CS 225

Data Structures

April 19 – Graph Traversals & MST

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Mid-Project Check-ins this week!

Discuss:

Current Progress (First deliverable done?)

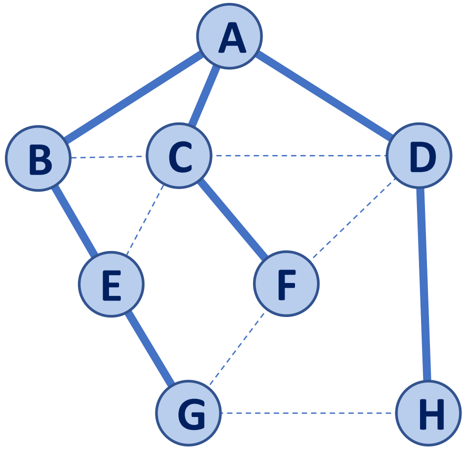
Future Progress (What do you have left to do?)

Group Cohesion (Any issues or concerns?)

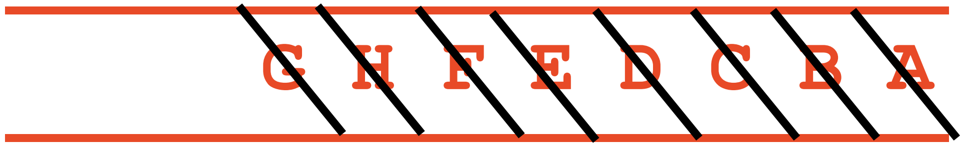
Learning Objectives

- Review BFS and discuss pseudo-code for DFS on graphs
- Analyze and contrast BFS/DFS algorithms
- Introduce Minimum Spanning Tree (MST) problem

Traversal: BFS



v	d	P	Adjacent Edges
A	0	-	B C D
B	1	A	A C E
C	1	A	A B D E F
D	1	A	A C F H
E	2	C	B C G
F	2	C	C D G
G	3	E	E F H
H	2	D	D G



BFS Observations

Obs. 1: BFS can be used to count components.

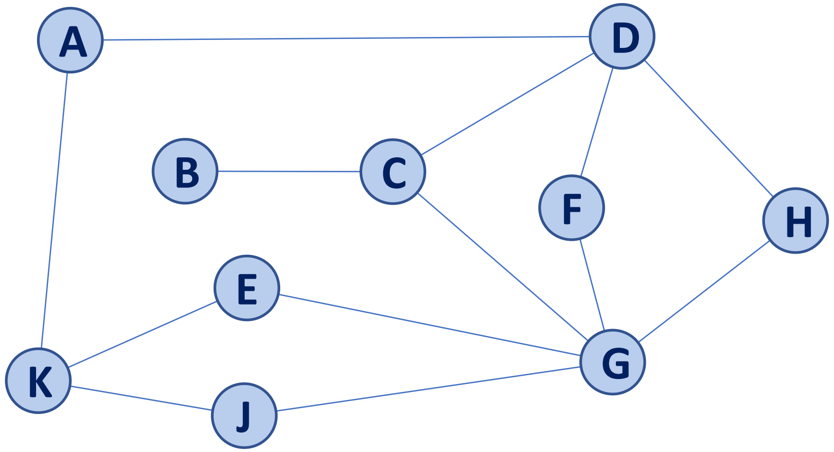
Obs. 2: BFS can be used to detect cycles.

Obs. 3: In BFS, **d** provides the shortest distance to every vertex.

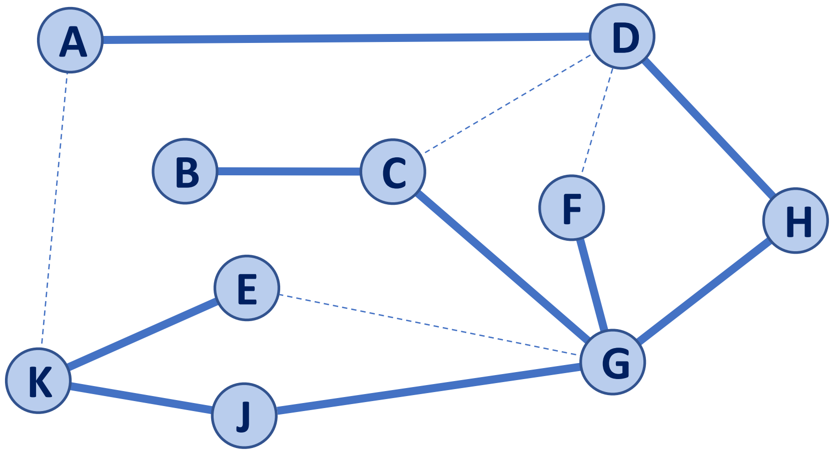
Obs. 4: In BFS, the endpoints of a cross edge never differ in distance, **d**, by more than 1:

$$|d(u) - d(v)| \leq 1$$

Traversal: DFS



Traversal: DFS



Discovery Edge

Back Edge

```
BFS(G):
  Input: Graph, G
  Output: A labeling of the edges on
      G as discovery and cross edges

  foreach (Vertex v : G.vertices()):
    setLabel(v, UNEXPLORED)
  foreach (Edge e : G.edges()):
    setLabel(e, UNEXPLORED)
  foreach (Vertex v : G.vertices()):
    if getLabel(v) == UNEXPLORED:
       BFS(G, v)
```

```
BFS(G, v):
  Queue q
  setLabel(v, VISITED)
  q.enqueue(v)

  while !q.empty():
    v = q.dequeue()
    foreach (Vertex w : G.adjacent(v)):
      if getLabel(w) == UNEXPLORED:
         setLabel(v, w, DISCOVERY)
         setLabel(w, VISITED)
         q.enqueue(w)
      elseif getLabel(v, w) ==
UNEXPLORED:
         setLabel(v, w, CROSS)
```

```
DFS(G):
  Input: Graph, G
  Output: A labeling of the edges on
      G as discovery and back edges

  foreach (Vertex v : G.vertices()):
    setLabel(v, UNEXPLORED)
  foreach (Edge e : G.edges()):
    setLabel(e, UNEXPLORED)
  foreach (Vertex v : G.vertices()):
    if getLabel(v) == UNEXPLORED:
       DFS(G, v)
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    foreach (Vertex w : G.adjacent(v)):
      if getLabel(w) == UNEXPLORED:
         setLabel(v, w, DISCOVERY)
         setLabel(w, VISITED)
         DFS(G, w)
      elseif getLabel(v, w) ==
UNEXPLORED:
         setLabel(v, w, BACK)
```

DFS Observations

Obs. 1: DFS can be used to count components.

Obs. 2: DFS can be used to detect cycles.

Obs. 3: In DFS, **d** provides no clear meaning

DFS vs BFS

DFS:

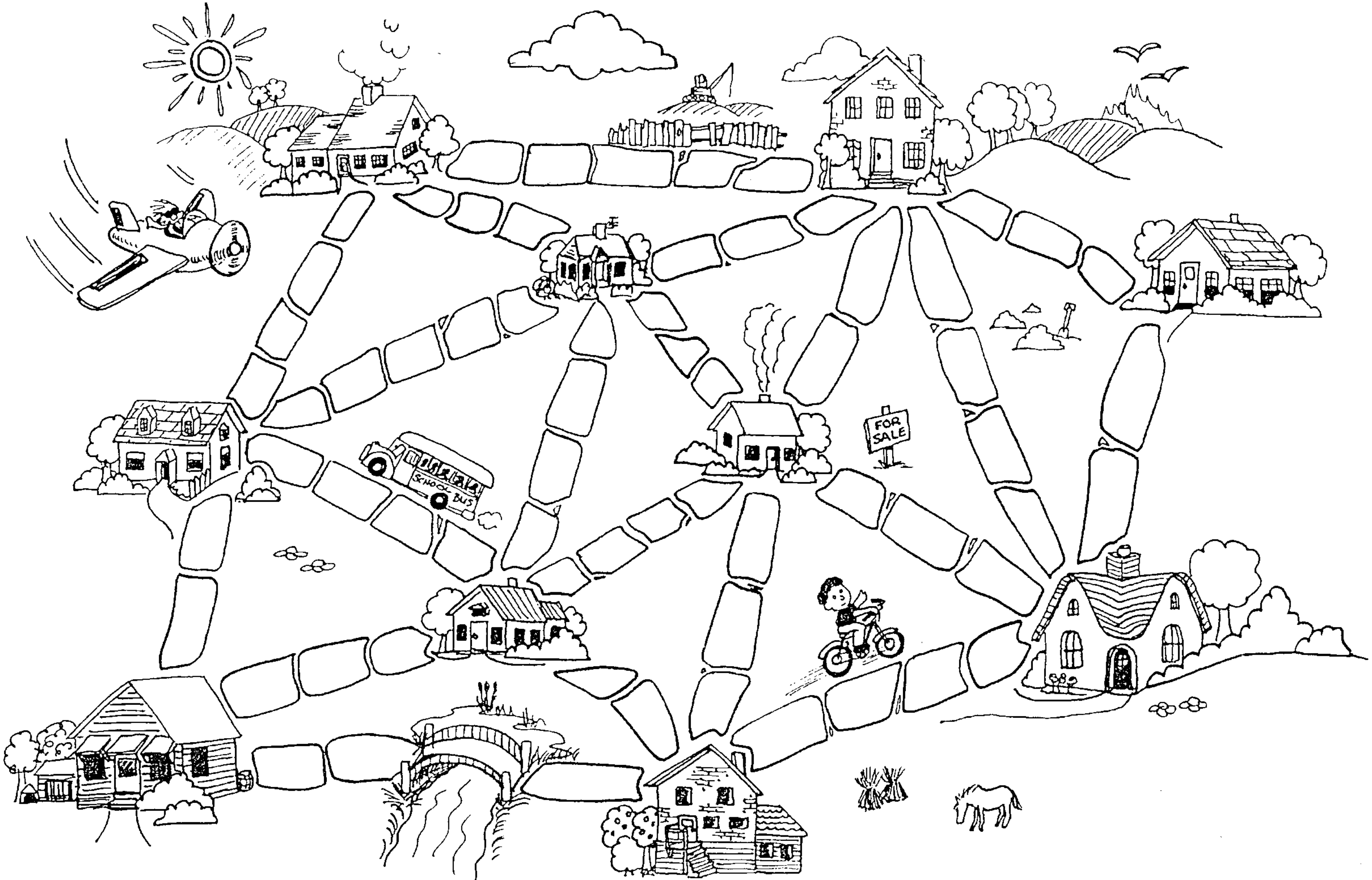
Pros:

Cons:

BFS:

Pros:

Cons: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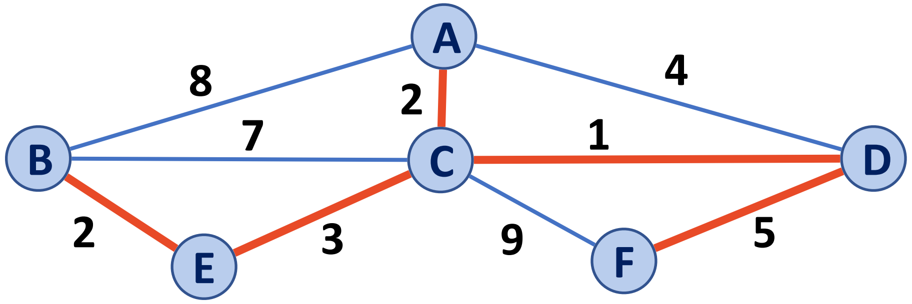
Minimum Spanning Tree Algorithms

Input: Connected, undirected graph **G** with edge weights (unconstrained, but must be additive)

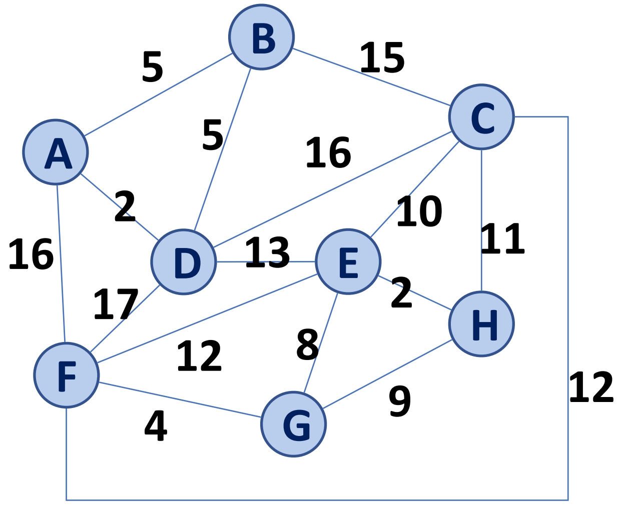
Output: A graph G’ with the following properties:

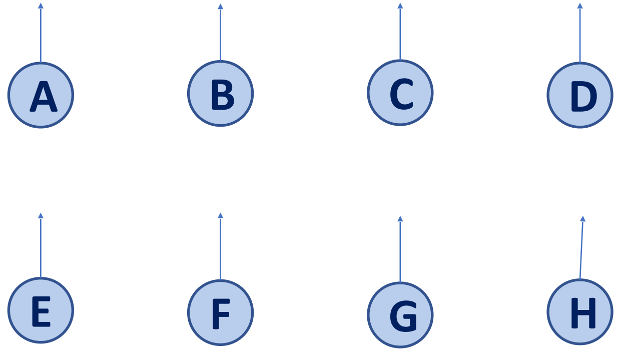
- G’ is a spanning graph of G
- G’ is a tree (connected, acyclic)
- G’ has a minimal total weight among all spanning trees



Kruskal's Algorithm

(A, D)
(E, H)
(F, G)
(A, B)
(B, D)
(G, E)
(G, H)
(E, C)
(C, H)
(E, F)
(F, C)
(D, E)
(B, C)
(C, D)
(A, F)
(D, F)






```
KruskalMST(G):
  DisjointSets forest
  foreach (Vertex v : G):
    forest.makeSet(v)

  PriorityQueue Q    // min edge weight
  foreach (Edge e : G):
    Q.insert(e)

  Graph T = (V, {})

  while |T.edges()| < n-1:
    Vertex (u, v) = Q.removeMin()
    if forest.find(u) != forest.find(v):
       T.addEdge(u, v)
       forest.union( forest.find(u),
                     forest.find(v) )

  return T
```

Kruskal's Algorithm

Priority Queue:	Heap	Sorted Array
Building (Line 6-8)		
Each removeMin (Line 13)		

```
1  KruskalMST(G):
2    DisjointSets forest
3    foreach (Vertex v : G):
4      forest.makeSet(v)
5
6    PriorityQueue Q     // min edge weight
7    foreach (Edge e : G):
8      Q.insert(e)
9
10   Graph T = (V, {})
11
12   while |T.edges()| < n-1:
13     Vertex (u, v) = Q.removeMin()
14     if forest.find(u) != forest.find(v):
15        T.addEdge(u, v)
16        forest.union( forest.find(u),
17                      forest.find(v) )
18
19   return T
```

Kruskal's Algorithm

Priority Queue:	Total Running Time
Heap	
Sorted Array	

```
KruskalMST(G):
  DisjointSets forest
  foreach (Vertex v : G):
    forest.makeSet(v)

  PriorityQueue Q    // min edge weight
  foreach (Edge e : G):
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  Graph T = (V, {})

  while |T.edges()| < n-1:
    Vertex (u, v) = Q.removeMin()
    if forest.find(u) != forest.find(v):
       T.addEdge(u, v)
       forest.union( forest.find(u),
                     forest.find(v) )

  return T
```
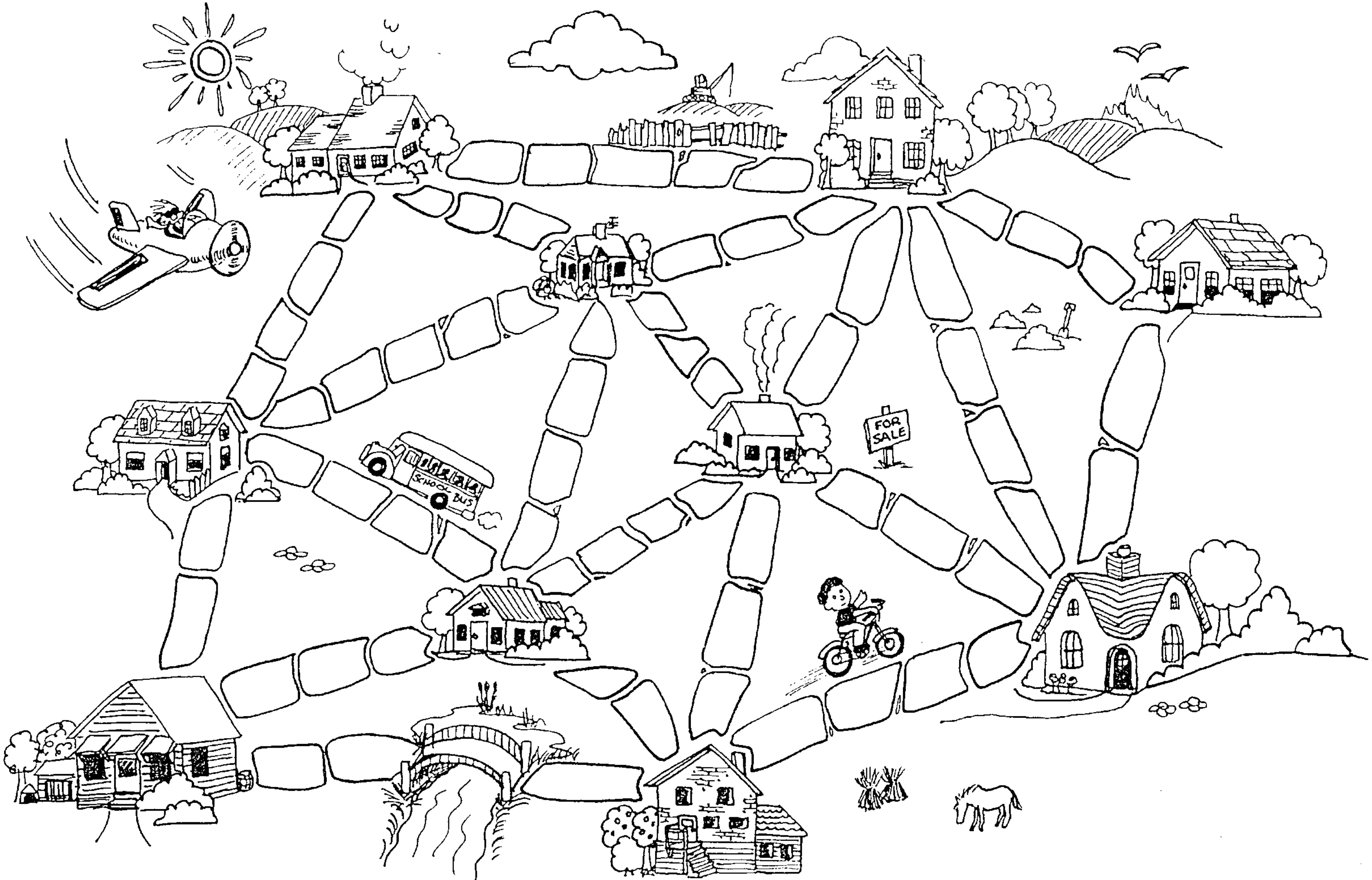

Kruskal’s Algorithm

Which Priority Queue Implementation is better for running Kruskal’s Algorithm?

- Heap:

- Sorted Array: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