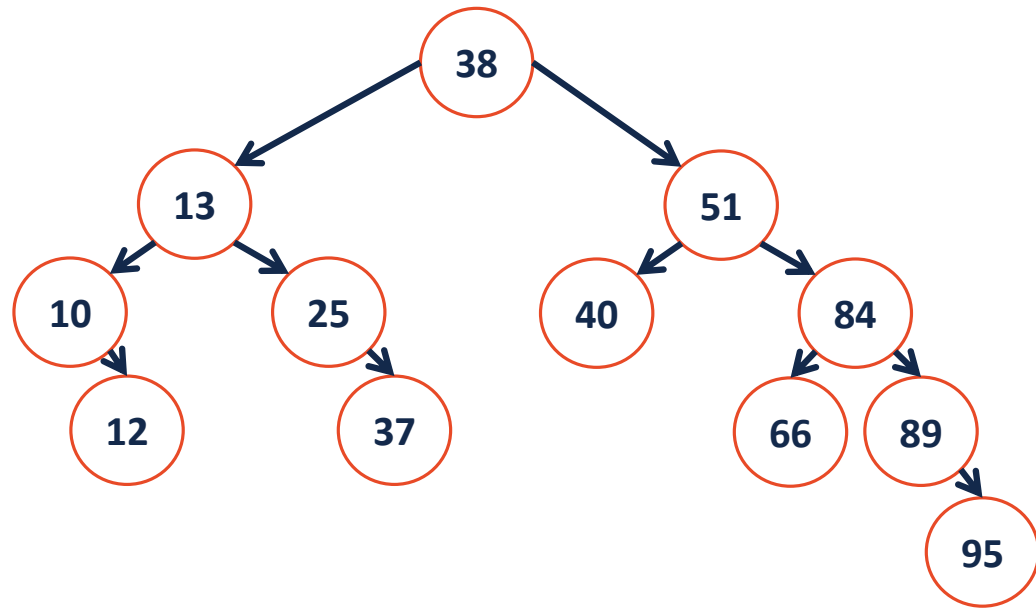


CS 225

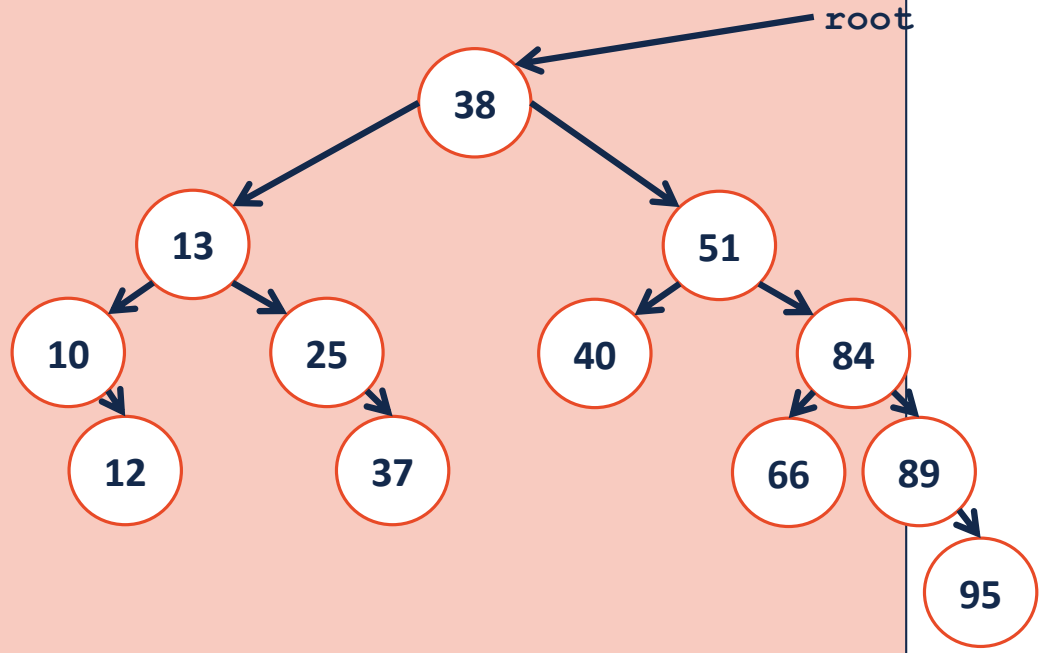
Data Structures

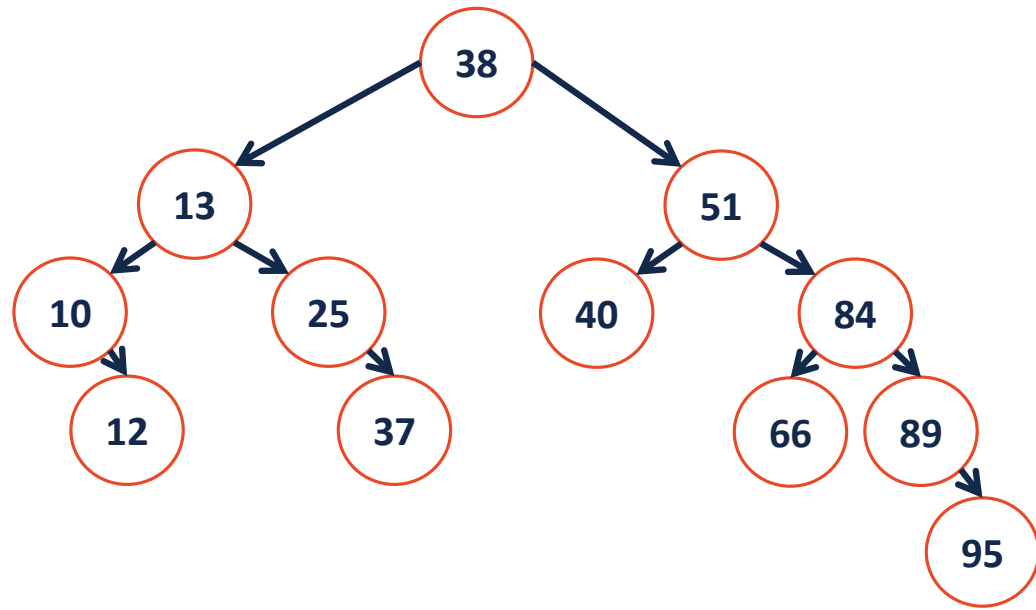
February 23 – BST

G Carl Evans

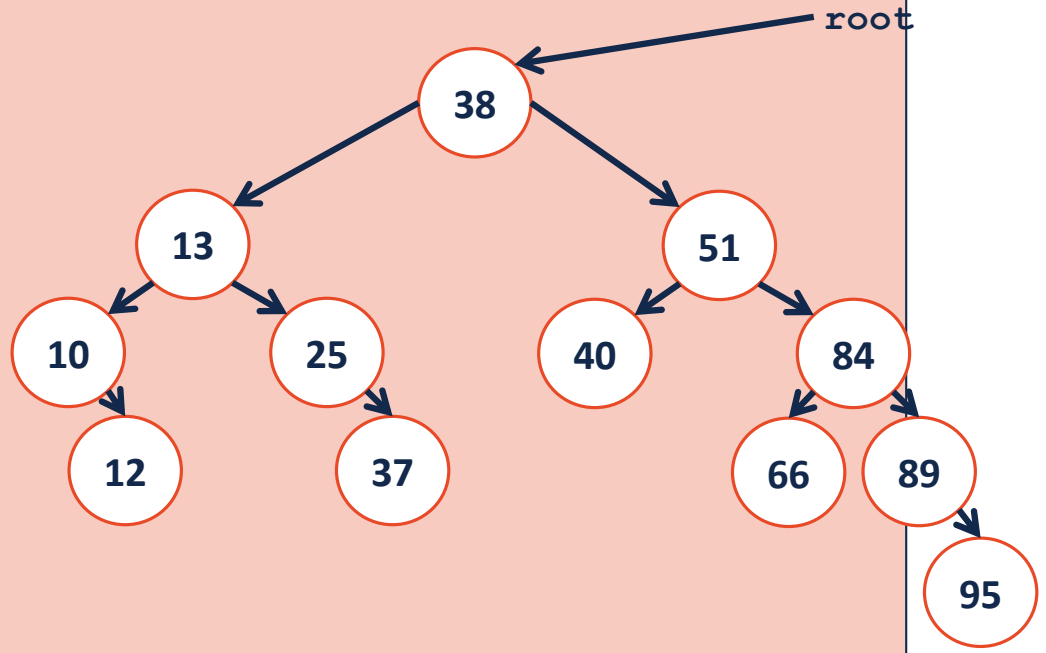


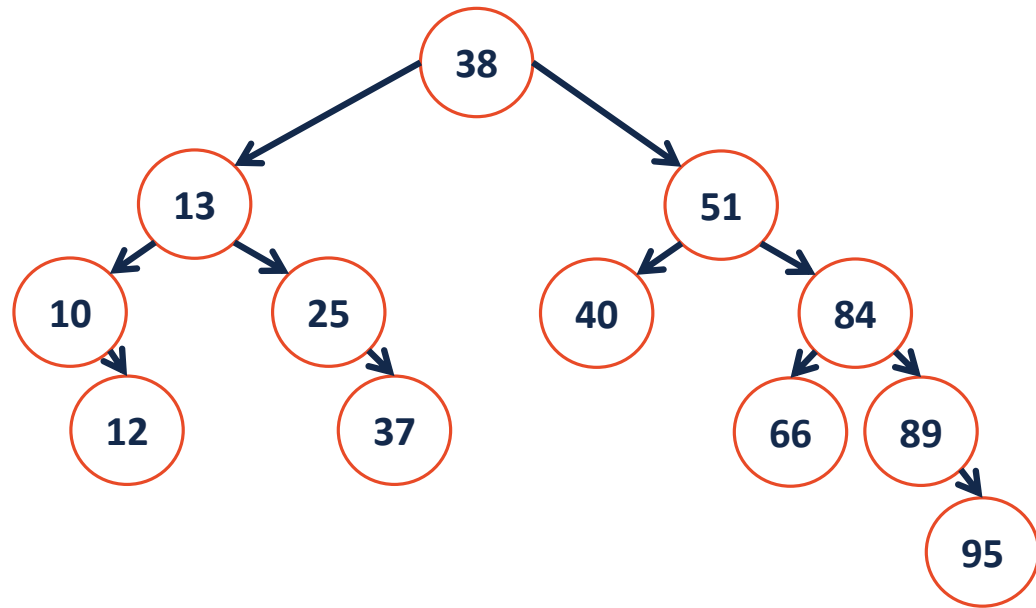
```
1  template<class K, class V>
2  _____ _insert(TreeNode *& root, const K & key) {
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26 }
```



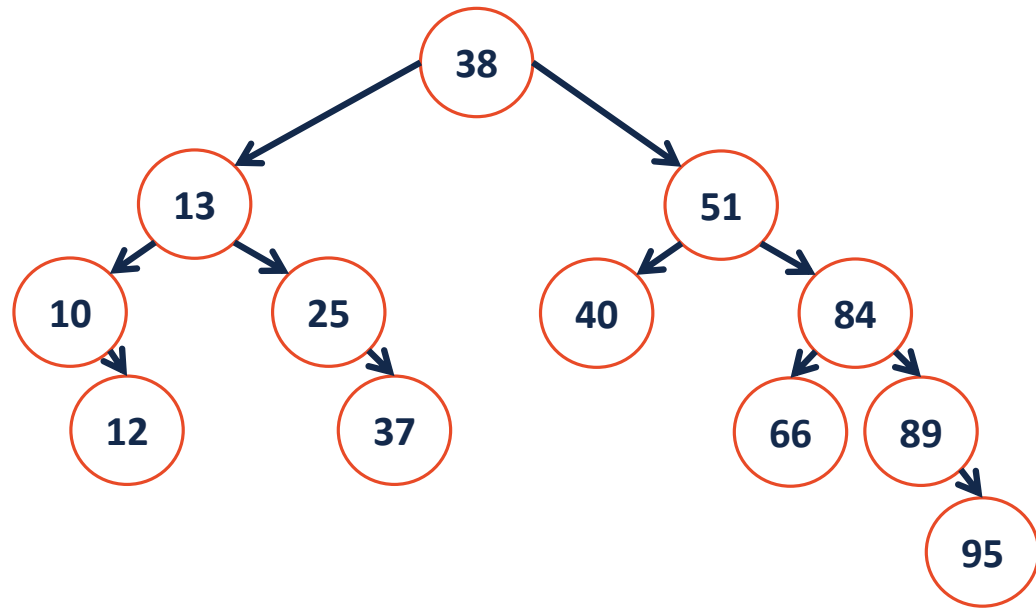


```
1  template<typename K, typename V>
2  _____ _remove(TreeNode *& root, const K & key) {
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26 }
```

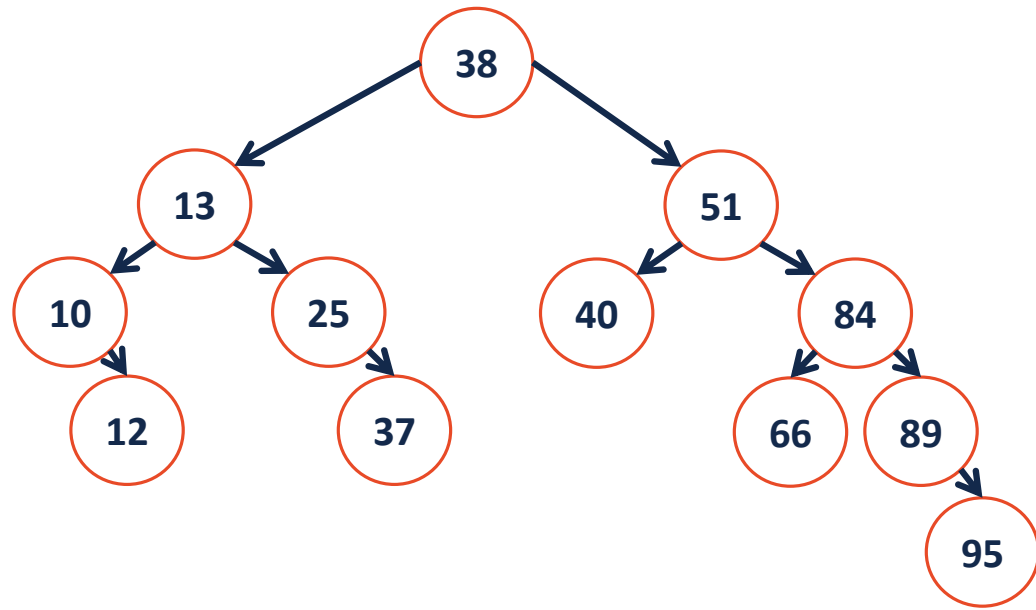




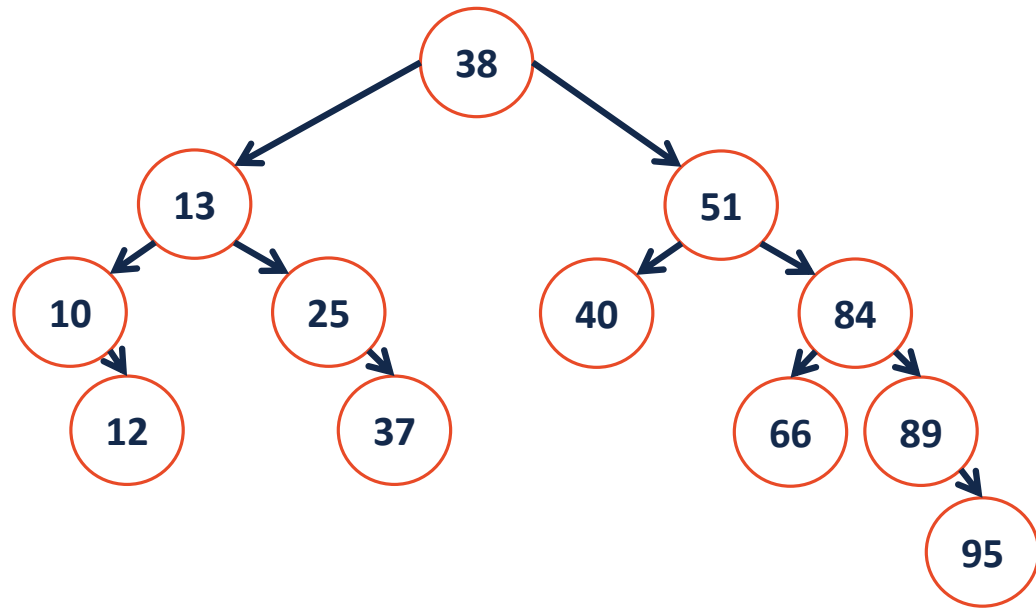
`remove (40) ;`



`remove (25) ;`



`remove(10);`



`remove (13) ;`

BST Analysis – Running Time

Operation	BST Worst Case
find	
insert	
delete	
traverse	



BST Analysis

Every operation that we have studied on a BST depends on the height of the tree: **$O(h)$** .

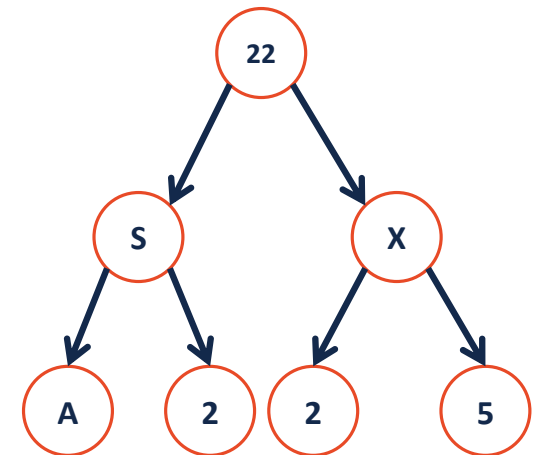
...what is this in terms of **n** , the amount of data?

We need a relationship between **h** and **n** :

$$f(h) \leq n \leq g(h)$$

BST Analysis

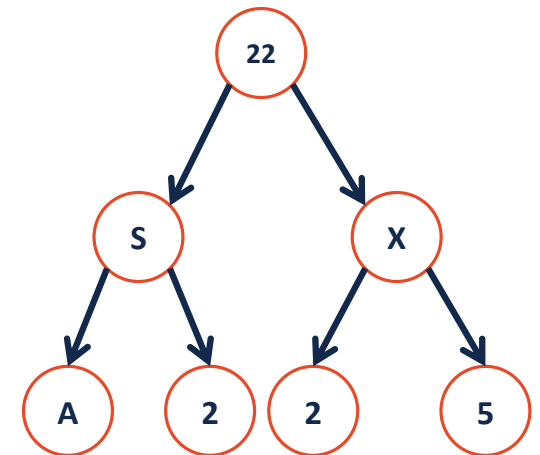
Q: What is the maximum number of nodes in a tree of height h ?



BST Analysis

Q: What is the minimum number of nodes in a tree of height h ?

What is the maximum height for a tree of n nodes?





BST Analysis

Therefore, for all BST:

Lower bound:

Upper bound:



BST Analysis

The height of a BST depends on the order in which the data is inserted into it.

ex: 1 3 2 4 5 7 6

vs.

4 2 3 6 7 1 5

Q: How many different ways are there to insert keys into a BST?

Q: What is the average height of all the arrangements?



BST Analysis

Q: How many different ways are there to insert keys into a BST?

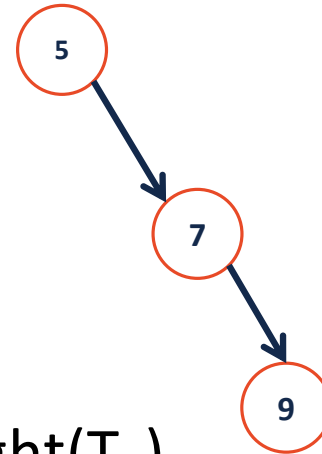
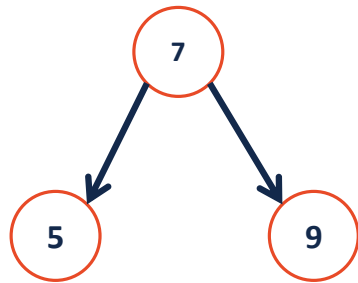
Q: What is the average height of all the arrangements?

BST Analysis – Running Time

Operation	BST Average case	BST Worst case	Sorted array	Sorted List
find				
insert				
delete				
traverse				

Height-Balanced Tree

What tree makes you happier?



Height balance: $b = \text{height}(T_L) - \text{height}(T_R)$

A tree is height balanced if: