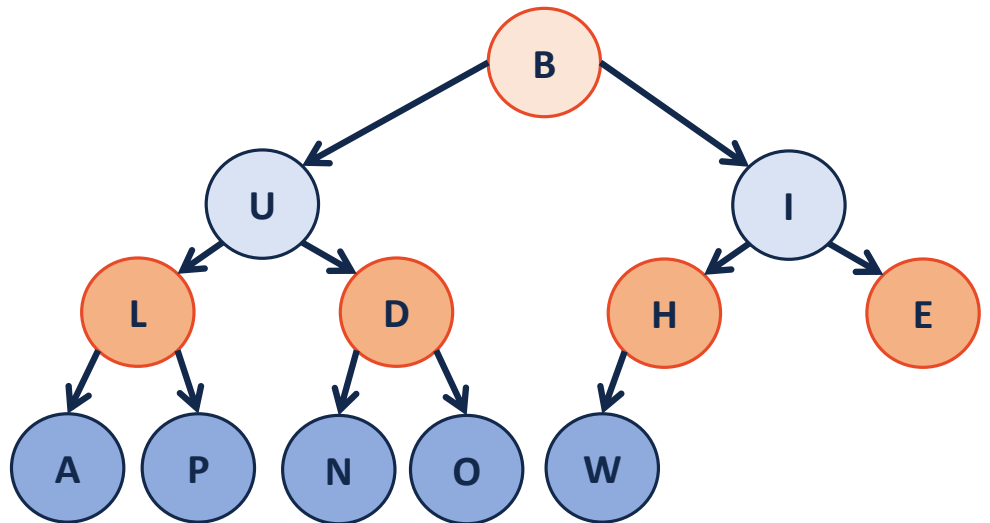
$S(h)$ : Sum of the heights of all nodes in a complete tree of height  $h$ .

$S(0) =$

$S(1) =$

$S(2) =$

$S(h) =$





# Proving buildHeap Running Time

**Proof the recurrence:**

Base Case:

IH:

General Case:



# Proving buildHeap Running Time

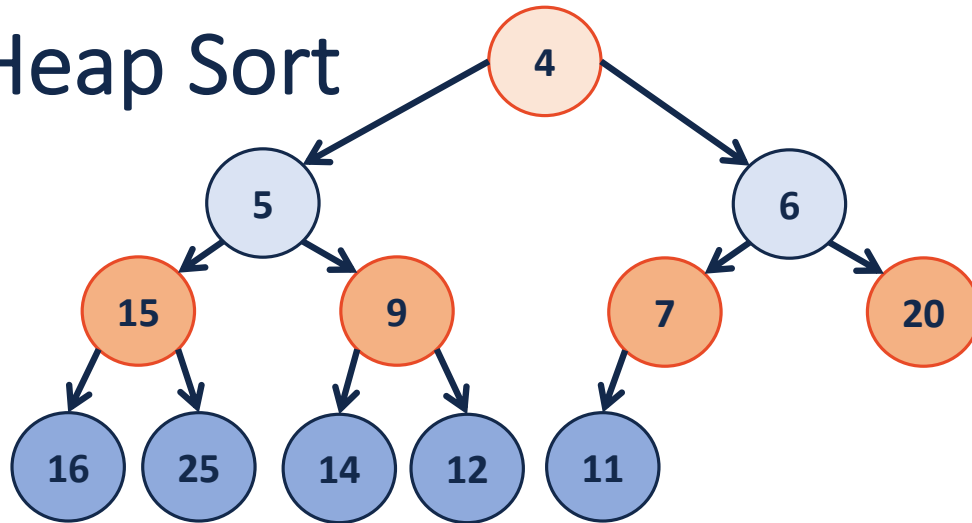
**From  $S(h)$  to RunningTime(n):**

$S(h)$ :

Since  $h \leq \lg(n)$ :

RunningTime(n)  $\leq$

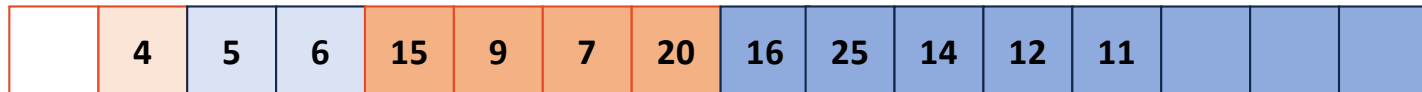
# Heap Sort



1.

2.

3.



Running Time?

Why do we care about another sort?